Binding energy of light nucleus from lattice QCD

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Outline

- Introduction
- Important condition of binding energy calculation
- Preliminary result of $N_f = 0 \ m_\pi = 0.8 \ \text{GeV}$
- $N_f = 2 + 1$ QCD results
- Summary and future work
Introduction

Binding force \( \{ \)
- protons and neutrons \( \rightarrow \) nuclei
- quarks and gluons \( \rightarrow \) protons and neutrons

both from fundamental strong interaction of quark and gluon
well known, but hard to prove

quark and gluon \( \rightarrow \) proton and neutron \( \rightarrow \) nucleus
Introduction

Binding force \{ 
\begin{align*} 
\text{protons and neutrons} & \rightarrow \text{nuclei} \\
\text{quarks and gluons} & \rightarrow \text{protons and neutrons} 
\end{align*} 
\}

both from fundamental strong interaction of quark and gluon well known, but hard to prove

Spectrum of proton and neutron (nucleons) 
success of non-perturbative lattice QCD calculation 
degrees of freedom of quarks and gluons 
\begin{align*} 
\text{quark and gluon} & \rightarrow \text{proton and neutron} \rightarrow \text{nucleus} 
\end{align*}
Hadron spectrum in $N_f = 2 + 1$ QCD

Lattice 2015, Ukita for PACS Collaboration PoS(LATTICE2015)075

$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm at $a^{-1} = 2.33$ GeV (SPIRE Field 5)

using reweighting $m_{ud}, m_s$ $+$ extrapolation $\rightarrow$ physical $m_\pi$ and $m_K$

$m_\pi \sim 0.145$ GeV

physical point

\begin{align*}
\pi & \quad K & \quad \eta_{ss} & \quad \rho & \quad K^* & \quad \phi & \quad N & \quad \Lambda & \quad \Sigma & \quad \Xi & \quad \Delta & \quad \Sigma^* & \quad \Xi^* & \quad \Omega \\
\end{align*}
Hadron spectrum in $N_f = 2 + 1$ QCD

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$m_\pi \sim 0.145$ GeV on $L \sim 8$ fm at $a^{-1} = 2.33$ GeV (SPIRE Field 5)
using reweighting $m_{ud}, m_s +$ extrapolation $\rightarrow$ physical $m_\pi$ and $m_K$

$$\bar{l}_3 = 2.87(62), \quad \bar{l}_4 = 4.38(33)$$
FLAG2013: $\bar{l}_3 = 3.05(99), \quad \bar{l}_4 = 4.02(28)$ at $\mu = m_\pi^{\text{phys}}$

$$m_{ud}^{\overline{\text{MS}}} = 3.142(26)(35)(28) \text{MeV}, \quad m_s^{\overline{\text{MS}}} = 88.59(61)(98)(79) \text{MeV}$$
FLAG2013: $m_{ud}^{\overline{\text{MS}}} = 3.42(6) \text{MeV}, \quad m_s^{\overline{\text{MS}}} = 93.8(1.5)(1.9) \text{MeV}$

$$f_\pi = 131.79(80)(90)(1.25) \text{MeV}, \quad f_K = 155.55(68)(1.06)(1.48) \text{MeV}$$
FLAG2013: $f_\pi = 130.2(1.4) \text{MeV}, \quad f_K = 156.3(0.9) \text{MeV}$

reasonably consistent

investigation of $a \rightarrow 0$ limit necessary
Introduction

Binding force \{ 
protons and neutrons \rightarrow nuclei
quarks and gluons \rightarrow protons and neutrons
\}
both from fundamental strong interaction of quark and gluon
well known, but hard to prove

Spectrum of proton and neutron (nucleons)
success of non-perturbative lattice QCD calculation
degrees of freedom of quarks and gluons
quark and gluon \rightarrow proton and neutron \rightarrow nucleus
Introduction

Binding force \[ \{ \text{protons and neutrons} \rightarrow \text{nuclei} \]
\[ \text{quarks and gluons} \rightarrow \text{protons and neutrons} \]
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Spectrum of proton and neutron (nucleons)
success of non-perturbative lattice QCD calculation
degrees of freedom of quarks and gluons

\[ \text{quark and gluon} \rightarrow \text{proton and neutron} \rightarrow \text{nucleus} \]

goal: quantitatively understand property of nucleus from QCD

So far not many studies for multi-baryon bound states
\[ \rightarrow \text{Can we reproduce binding energy of light nuclei?} \]
Ultimate goal of lattice QCD

http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/ ; figure from Irie-san

quantitatively understand property of nuclei from QCD
Multi-baryon bound state from lattice QCD

1. $^4$He and $^3$He

  '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)

  '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV $^4$He

  '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

  '12 TY et al. $N_f = 2 + 1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)

  '15 TY et al. $N_f = 2 + 1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)

2. H dibaryon in $\Lambda\Lambda$ channel (S=−2, I=0)

  '88 Iwasaki et al. $N_f = 0$ $m_\pi = 0.7$–1.9 GeV

  '11, '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV, $N_f = 3$ $m_\pi = 0.81$ GeV

  '11, '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.02 GeV

  '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV

  '14, '15, '16 Mainz $N_f = 2$ $m_\pi = 0.45, 1.0$ GeV

3. NN

  '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)

  '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV (Possibility)

  '12 NPLQCD, '15 CalLat $N_f = 3$ $m_\pi = 0.81$ GeV

  '12 TY et al. $N_f = 2 + 1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)

  '15 TY et al. $N_f = 2 + 1$ $m_\pi = 0.30$ GeV PRD92:014501(2015)

  '15 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.45$ GeV

  Other states: $\Xi\Xi$, '12 NPLQCD; spin-2 $N\Omega$, $^{16}$O and $^{40}$Ca, '14 HALQCD, ...
Identification of bound state from volume dependence of $\Delta E$

$\Delta E_{4\text{He}} = 27.7(7.8)(5.5) \text{ MeV}$

1. Observe bound state in both channels
2. Same order of $\Delta E$ to experiment

Several systematic errors included, e.g., $N_f = 0$, $m_\pi = 0.8 \text{ GeV}$

Go to realistic calculation: $N_f = 2 + 1$, $m_\pi \sim 0.135 \text{ GeV}$
Multi-baryon bound state from lattice QCD

1. \(^4\)He and \(^3\)He

\[ \begin{align*}
\text{'10 PACS-CS } & N_f = 0 \ m_\pi = 0.8 \ \text{GeV PRD81:111504(R)(2010)} \\
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\text{'12 NPLQCD } & N_f = 3 \ m_\pi = 0.81 \ \text{GeV} \\
\text{'12 TY et al. } & N_f = 2 + 1 \ m_\pi = 0.51 \ \text{GeV PRD86:074514(2012)} \\
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\text{'14, '15, '16 Mainz } & N_f = 2 \ m_\pi = 0.45, 1.0 \ \text{GeV}
\end{align*} \]

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\end{align*} \]

Other states: ΞΞ, '12 NPLQCD; spin-2 NΩ, \(^{16}\)O and \(^{40}\)Ca, '14 HALQCD, …
Calculation method of multi-nucleon bound state

Traditional method: example $^4$He channel

$$\langle 0|O_{^4\text{He}}(t)\overline{O}_{^4\text{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\text{He}}|n\rangle \langle n|\overline{O}_{^4\text{He}}|0\rangle e^{-E_n t} \xrightarrow{t \gg 1} A_0 e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp \left( N_N \left[ m_N - \frac{3}{2} m_\pi \right] t \right) \rightarrow \text{large} \left\{ \begin{array}{l} \text{for } m_\pi, m_N \rightarrow \text{small} \\ \text{for } t \rightarrow \text{large} \\ \text{for } N_N \rightarrow \text{large} \end{array} \right.$

2. Calculation cost

Wick contraction for $^4$He $= p^2 n^2 = (udu)^2 (dud)^2$: 518400
proton $= p = (udu)$: 2

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$
Calculation method of multi-nucleon bound state

Traditional method: example $^4$He channel

$$
\langle 0|O_{^4\text{He}}(t)\overline{O}_{^4\text{He}}(0)|0 \rangle = \sum_n \langle 0|O_{^4\text{He}}|n \rangle \langle n|\overline{O}_{^4\text{He}}|0 \rangle e^{-E_n t} \quad \text{for } t \gg 1 \quad A_0 e^{-E_0 t}
$$

Difficulties for multi-nucleon calculation

1. Statistical error
   Statistical error $\propto \exp\left( N_N \left[ m_N - \frac{3}{2} m_\pi \right] t \right)$
   $\rightarrow$ heavy quark $m_\pi = 0.8-0.3$ GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)
   Wick contraction for $^4$He $= p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107
   $\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$
   + block of 3 quark props(parallel) and contraction(workstation)
   '12 Doi and Endres; Detmold and Orginos; '13 Günther et al.; '15 Nemura

3. Identification of bound state on finite volume
   attractive scattering state $\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0$
   '86,'91 Lüscher, '07 Beane et al.
   $\rightarrow$ Volume dependence of $\Delta E_L \rightarrow \Delta E_\infty \neq 0 \rightarrow$ bound state
   Spectral weight: '04 Mathur et al., Anti-PBC '05 Ishii et al.
Calculation method of multi-nucleon bound state

Wall and Exp sources give different $\Delta E$  '16 HALQCD

$\Delta E_{\Xi\Xi}(t) = \log \sum_n b_n \exp(-\Delta E_n t) / \sum_n b_n \exp(-\Delta E_n (t+1))$

-15 -10 -5 0 5 10

$t [a]$ 0 5 10 15 20

Wall src. $\Xi\Xi(^1S_0)$
smeared src. $\Xi\Xi(^1S_0)$

$b_{wall}^{\Xi\Xi}_{n=0\ldots2}$ at $t = 14$
$b_{smear}^{\Xi\Xi}_{n=0\ldots2}$ at $t = 14$

figures from slide by Iritani at INT workshop “Nuclear Physics from Lattice QCD”
Calculation method of multi-nucleon bound state

\[ C^f_N(t) = \langle 0 | O_N(x, t) \overline{O}^f_N(0) | 0 \rangle, \quad \overline{O}^f_N(x, t) = \bar{q}^f(x, t) \bar{q}^f(x, t) \bar{q}^f(x, t) \]

Smeared quark \( \bar{q}^f(x, t) = \sum_y f(|x - y|) \bar{q}(y, t) \)

Local \( f(x) = \delta(x) \)

Exp \( f(x) = e^{-Ax} \)

Wall \( f(x) = 1 \)

\[ C^f_N(t) = A_0^f e^{-m_N t} + A_1^f e^{-E_1 t} + \cdots \]

\( A_i^f \) depends on \( f(x) \), but \( m_N, E_i \) are independent.

Prefer large \( A_0^f \) to obtain \( m_N \) in small \( t \) region \( \leftarrow \) error large in large \( t \)

Well known: Wall needs longest \( t \) to have plateau of nucleon
Calculation method of multi-nucleon bound state

Wall and Exp sources give different $\Delta E$  '16 HALQCD

$\Delta E_{\text{eff}}(t) = \log \sum_n b_n \exp (-\Delta E_n t) / \sum_n b_n \exp (-\Delta E_n (t+1))$

"direct measurement" — well reproduced by low-lying 3 eigenmodes

"fake plateau" of smeared src. around $t = 14 - 18$

figures from slide by Iritani at INT workshop “Nuclear Physics from Lattice QCD”

Solid curves: expected effective $\Delta E$ by HALQCD potential
$\Delta E$ of Wall source $\approx \Delta E$ from HALQCD potential

Our understanding
Wall will agree Exp when important condition is satisfied
($\Delta E$ is determined from each plateau region)
Important condition of binding energy calculation

Traditional method in lattice QCD (NN channel)

nucleon correlation function

\[ C_N(t) = \langle 0|N(t)\overline{N}(0)|0\rangle = \sum_n \langle 0|N|n\rangle\langle n|\overline{N}|0\rangle e^{-E^N_n t} \quad \text{as} \quad t \geq t_N \gg 1 \rightarrow A_0^N e^{-m_N t} \]

NN correlation function

\[ C_{NN}(t) = \langle 0|O_{NN}(t)\overline{O}_{NN}(0)|0\rangle = \sum_n \langle 0|O_{NN}|n\rangle\langle n|\overline{O}_{NN}|0\rangle e^{-E_{nt}} \quad \text{as} \quad t \geq t_{NN} \gg 1 \rightarrow A_0 e^{-E_{NN} t} \]

Ratio of correlation functions

\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \quad \text{as} \quad t \geq t_R \gg 1 \rightarrow A'_0 e^{-\Delta E t}, \quad \Delta E = E_{NN} - 2m_N \]

Important condition: \( t_R \geq t_N, t_{NN} \)

i.e. \( C_N(t) \) and \( C_{NN}(t) \) are written by each ground state in \( t \geq t_R \)
Simulation parameter

High precision calculation in $N_f = 0$ $m_\pi = 0.8$ GeV

Iwasaki gauge ($\beta = 2.416$, $a^{-1} = 1.541$ GeV) + tadpole improved Wilson fermion actions
same action as ’02 CP-PACS, PRD81:111504(R)(2010); PRD84:054506(2011)

Comparison exponential and wall sources in $NN^3S_1$ channel
point sink (each $N$ with $p = 0$)

<table>
<thead>
<tr>
<th>$L$</th>
<th>$T$</th>
<th>source</th>
<th>$N_{\text{meas}}$</th>
</tr>
</thead>
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<tr>
<td>16</td>
<td>64</td>
<td>Exp</td>
<td>12,544,000</td>
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<tr>
<td></td>
<td></td>
<td>Wall</td>
<td>8,307,200</td>
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<tr>
<td>20</td>
<td>64</td>
<td>Exp</td>
<td>5,504,000</td>
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<tr>
<td></td>
<td></td>
<td>Wall</td>
<td>8,960,000</td>
</tr>
<tr>
<td>32</td>
<td>64</td>
<td>Exp</td>
<td>3,200,000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Wall</td>
<td>4,473,600</td>
</tr>
</tbody>
</table>

All $N_f = 0$ results are preliminary.

Computational resources (HPCI System Research Project: hp160124)
COMA and HA-PACS(U. of Tsukuba), FX10 and Reedbush (U. of Tokyo), Tatara (Kyushu U.),
FX100 and CX400(Nagoya U.), OFP(JCAHPC)
$$R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \text{ in } L = 20$$

Effective mass: $$m^{\text{eff}} = \log\left(\frac{C(t)}{C(t+1)}\right) \xrightarrow{t \gg 1} m$$

Effective $2m_N$ and $E_{NN}$

---

**Graphs:**
- **exp source**
  - $2m_N^{\text{eff}}$
  - $E_{NN}^{\text{eff}}$
- **wall source**
  - $2m_N^{\text{eff}}$
  - $E_{NN}^{\text{eff}}$
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \] in \( L = 20 \)

Effective mass: \[ m^{\text{eff}} = \log\left(\frac{C(t)}{C(t+1)}\right) \xrightarrow{t \gg 1} m \]

Effective \( 2m_N \) and \( E_{NN} \)

- \( t_N \sim t_{NN} \sim 12 \)
- \( t_{NN} \sim 16, \ t_N \sim 17 \)
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \text{ in } L = 20 \]

Effective mass: \[ m^{\text{eff}} = \log\left(\frac{C(t)}{C(t+1)}\right) \xrightarrow{t \gg 1} m \]

Effective \(2m_N\) and \(E_{NN}\)

\[ t_N \sim t_{NN} \sim 12 \]

\[ t_{NN} \sim 16, \quad t_N \sim 17 \]

dashed lines determined from exponential source

consistent with each other in plateau region
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \text{ in } L = 20 \]

Effective \(2m_N\) and \(E_{NN}\)

\[ t_N \sim t_{NN} \sim 12 \]

Effective \(\Delta E_{NN} = E_{NN} - 2m_N\)

exp: reasonable plateau in \(t > t_{N,NN}\)

wall: non-monotonic behavior consistent with exp in \(t > t_N\) with large error

Similar results on other volumes
Volume dependence of wall source

Preliminary result + pilot study of PRD84:054506(2011): \( N_f = 0 \) \( m_\pi = 0.8 \) GeV

Clear volume dependence + bowl like structure in small \( t \)

\( \Leftarrow \) mistaken as plateau in large volume if statistics is not enough
Volume dependence of wall source

Preliminary result + pilot study of PRD84:054506(2011): $N_f = 0 \quad m_\pi = 0.8 \text{ GeV}$

$$N_f = 0 \quad \pi = 0.8 \text{ GeV}$$

Clear volume dependence + bowl like structure in small $t$

$\Leftarrow$ mistaken as plateau in large volume if statistics is not enough

Different behavior from HALQCD expectation

Our expectation

In large statistics similar behavior will appear
Simulation parameters

\(N_f = 2 + 1\) QCD \(\beta = 1.90, a^{-1} = 2.194\) GeV with \(m_\Omega = 1.6725\) GeV, ’10 PACS-CS

Iwasaki gauge + non-perturbative \(O(a)\)-improved Wilson fermion actions

\[m_\pi = 0.51\ \text{GeV and } m_N = 1.32\ \text{GeV} \quad \text{PRD86:074514(2012)}\]
\[m_\pi = 0.30\ \text{GeV and } m_N = 1.05\ \text{GeV} \quad \text{PRD92:014501(2015)}\]

\[m_s \sim \text{physical strange quark mass}\]

\(^4\text{He}, \ ^3\text{He}, \ ^{\text{NN}(3S_1 \text{ and } 1S_0)}\)

<table>
<thead>
<tr>
<th>(L)</th>
<th>(L) [fm]</th>
<th>(N_{\text{conf}})</th>
<th>(N_{\text{meas}})</th>
<th>(N_{\text{conf}})</th>
<th>(N_{\text{meas}})</th>
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<tbody>
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<td>192</td>
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<tr>
<td>40</td>
<td>3.6</td>
<td>200</td>
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</tr>
<tr>
<td>48</td>
<td>4.3</td>
<td>200</td>
<td>192</td>
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<tr>
<td>64</td>
<td>5.8</td>
<td>190</td>
<td>256</td>
<td>160</td>
<td>1536</td>
</tr>
</tbody>
</table>

\[R = \frac{(N_{\text{conf}} \cdot N_{\text{meas}})_{0.3\text{GeV}}}{(N_{\text{conf}} \cdot N_{\text{meas}})_{0.5\text{GeV}}}\]

Exponential smeared source and point sink \((N\ \text{with } p = 0)\) operators

Computational resources

PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS
$NN$ channels (binding energy calculation)  

\[ \Delta E = E_{NN} - 2m_N \]

existence of bound states in $^3S_1$ and $^1S_0$
consistent with experiment due to larger $m_\pi$ (\(?\))

Investigation of $m_\pi$ dependence $\rightarrow m_\pi \sim 0.145$ GeV on $L \sim 8$ fm
\(NN\) channels (binding energy calculation) \(\Delta E = E_{NN} - 2m_N\)

\[
\Delta E^3_{S_1}\text{[MeV]}
\]

\[
\Delta E^{1S_0}\text{[MeV]}
\]

Investigations of \(m_\pi\) dependence \(\rightarrow m_\pi \sim 0.145\) GeV on \(L \sim 8\) fm

Large finite volume effect expected even on \(L \sim 8\) fm

\(3S_1: \Delta E_{\exp} = 2.2\) MeV

\[
\Delta E_L = -(\Delta E_{\exp} + \mathcal{O}(\exp(-L\sqrt{m_N\Delta E_{\exp}}))) \lesssim -4\text{ MeV}
\]

\(1S_0: a_0^{\exp} = 23.7\) fm

\[
\Delta E_L = -\frac{4\pi a_0^{\exp}}{m_NL^3} + \mathcal{O}(1/L^4) \lesssim -2\text{ MeV}
\]
$^4$He and $^3$He channels

$$\Delta E(^4\text{He}) = E_{^4\text{He}} - 4m_N$$

$$\Delta E(^3\text{He}) = E_{^3\text{He}} - 3m_N$$

Light nuclei likely formed in $0.3 \text{ GeV} \leq m_\pi \leq 0.8 \text{ GeV}$

Same order of $\Delta E$ to experiments
**4He and ³He channels**

\[ \Delta E(^{4}\text{He}) = E_{^{4}\text{He}} - 4m_{N} \]

\[ \Delta E(^{3}\text{He}) = E_{^{3}\text{He}} - 3m_{N} \]

Light nuclei likely formed in \(0.3 \text{ GeV} \leq m_{\pi} \leq 0.8 \text{ GeV}\)

Same order of \(\Delta E\) to experiments \(\rightarrow\) relatively easier than \(NN\)

large \(|\Delta E|\) makes less \(V\) dependence at physical \(m_{\pi}\)

touchstone of quantitative understanding of nuclei from lattice QCD

Investigations of \(m_{\pi}\) dependence \(\rightarrow m_{\pi} \sim 0.145 \text{ GeV}\) on \(L \sim 8 \text{ fm}\)
Preliminary results of effective $\Delta E$ at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

need more statistics to obtain clear signal

Computational resources (HPCI System Research Project: hp160124)
HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN
Preliminary results of effective $\Delta E$ at $m_\pi \sim 0.145$ GeV on $L \sim 8$ fm

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HA-PACS, COMA @Univ. of Tsukuba, K @AICS, FX100 @RIKEN
Summary

Binding energy calculation of light nucleus $NN(3S_1, 1S_0), 3\text{He}, 4\text{He}$

$N_f = 0$ QCD $m_\pi = 0.8$ GeV

Wall source gives consistent result with exp source if important condition is satisfied

$\Delta E_{NN}^{\text{eff}}$: non-monotonic structure in small $t$ region → different from HALQCD expectation

more sophisticated method (GEVP) necessary for more reliable result

$N_f = 2 + 1$ QCD $m_\pi = 0.5, 0.3$ GeV

bound state in $4\text{He}, 3\text{He}, 3S_1$ and $1S_0$

- $\Delta E$ larger than experiment and small $m_\pi$ dependence
- Bound state in $1S_0$ not observed in experiment, but similar to $N_f = 3 \ m_\pi = 0.8$ GeV by NPLQCD and CalLat; $N_f = 2 + 1 \ m_\pi = 0.45$ GeV by NPLQCD

Need further investigations of systematic errors e.g. large $m_\pi$, finite lattice spacing, excited state

$N_f = 2 + 1 \ m_\pi \sim 0.145$ GeV on $L \sim 8$ fm
Back up
Volume dependence of wall source

Preliminary result + pilot study of PRD84:054506(2011): $N_f = 0 \ m_\pi = 0.8 \ GeV$

Clear volume dependence + bowl like structure in small $t$

← mistaken as plateau in large volume if statistics is not enough

exp source: milder volume dependence and constant in $L \rightarrow \infty$
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \text{ in } L = 16 \]

**Effective 2m_N and E_{NN}**

\[ t_N \approx t_{NN} \approx 12 \]

\[ t_{NN} \approx 14, \ t_N \approx 15 \]

dashed lines determined from exponential source

reasonably consistent with each other in plateau region

wall data: little smaller than exp data due to statistics, probably
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \] in \( L = 16 \)

**Effective \( 2m_N \) and \( E_{NN} \)**

\[ t_N \sim t_{NN} \sim 12 \]

\[ t_{NN} \sim 14, \quad t_N \sim 15 \]

**Effective \( \Delta E_{NN} = E_{NN} - 2m_N \)**

*exp*: reasonable plateau in \( t \gtrsim t_{N,NN} \)

*wall*: non-monotonic behavior consistent with exp in \( t \gtrsim t_N \) with large error
$R(t) = C_{NN}(t)/(C_N(t))^2$ in $L = 32$

Effective $2m_N$ and $E_{NN}$

$t_N \sim t_{NN} \sim 14$  \hspace{1cm} $t_{NN} \approx 20$, $t_N \sim 20$

dashed lines determined from exponential source

hard to obtain consistent $E_{NN}$ in this statistics

wall source

overlap to scattering state becomes larger as volume increases.

$\rightarrow$ need longer $t$ to reach plateau

$\rightarrow$ assume $t_{NN} = 20$ in this study
\[ R(t) = \frac{C_{NN}(t)}{(C_N(t))^2} \] in \( L = 32 \)

**Effective 2m\(_N\) and \( E_{NN} \)**

\begin{align*}
\text{exp source} & : t_N \sim t_{NN} \sim 14 \\
\text{wall source} & : t_{NN} \approx 20, \ t_N \sim 20
\end{align*}

**Effective \( \Delta E_{NN} = E_{NN} - 2m_N \)**

\begin{align*}
\text{exp: reasonable plateau in } t \gg t_{N,NN} \\
\text{wall: non-monotonic behavior consistent with exp in } t \gg t_N \\
\text{with large error}
\end{align*}
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Clear volume dependence $+$ bowl like structure in small $t$

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Plateau of $\Delta E_{NN}$ is expected to be obtained in shaded region.
Volume dependence of wall source

Preliminary result + pilot study of PRD84:054506(2011): $N_f = 0 \ m_\pi = 0.8 \text{ GeV}$

Clear volume dependence $\rightarrow$ bowl like structure in small $t$
$\Leftarrow$ mistaken as plateau in large volume if statistics is not enough

Plateau of $\Delta E_{NN}$ is expected to be obtained in shaded region.

Positive effective $\Delta E_{NN}$ might be obtained in large volume.
$\rightarrow$ overlap to scattering state becomes larger with volume
and $\Delta E_{NN}^{\text{scat}} > 0$ when bound state exists.