Towards a full QED + QCD calculation of the leading order HVP in the muon's anomalous magnetic moment

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QCD Downunder, July 2017

Outline

Whats to come

Background

Background

The magnetic moment of a charged particle is defined as

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{s} \qquad (1)$$

with charge e, spin $\vec{s}$ and gyromagnetic ratio g.

The anomalous magnetic moment is

$$a = \frac{g-2}{2} \qquad (2)$$

Background

Experemintally measured to 0.5ppm

$$a_\mu^{exp} = 116592080 \pm 63 \times 10^{-11} \tag{3}$$

Current work underway to increase the precision by a factor of 4

Standard Model calculations also to high accuracy

$$a_\mu^{SM} = 116591828 \pm 50 \times 10^{-11} \tag{4}$$

We see there is a discrepancy here of $\sim 3\sigma$

Sign of new physics?

Current Values

SM Contribution	Value $\times 10^{11}$	Error $\times 10^{11}$	Ref
QED (5 loops)	116584718.951	0.080	[Aoyama et al., 2012]
EW (2 loops)	153.6	1.0	[Gnendiger et al., 2013]
HVP LO	6949	43	[Hagiwara et al., 2011]
HVP NLO	-98.4	0.7	[Hagiwara et al., 2011]
HLbL	105	26	[Prades et al., 2009]
SM Total	116591828	50	[Hagiwara et al., 2011]
Experimental	116582080	63	[Bennett et al., 2006]

Majority of SM uncertainty comes from the hadronic term

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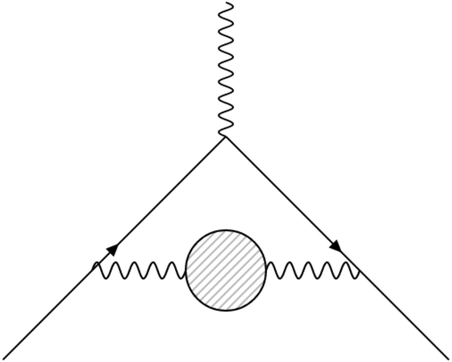
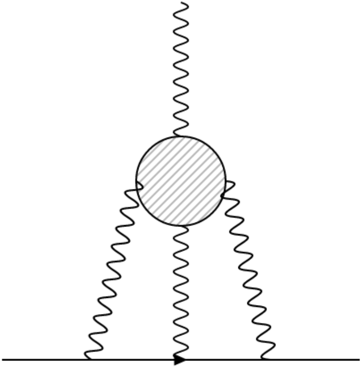
Hadronic Contributions

Majority of uncertainty comes from HVP and hadronic light-by-light





We will focus on the HVP now

HVP Calculation

HVP Calculation

HVP calculated from experimental results. $e^+e^- \rightarrow hadrons$ cross-section.

Current best estimate for a_μ^{HVP}

Some discrepancies between results from different data sets

Independent computation from first principles is desirable

Due to gluon coupling to hadrons, cant use perturbative methods to calculate HVP

We can use Lattice Gauge Theory

Calculating HVP with Lattice Gauge Theory

The leading order HVP correction to a_μ is given by

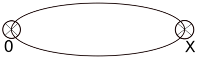
$$a_\mu^{HVP} = 4\alpha^2 \int_0^\infty dQ^2 f(Q^2)\hat{\Pi}(Q^2) \tag{5}$$

- $\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0)$ is the vacuum subtracted polarisation
- $f(Q^2)$ is an analytically known kernel function.

$\Pi(Q^2)$ can be computed on the lattice using

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{iQx} \langle J_\mu(x) J_\nu(0) \rangle = (Q^2\delta_{\mu\nu} - Q_\mu Q_\nu)\Pi(Q^2) \tag{6}$$

where $J_\mu(x)$ is the electromagnetic current



$$J_\mu(x) = q_d \bar{d}\gamma_\mu d + q_u \bar{u}\gamma_\mu u + q_s \bar{s}\gamma_\mu s \tag{7}$$

Lattice Details

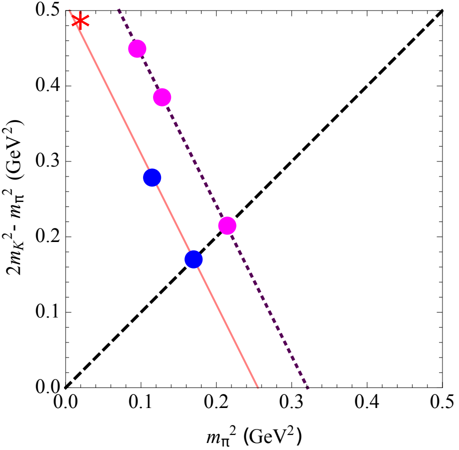
$32^3 \times 64$ lattice

a $= 0.074$ fm

$\mathcal{O}(a)$ improved action

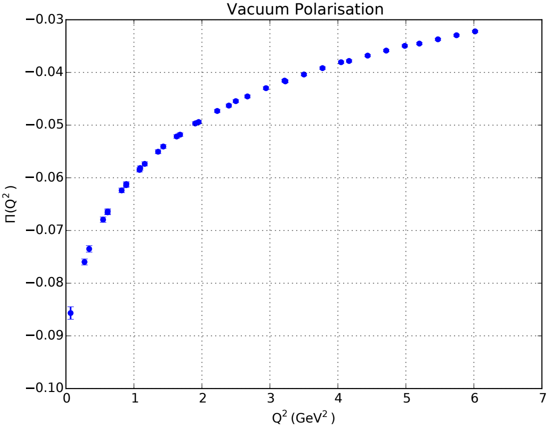
$N_f = 2 + 1$

Tune quark masses by keeping singlet quark mass fixed at physical value $\bar{m}^R = \frac{1}{3}\left(2m_l^R + m_s^R\right)$



Fitting to Lattice Data

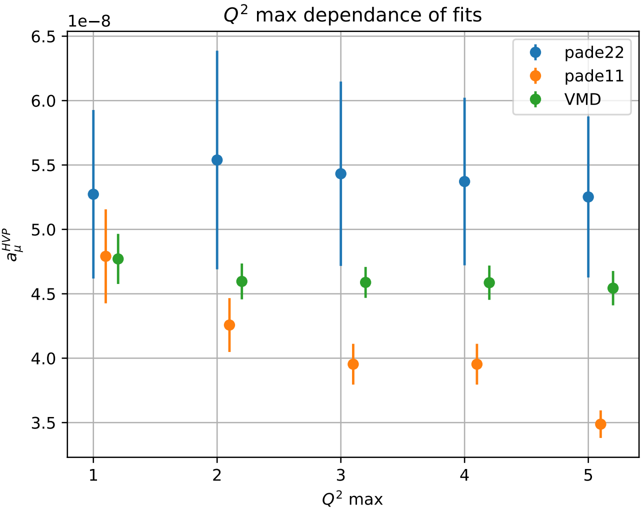
We have lattice results for $\Pi(Q^2)$
Need to fit a function to data



Try Padé functions, and VMD

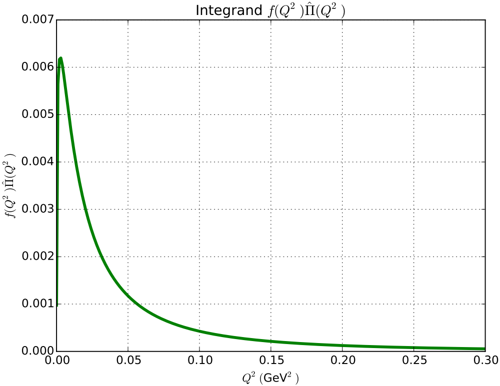
Fitting to Lattice Data

Compare fits by fitting upto different Q^2_{max}.



Computing the Integrand

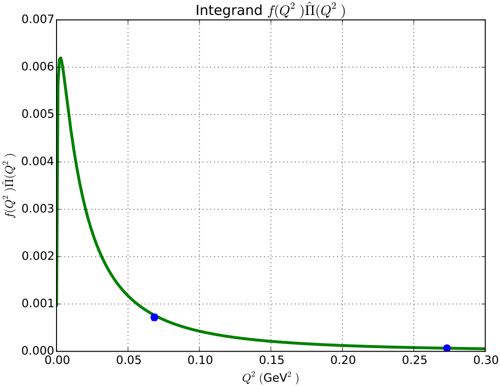
$\Pi(Q^2)$ is fairly straight forward to calculate on the lattice.
a_μ^{HVP} is not, however, due to the kernel function $f(Q^2)$.
This is strongly peaked at $Q^2 \sim \frac{m_\mu^2}{4}$



Computing the Integrand

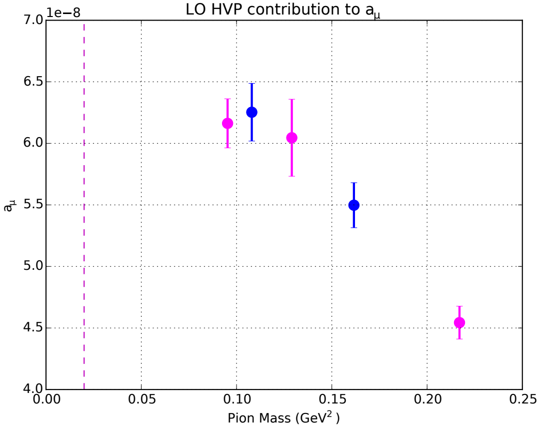
Due to lattice discretisation and periodic boundary conditions, restriction on momentum choices based on lattice volume.
Momentum restricted to

$$q_i = \frac{2\pi}{L} n_i, \quad q_4 = \frac{2\pi}{T} n_4 \tag{8}$$



QCD HVP Calculation

Use model of $\Pi(Q^2)$ to calculate a_μ^{HVP}

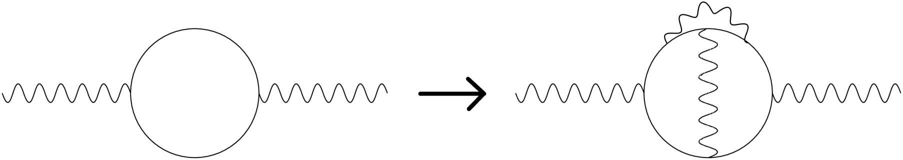


Done at various pion masses

QED Inclusion

QED Inclusion

Now that we have a pure QCD baseline, can look at QED corrections.



Include U(1) photon gauge action (S_A) to the lattice action

$S = S_g + S_q \rightarrow S = S_g + S_A + S_q$

QED+QCD Lattice

Gauge coupling turned up ~ 10 times. $\alpha_{QED} = 0.1$
Lattice Volume $32^3 \times 64$, $a = 0.068fm$
Quark masses tuned to SU(3)$_{sym}$ point (Dashen Scheme)

$$m_\pi^{u\bar{u}} = m_\pi^{d\bar{d}} = m_\pi^{n\bar{n}} \tag{9}$$

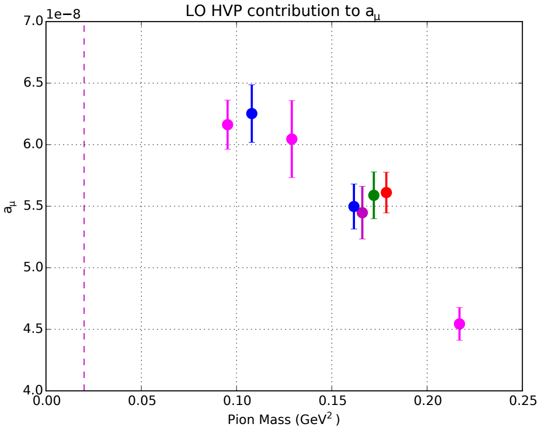
$$u : +\tfrac{2}{3} \quad m_\pi^{u\bar{u}} = 407(3) \text{ MeV}$$

$$d : -\tfrac{1}{3} \quad m_\pi^{d\bar{d}} = 409(1) \text{ MeV}$$

$$n : 0 \quad m_\pi^{n\bar{n}} = 408(3) \text{ MeV}$$

QCD+QED HVP Contribution

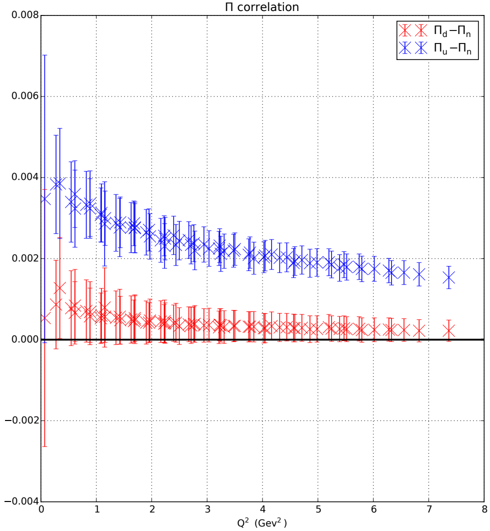
Now we repeat the HVP calculation with QED corrections included.
Did this for Up, Down and neutral quark at same pion mass.



Unfortunatly, we can't see much from this. Need to look elsewhere

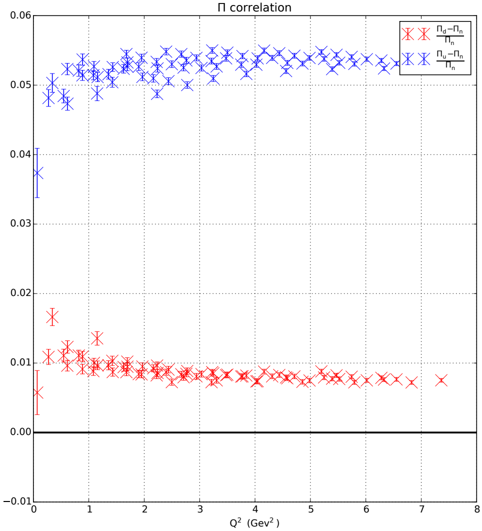
HVP Correction

Look at the HVP term itself!



Can see the charge clearly has some effect here!

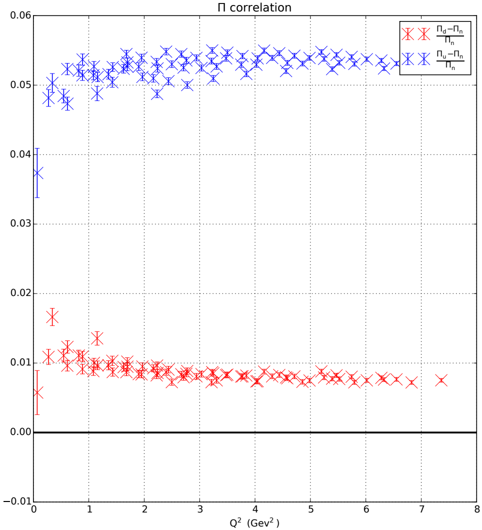
HVP Correction



Effect non zero for both up and down quark

Correction of ∼5% for up quark, ∼1% for down quark.

HVP Correction



Effect non zero for both up and down quark
Correction of ∼0.5% for up quark, ∼0.1% for down quark.
Remember that our QED coupling is 10 times larger than physical

HVP Correction

Recalling the Standard Model contributions

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The current uncertainty in LO HVP $\sim 0.6\%$
QED corrections are of similar order to uncertainty of LO HVP

Summary

QED corrections to HVP are non trivial.

Become increasingly important as we attempt to increase the precision of a_μ^{HVP} calculations.

Further investigation needed. Lighter Pion mass. Larger Lattice volumes.

QED effects sensitive to finite volume effects.

Thankyou