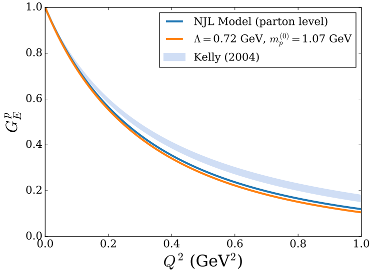
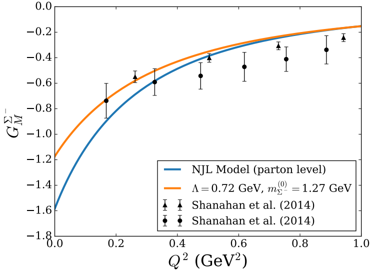
Chiral Corrections to Electromagnetic Form Factors in the NJL Model

Manuel Carillo Serrano, Robert Perry, Anthony W. Thomas

Overview

- We calculated electromagnetic form factors for the Nucleon, and the $\Sigma^{\pm}$ hyperons.
- We utilized the NJL Model to predict our 'bare' electromagnetic form factors.
- We incorporated long range pionic effects using the Light Front Cloudy Bag Model.





Talk Outline

- Motivation
- Baryon Electromagnetic Form Factors
- Pion-Nucleon Effective Field Theory
- Nucleon Results
- Generalizations
- Hyperon Electromagnetic Form Factors.
- Conclusion

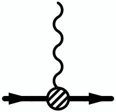
Motivation

One of the fundamental goals of nuclear physics is to understand the structure and behavior of strongly interacting matter in terms of its basic constituents, quarks and gluons. An important step towards this goal is the characterization of the internal structure of the nucleon; the elastic electric and magnetic form factors of the proton and neutron are key ingredients of this characterization.

- JLab Research Highlights
`jlab.org/highlights/phys.html`

Baryon Electromagnetic Form Factors

- Form factors contain information about the structure of the baryon.
  - Requires a microscopic description of the baryon.



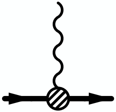
$$= \overline{u}(p')\Gamma^{\mu}(p',p)u(p) = \overline{u}(p')\left[\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m_N}F_2(q^2)\right]u(p)$$

- Common to use the Sachs Parametrisation.

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Baryon Electromagnetic Form Factors

- Form factors contain information about the structure of the baryon.
  - Requires a microscopic description of the baryon.



$$= \overline{u}(p')\Gamma^{\mu}(p',p)u(p) = \overline{u}(p')\left[\gamma^{\mu}F_1(q^2)+\frac{i\sigma^{\mu\nu}q_{\nu}}{2m_N}F_2(q^2)\right]u(p)$$

- Common to use the Sachs Parametrisation.

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Low Energy QCD

- QCD is non-perturbative at low energies.
  - Build realistic models by examining symmetries of parent theory.
- Microscopic description of system essential for form factors
  - Requires quark model.
- The pion, being the lightest meson, is known to play an important role in low-energy nucleon phenomena.
  - Incorporation of pion cloud leads to corrections to observables.

Incorporating Pion Effects

- Two papers due to Thomas and Krein:
  - *Chiral Corrections in Hadron Spectroscopy* (1999)
  - *Chiral Aspects of Hadron Structure* (2000)

Parton Level

- Calculate pion effects from quark-pion coupling
- Idea goes back to Manohar and Georgi: *Chiral Quarks and the Non-Relativistic Quark Model* (1985)

Hadron Level

- Project onto baryon states, and calculate in nucleon-pion EFT.
- Implemented in Cloudy Bag Model, and Light Front Cloudy Bag Model

Can We Observe the Difference?

- While the theoretical conclusion is clear, we wished to determine the effects of the differing approaches.
- Cloët, Bentz and Thomas wrote paper - *Role of diquark correlations and the pion cloud in nucleon elastic form factors* (2014)
  - Pion loops calculated at parton level.
    - Theoretical arguments suggest that this approach is less well motivated (although it is often done in literature).
- Two Step Process:
  1. Propose to calculate bare (pionless) form factors in NJL Model.
  2. Perform chiral loop corrections at hadron level (use Light Front Cloudy Bag Model).

Calculating Bare Form Factors in the NJL Model

The Nambu–Jona-Lasinio (NJL) Model: Bare Form Factors

- Bare form factors calculated in NJL Model.
  - Low energy approximation of QCD: 4 fermion contact interaction
- Confinement failure of basic model, but imposed via Proper Time Regularisation & infra-red cutoff.

$$\frac{1}{X^n} = \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau X}$$

- Parameters are fitted to give correct experimental observables including the nucleon mass…

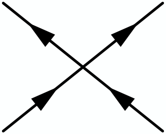


Figure 2: The NJL Model is based on a 4 fermion contact interaction, c.f. Fermi Theory.

The Nambu–Jona-Lasinio (NJL) Model: Bare Form Factors

- Bare form factors calculated in NJL Model.
  - Low energy approximation of QCD: 4 fermion contact interaction
- Confinement failure of basic model, but imposed via Proper Time Regularisation & infra-red cutoff.

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau \tau^{n-1} e^{-\tau X}$$

- Parameters are fitted to give correct experimental observables including the nucleon mass. . .

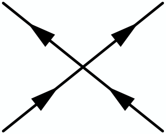


Figure 2: The NJL Model is based on a 4 fermion contact interaction, c.f. Fermi Theory.

Baryon Self Energy

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$

Dressed State

Calculate chiral loops

$= \frac{iZ}{\not{p} - m_N + i\epsilon}$

- Must fit NJL model parameters to Bare Mass
- Related to physical mass via

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Baryon Self Energy

Bare Calculation → **Dressed State**

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Calculate chiral loops

$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

- Must fit NJL model parameters to Bare Mass
- Related to physical mass via

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Baryon Self Energy

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops

$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

- Must fit NJL model parameters to Bare Mass
- Related to physical mass via

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Performing Chiral Corrections to NJL Model at Hadron Level

Pion-Nucleon Effective Field Theory

- Work with a pseudoscalar pion-nucleon interaction:

$$\mathcal{L}_{N\pi} = -ig_0 \overline{\psi}_N \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi_N$$

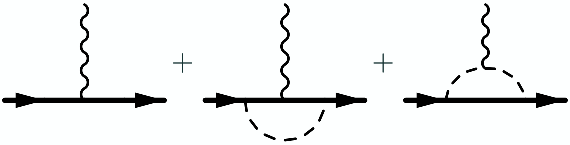
- After minimal substitution, one has three diagrams at first loop order.

Pion-Nucleon Effective Field Theory

- Work with a pseudoscalar pion-nucleon interaction:

$$\mathcal{L}_{N\pi} = -ig_0\overline{\psi}_N\gamma_5\vec{\tau}\cdot\vec{\pi}\psi_N$$

- After minimal substitution, one has three diagrams at first loop order.



Cutoff Dependence

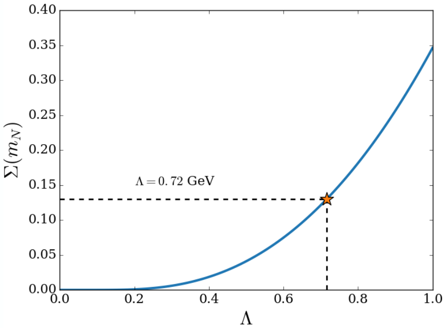
- Incorporate strong form factor at πN vertex
  - Utilize a t-dependent Form Factor

$$G(t) = \exp\left[\frac{(t - m_\pi^2)}{\Lambda^2}\right]$$

  to parameterize extended particle structure.
- All observables become cutoff (Λ) dependent.
  - Self energy cutoff dependent.
  - $m_N^{(0)} = m_N - \Sigma(m_N, \Lambda)$
  - Bare mass (used in NJL Calculation) varies with Λ.

Fitting Model Parameters

- Self Energy not observable, so must take guidance from other models.
- CBM predicts total pion self energy contribution ($\Sigma(m_N)$) in the range 0.1 GeV to 0.3 GeV.
- In practice, this, along with the light quark mass (required to be chosen in the NJL Model) were scanned over.



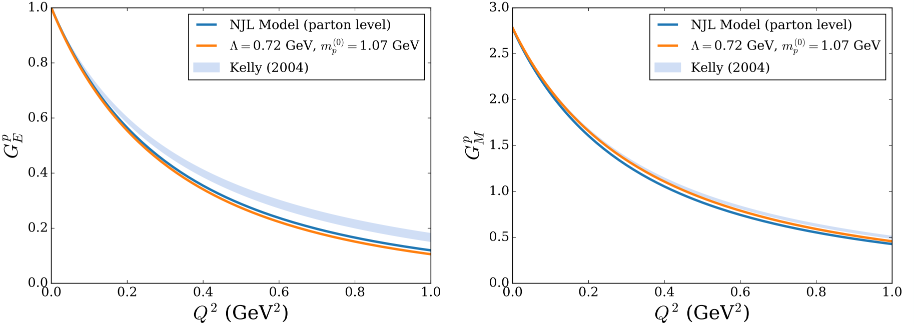


Figure 3: G_E^p and G_M^p (arXiv preprint: 1703.01032).

Nucleon Results

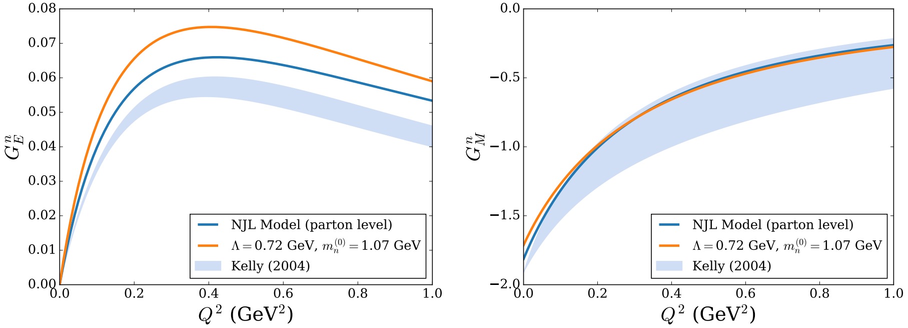
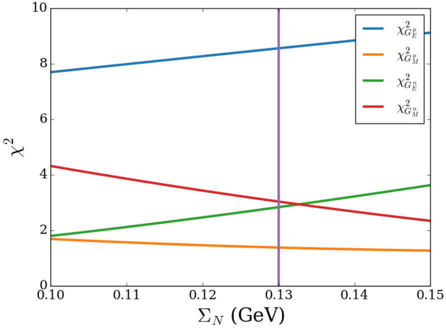


Figure 4: G_E^n and G_M^n (arXiv preprint: 1703.01032).

Variation of Nucleon Self Energy

- How do fits change if we vary our self energy?
  - This corresponds to modifying the parameter Λ in our pion-nucleon form factor $G(t)$.
- Results are reasonable stable for self energies between 100 and 150 MeV.
- Our choice of $\Sigma = 130$ MeV is seen to be reasonable.



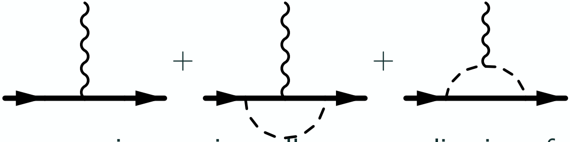
Generalizing Results to Hyperons

Generalizing the Light Front Cloudy Bag Model

- Due to approximate $SU(3)$ symmetry, one has relations between nucleon-pion and hyperon-pion couplings.

$$g_{\Lambda\Sigma\pi} = \frac{2}{\sqrt{3}}(1-\alpha)g_{NN\pi}; \qquad g_{\Sigma\Sigma\pi} = 2\alpha g_{NN\pi}$$

- Although the particles themselves are different, topology of contributing diagrams are the same.



- Simple replacements in equations allows generalization of the equations to consider the hyperons.

Hyperon Results

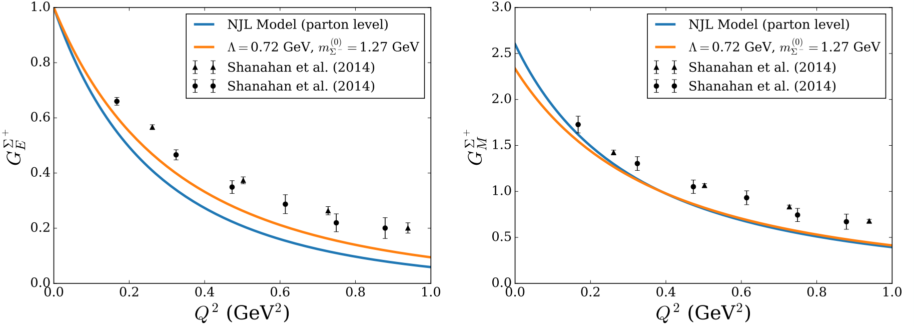


Figure 5: $G_E^{\Sigma^+}$ and $G_M^{\Sigma^+}$ (arXiv preprint: 1703.01032).

Hyperon Results

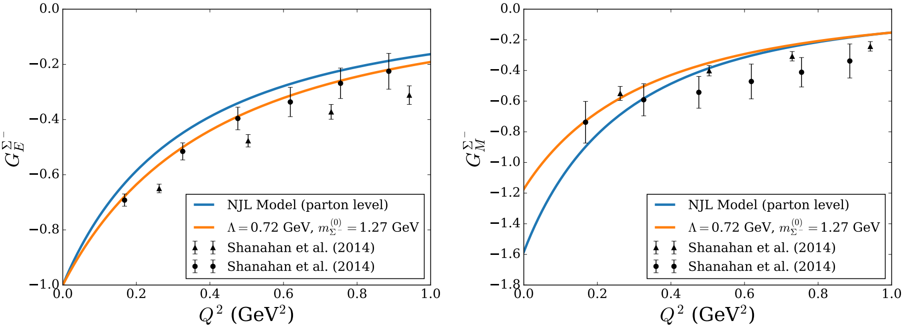


Figure 6: $G_E^{\Sigma^-}$ and $G_M^{\Sigma^-}$ (arXiv preprint: 1703.01032).

Magnetic Moments and Charge Radii

Magnetic Moments (μ)				
	p	n	Σ^-	Σ^+
This Work	2.78	-1.71	-1.17	2.33
Prev. NJL	2.78	-1.81	-1.58	2.60
Exp.	2.793	-1.913	-1.160(25)	2.458(10)

Table 1: Experimental results are taken from [?, ?]. Magnetic moments are in units of nuclear magnetons ($\mu_N = e/2m_N$).

Electric Charge Radii $\left(\langle r^2 \rangle^{\frac{1}{2}}\right)$				
	p	n	Σ^-	Σ^+
This Work	0.89	0.41	0.78	0.88
Prev. NJL	0.87	0.38	0.86	0.97
Exp.	0.84 [?]	0.335	0.780	0.61(8) [?]

Table 2: Experimental results are taken from [?, ?, ?], except for the Σ^+ charge radius. Here a recent lattice QCD result [?] is given instead. Charge radii are quoted in femtometres.

Conclusion

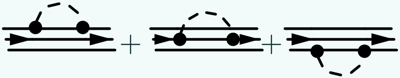
In Conclusion...

- Form factors are important ingredient in characterisation of the structure of the hadron.
- Earlier work has shown the correct way is at the hadron level.
- Calculated chiral loop corrections to the NJL model at the Hadron Level.
- Similar fit to Kelly Parameterization for Nucleon system for both approaches.
- Hadron level corrections offer a better prediction for the Σ^- Magnetic Moment over the parton level corrections.

Thanks for listening!

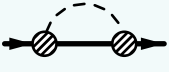
Incorporating Pion Effects

Parton Level



$$\propto |g_{\pi q_1 q_1}|^2 + |g_{\pi q_2 q_2}|^2 + |g_{\pi q_3 q_3}|^2$$

Hadron Level



$$\propto |g_{\pi NN}|^2 = |g_{\pi q_1 q_1}|^2 + |g_{\pi q_2 q_2}|^2 + |g_{\pi q_3 q_3}|^2 + \text{cross terms}$$

The Self Energy

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops

$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

 $= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$

Dressed State

Calculate chiral loops

 $= \frac{iZ}{\not{p} - m_N + i\epsilon}$

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Bare Calculation

Calculate bare (pionless) form factors in NJL Model

$$= \frac{i}{\not{p} - m_N^{(0)} + i\epsilon}$$

Dressed State

Calculate chiral loops

$$= \frac{iZ}{\not{p} - m_N + i\epsilon}$$

$$m_N = m_N^{(0)} + \Sigma(\not{p})\Big|_{\not{p}=m_N}$$

Scaling the Form Factors

- The calculation at non-physical mass requires one to modify the equations for the loop corrections.
- The outputs of NJL Model are $F_1^{(0)}(Q^2)$, and $F_2^{(0)}(Q^2)$, and come from separating the gamma function Γ^μ

$$\Gamma^\mu = F_1^{(0)}(Q^2)\gamma^\mu + F_2^{(0)}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N^{(0)}}$$

- When inserting this into the chiral correction equations, one must rescale $F_2^{(0)}(Q^2)$, as

f

Equations

$$F^p_{1,b} = \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} d\tau \Big([4x(x+y)m_N^2 - 2(x+y)m_N^2 + \frac{1}{2}(x+$$

$$\times (F^n_{1,a} + \frac{1}{2}F^p_{1,a}) + [x(x+y)q^2](F^n_{2,a} + \frac{1}{2}F^p_{2,a})\Big) e^{-\tau\Delta}$$

$$+ \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \frac{d\tau}{\tau}\frac{1}{1-x}(F^n_{1,a} + \frac{1}{2}F^p_{1,a})e^{-\tau\Delta'}$$

$$F^p_{2,b} = \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} d\tau \Big(- [4x(x+y)m_N^2](F^n_{1,a} + \frac{1}{2}F^p_{1,a})$$

$$+ [\frac{1}{2}(x+y)q^2 - 2(x+y)m_N^2 - \frac{2}{\tau} - 2xyq^2](F^n_{2,a} + \frac{1}{2}F^p_{2,a})\Big) e^{-\tau\Delta}$$

$$+ \frac{Zg_0^2}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \frac{d\tau}{\tau}\frac{1}{1-x}(F^n_{2,a} + \frac{1}{2}F^p_{2,a})e^{-\tau\Delta'}$$

Equations

$$F^p_{1,c} = \frac{Zg_0^2}{(4\pi)^2} F_\pi(q^2) \int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \left[\frac{1}{\tau} - 2m_N^2 z^2\right] e^{-\tau\Delta}$$

$$F^p_{2,c} = \frac{Zg_0^2}{(4\pi)^2} F_\pi(q^2) \int_0^1 dx \int_0^{1-x} dz \int_{r_0^2}^{\infty} \left[2m_N^2 z^2\right] e^{-\tau\Delta}$$

REFERENCES