TOWARDS A COMPETITIVE $|V_{us}|$ DETERMINATION FROM STRANGE HADRONIC τ DECAY DATA AND LATTICE HVPs

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QCD Down Under III

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OUTLINE

- *Background/motivation*
  - *The long-standing FB FESR τ V_{us} puzzle*
  - *Systematic issues in the conventional implementation, a new implementation (and solution) + current practical limitations to this new approach*

- *A new lattice+inclusive us V+A τ data strategy: dispersive formulation, results, and future prospects*

CONTEXT

- The puzzle: τ vs. non-τ determinations

$\vert V_{us}\vert$	Source
0.2258(9)(?)	3-family unitarity, HT14 $\vert V_{ud}\vert$
$0.2231(4)_{exp}(7)_{latt}$	$K_{\ell 3}$, 2+1+1 lattice $f_+(0)$
$0.2253(4)_{exp}(6)_{latt}$	$\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$, lattice f_K/f_π
$0.2186(18)_{exp}(10?)_{th}$	τ FB FESRs (HFAG17)

- τ result is from restrictive "conventional implementation" of more general inclusive FB FESR framework

- Interesting (if true) given other hints of lepton universality breakdown [see e.g. Nature 546 (2017) 227]

  - Combined Belle, BaBar, LHCb results for $R[D^{(*)}] \equiv B[\bar{B}^0 \to D^{(*)+}\tau^-\nu_\tau]/B[\bar{B}^0 \to D^{(*)^+}\mu^-\nu_\mu]$ (2nd, 3rd generation quarks; 3rd vs 2nd generation leptons) *combined* $\sim 3.9\sigma$ above SM expectations

  - LHCb results for $R_{K^{(*)}} \equiv B[B^+ \to K^{(*)+}\mu^+\mu^-]/B[B^+ \to K^{(*)+}e^+e^-]$ (2nd, 3rd generation quarks; 2nd vs 1st generation leptons) $\sim 2.5\sigma$ below SM expectations

- C.f. non-strange, strange τ decays: 3rd generation lepton; 1st, 2nd generation quarks

BASICS: HADRONIC τ DECAYS IN THE SM

- $R_{ij;V/A} \equiv \Gamma[\tau \to \nu_\tau \,\mathrm{hadrons}_{ij;V/A}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$

- With $y_\tau \equiv s/m_\tau^2$, flavor ij decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2\,|V_{ij}|^2 S_{EW}}{m_\tau^2}\,[1-y_\tau]^2\,\tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1+2y_\tau)\,\rho^{(J=1)}_{ij;V+A}(s) + \rho^{(J=0)}_{ij;V+A}(s)\right]$$

kinematic weight : $w_\tau(y) = (1-y)^2(1+2y)$

THE INCLUSIVE FB τ $|V_{us}|$ DETERMINATION

- FESRs for $\Pi = \Pi^{(J=0+1)}_{ud-us;V+A}(Q^2)$, $\rho = \rho^{(J=0+1)}_{ud-us;V+A}(s)$

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$

- Experiment: $|V_{ij}|^2\rho^{(0+1)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$

- $R^w_{ij;V/A}(s_0)$: re-weighted $R_{ij;V/A}$ analogue

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

- FB differences $\delta R^w(s_0) \equiv \frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - \frac{R^w_{us;V+A}(s_0)}{|V_{us}|^2}$

- FESR, OPE for $\delta R^w(s_0)$, input $|V_{ud}|$ $\Rightarrow$

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}$$

Self-consistency: $|V_{us}|$ independent of s_0, w

- **The conventional implementation** [Gamiz et al. JHEP03(2003)060]
  - $s_0 = m_\tau^2$, $w = w_\tau$ only [spectral integrals from inclusive ud, us BFs, **but no self-consistency tests**]
  - w_τ degree 3 $\Rightarrow$ OPE to $D = 8$
  - Strong assumptions re $D = 6$ (VSA), $D = 8$ (~ 0)

- $|V_{us}|$ results from extended variable-s_0, -w FB FESR analyses with same $D = 6, 8$ assumptions show [see KM, JZ et al arXiv:1702.01767]

  - strong unphysical s_0 dependence
  - strong unphysical w dependence even at $s_0 = m_\tau^2$
  - apparent convergence toward common value for different w beyond $s_0 = m_\tau^2$

- Observed unphysical behavior strongly suggests breakdown of $D = 6, 8$ OPE assumptions

AN ALTERNATE FB FESR IMPLEMENTATION

- Theory side
  - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data **(N.B. requires variable s_0)**
  - 3-loop-truncated FOPT $D = 2$, standard $D = 2+4$ error estimates [from comparison to lattice data]
  - C_{2N+2} ($D = 2N + 2$ condensate), $|V_{us}|$ both from $w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ FESR ($y = s/s_0$)
  - $|V_{us}|$ from different w_N as self-consistency check

- Experimental input (also for new lattice approach)
  - Updated/corrected 2013 ALEPH for ud V+A
  - us V+A from sum over exclusive modes
    * K from $K_{\mu 2}$ or $B[\tau \to K\nu_\tau]$
    * $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$: BaBar, Belle unit-normalized distributions, BFs

      [Note: HFAG $B[K^-\pi^0] + B[\bar{K}^0\pi^-] = 0.01273(21)$ c.f. 0.01327(48) with additional dispersive $K\pi$ ff constraints [ACLP 2013, JHEP 1310 (2013) 070]]
    * Remaining ("residual modes") from 1999 ALEPH (note: $\sim 25\%$ errors, some MC)

NEW FB FESR RESULTS

- With fitted $D > 4$ condensates
  - unphysical s_0-, $w(y)$-dependence problems resolved
  - $|V_{us}|$ increased by ~ 0.0020 (0.2186(21) $\rightarrow$ 0.2209(23))
  - Non-trivial $K\pi$ normalization uncertainty: with alternate ACLP choice, 0.2209(23) $\rightarrow$ 0.2233(28) (c.f. 0.2207(27) from conventional implementation)
- Very favorable (~ 0.0005) theory error situation
- us spectral integral uncertainty dominates current error

- *Theory error* $\Rightarrow$ *competitive with* $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ **with sufficient** us **experimental error improvement**

- us experimental uncertainties currently BF dominated

- BFs more easily improved experimentally than exclusive mode dR/ds distribution contributions

- Near-term low-multiplicity mode progress likely [combined BaBar, Belle (+Belle II) effort on spectral functions from existing B-factory data under way]

- **However** sub-0.5% $|V_{us}|$ needs sub-% $R^w_{us;V+A}$ error

- Exclusive us mode w_N spectral integral contributions

Relative exclusive mode $R^w_{us:V+A}$ contributions

Wt	s_0 $[GeV^2]$	K	$K\pi$	$K\pi\pi$ (B-factory)	Other
w_2	2.15	0.496	0.426	0.062	0.010
	3.15	0.360	0.414	0.162	0.065
w_3	2.15	0.461	0.446	0.073	0.019
	3.15	0.331	0.415	0.182	0.074
w_4	2.15	0.441	0.456	0.082	0.021
	3.15	0.314	0.411	0.194	0.081

- **"Other":** 1999 ALEPH data/MC, $\sim 25\%$ error

  $\Rightarrow$ **"sufficient improvement" includes experimentally (much) more challenging higher-multiplicity modes**

A LATTICE τ-BASED ALTERNATIVE

- Work with R. Lewis, J. Hudspith (York), P. Boyle, T. Izubuchi, A. Jütter, C. Lehner, H. Ohki, A. Portelli, M. Spraggs (RBC/UKQCD)

- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ us V+A polarizations with weights having poles at Euclidean Q^2
  - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$)
  - Theory: Lattice us 2-point function data (no OPE)
  - Weights tunable, allow suppression of larger-error, higher-multiplicity us spectral contributions

More on the lattice-inclusive us τ approach

- $|V_{us}|^2\, \tilde{\rho}_{us;V+A}(s)$ from experimental $dR_{us;V+A}/ds$

$$\tilde{\rho}_{us;V+A}(s) \equiv \left(1 + 2\frac{s}{m_\tau^2}\right) \rho^{(J=1)}_{us;V+A}(s) \,+\, \rho^{(J=0)}_{us;V+A}(s)$$

(no continuum us $J = 0$ subtraction required)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left(1 - 2\frac{Q^2}{m_\tau^2}\right) \Pi^{(J=1)}_{us;V+A}(Q^2) + \Pi^{(J=0)}_{us;V+A}(Q^2)$$

- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \to \infty$

- For weights $w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)}$, $N \geq 3$, obtain convergent, unsubtracted 'dispersion relation'

$$\int_{th}^{\infty} ds\, w_N(s)\tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\prod_{j\neq k}\left(Q_j^2 - Q_k^2\right)} \equiv \tilde{F}_{w_N}$$

  - Lattice data for $\tilde{\Pi}_{us;V+A}(Q_k^2)$ on RHS
  - LHS from experimental $dR_{us;V+A}/ds$, up to $|V_{us}|^2$
  - $w_N(s)$: rapid fall-off if all $Q_k^2 < 1\ GeV^2$

    $\Rightarrow$ K, $K\pi$ **dominate LHS, near-endpoint multi-particle,** $s > m_\tau^2$ **contributions strongly suppressed**
  - Optimization: increasing $\{Q_k^2\}$ decreases RHS lattice error, increases LHS experimental error

- A few details:
  - Near-physical-point RBC/UKQCD $n_f = 2+1$ DWF ensembles
    * $48^3 \times 96$, $1/a = 1.73\ GeV$, $m_\pi = 0.139\ GeV$, $m_K = 0.499\ GeV$
    * $64^3 \times 128$, $1/a = 2.36\ GeV$, $m_\pi = 0.139\ GeV$, $m_K = 0.508\ GeV$
    * (Small) retuning to physical valence masses [PQ]
  - Time-momentum representation for polarizations in $\tilde{F}_{w_N}$ sum (back-up slides)

- Weighted spectral integrals (include $|V_{us}|^2$ factor)

$$\tilde{R}_{w_N} \equiv \int_0^{m_\tau^2} \frac{m_\tau^2}{12\pi^2 S_{EW}(1-y_\tau)^2} \frac{dR_{us;V+A}(s)}{ds} w_N(s)ds$$

- $\tilde{F}^{pQCD}_{w_N} \equiv \int_{m_\tau^2}^{\infty} [\tilde{\rho}_{us}(s)]^{\rm pQCD}\, w_N(s)ds$

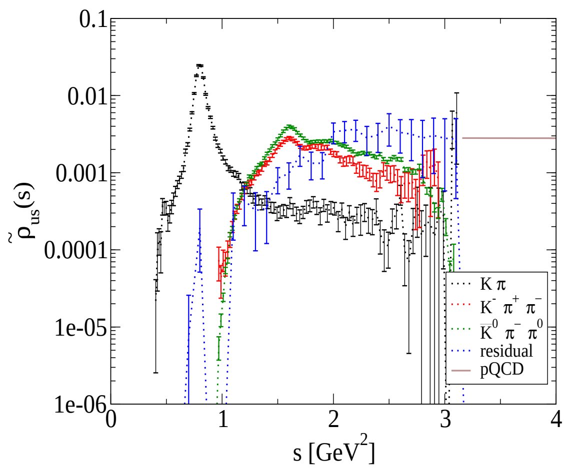
- $|V_{us}| = \sqrt{\tilde{R}_{us;w_N} / \left[\tilde{F}_{\omega_N} - \tilde{F}^{pQCD}_{w_N}\right]}$

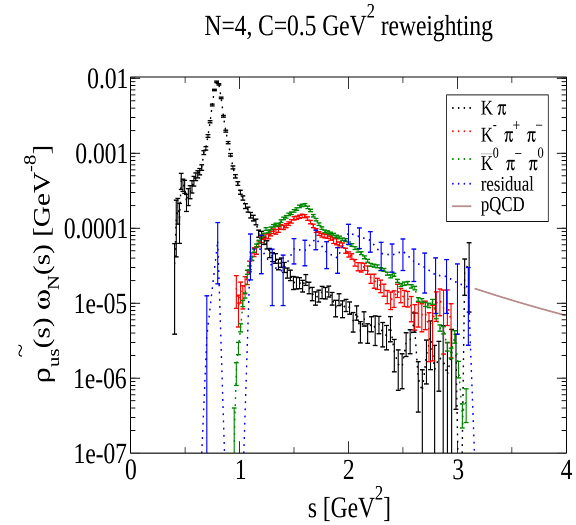
- Expect continuum spectral A, $J=0$ channel contributions negligible for w_N employed (confirmed by lattice data), hence exclusive A, $J=0$ analysis also possible (using only K spectral contribution)

- w_N below: uniform pole spacing Δ, centroid C

Reweighting of exclusive mode distributions

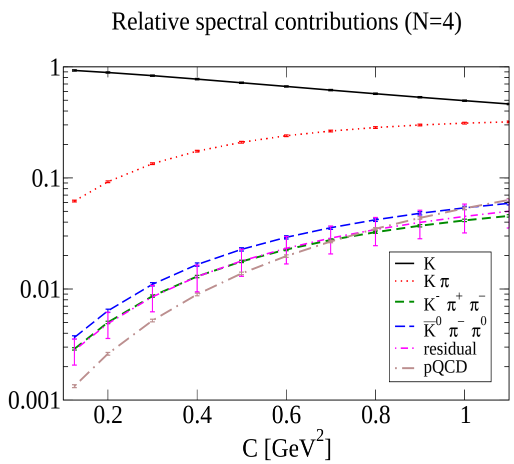
Left panel: un-re-weighted experimental data; right panel: $N = 4$, $C = 0.5\ GeV^2$ re-weighted version

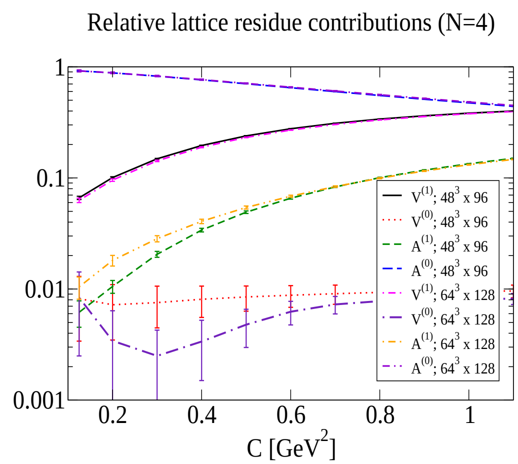




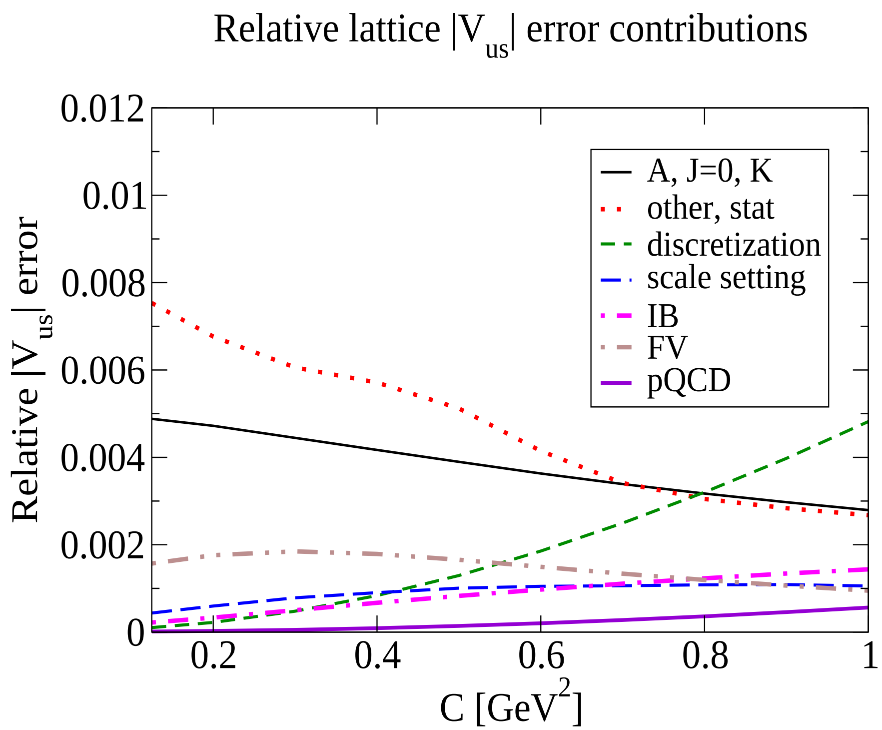
High-s spectral suppression, $N = 4$ example

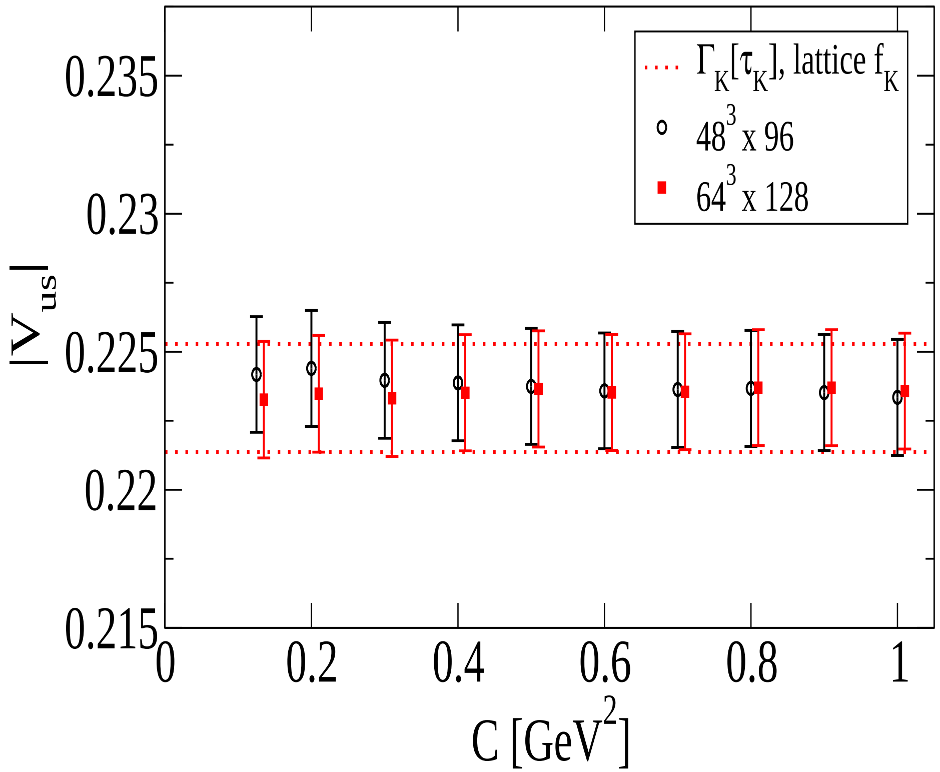
Left panel: relative exclusive mode spectral contributions
Right panel: relative lattice residue contributions





Lattice error breakdown vs. C, $N = 4$

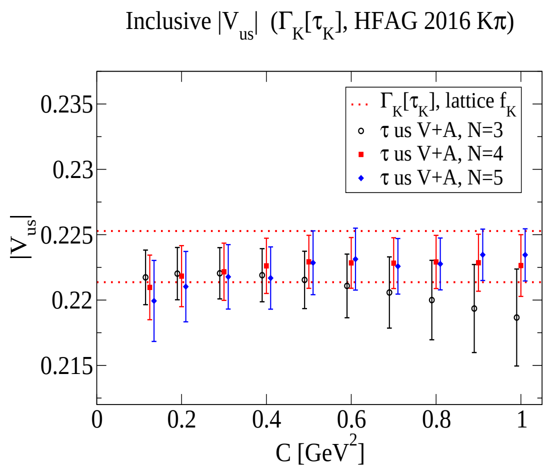


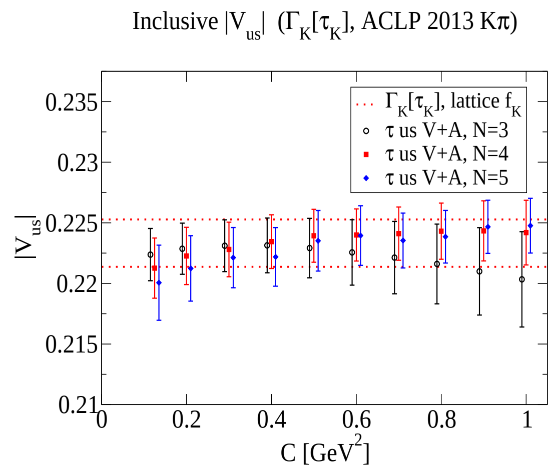


Exclusive A, $J = 0$ channel determination

Inclusive determinations

Left panel: HFAG $K\pi$ normalization; right panel: alternate, dispersively constrained ACLP $K\pi$ normalization





$|V_{us}|$ error budget [%], HFAG $K\pi$ normalization

	[N, C [GeV^2]]	[3, 0.3]	[3, 1]	[4, 0.7]	[5, 0.9]
theory	f_K	0.37	0.20	0.34	0.36
	others, stat.	0.41	0.19	0.34	0.41
	discretization	0.10	0.80	0.25	0.27
	scale setting	0.09	0.08	0.11	0.11
	IB	0.10	0.21	0.11	0.10
	FV	0.10	0.04	0.13	0.18
	pQCD	0.05	0.26	0.03	0.01
	total	0.59	0.91	0.58	0.65
experiment	K	0.48	0.27	0.44	0.47
	$K\pi$	0.20	0.32	0.23	0.22
	$K^-\pi^+\pi^-$	0.06	0.16	0.06	0.05
	$\bar{K}^0\pi^-\pi^0$	0.03	0.09	0.03	0.03
	residual	0.41	1.35	0.41	0.28
	total	0.66	1.43	0.65	0.59
Combined	total	0.88	1.70	0.87	0.88

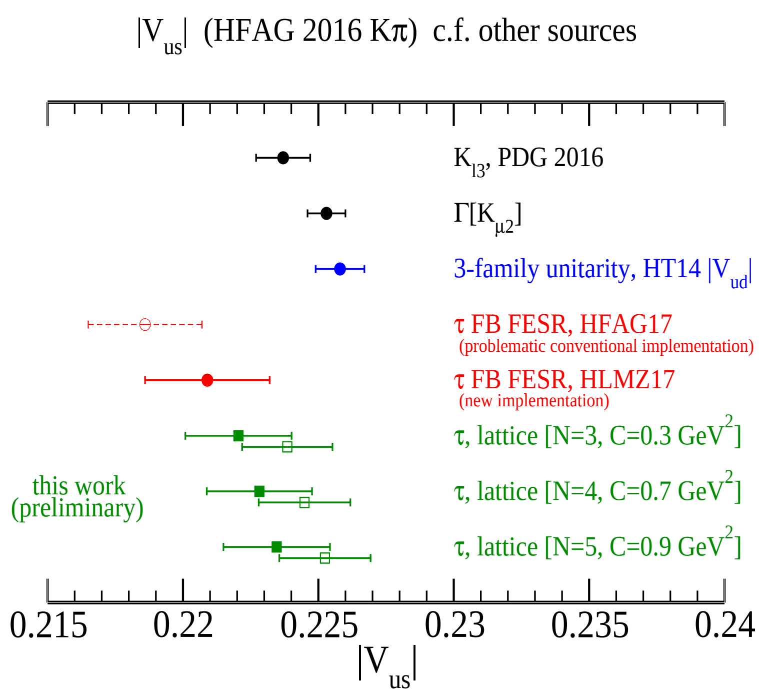
INCLUSIVE ANALYSIS RESULTS SUMMARY

- With $B[\tau \to K\nu_\tau]$ for K pole contribution:
  - HFAG $K\pi$ norm'n: $|V_{us}| = 0.2228(14)_{exp}(13)_{th}$
  - ACLP $K\pi$ norm'n: $|V_{us}| = 0.2241(18)_{exp}(13)_{th}$

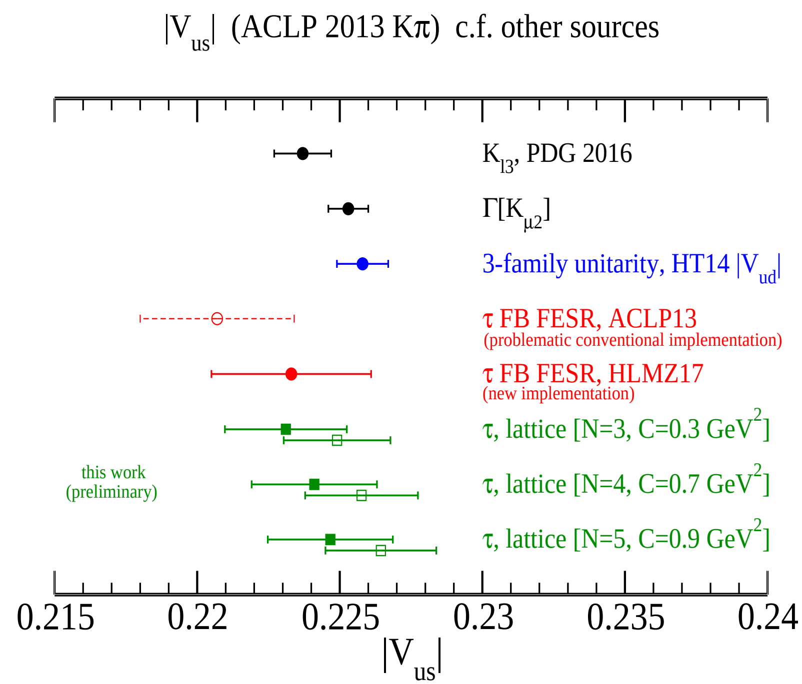
- With $\Gamma[K_{\mu 2}]$ for K pole contribution:
  - HFAG $K\pi$ norm'n: $|V_{us}| = 0.2245(10)_{exp}(13)_{th}$
  - ACLP $K\pi$ norm'n: $|V_{us}| = 0.2258(15)_{exp}(13)_{th}$

Comparison to results from other methods

With HFAG $K\pi$ normalization



With alternate ACLP $K\pi$ normalization



Comments/Prospects

- Lattice analysis confirms larger $|V_{us}|$ from τ decay data [as per alternate HMLZ FB FESR implementation]

- Significant error reduction from lattice approach c.f. FB FESR determination employing same data

- Theory uncertainty under better control for lattice than for OPE

- Improved suppression of high-s, higher-error spectral contributions in lattice approach *without blowing up theory errors*

- **Lattice thus superior to FB FESR approach and should replace it going forward**

- Trend to $|V_{us}|$ fall-off for $N = 3$, larger C compatible with missing high-s, higher-multiplicity spectral strength (larger impact on FB FESR than lattice results)

- Theory (lattice) errors straightforwardly reducible through improved statistics

- Signifiant experimental error reduction from improved K, $K\pi$ τ BFs even without unit-normalized distribution improvements (e.g., Belle II)

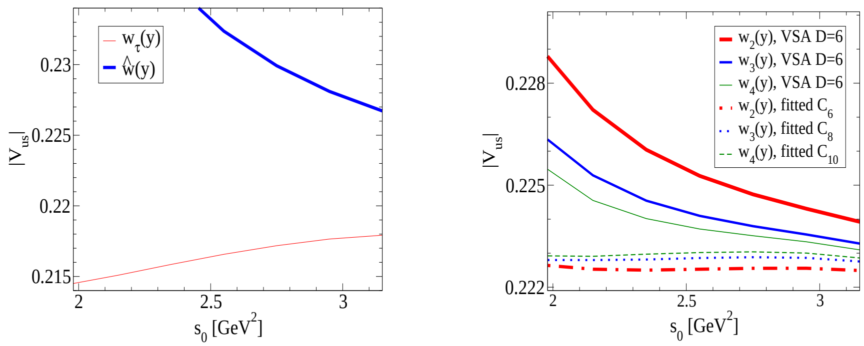
SUMMARY

- Old 3σ low inclusive FB τ FESR $|V_{us}|$ problem resolved
  - **Conventional FB FESR implementation using only inclusive BFs no longer tenable**
  - Alternate, no-assumptions implementation: $|V_{us}|$ higher by ~ 0.0020, compatible with other determinations
  - Near-term improvements via us exclusive BFs
  - Highly favorable theoretical error situation
  - However, competitive $|V_{us}|$ needs improvements to old ALEPH higher-multiplicity, low-statistics data

- Advantage of new lattice-inclusive us $V+A$ τ approach
  - Theory:
    - Lattice in place of OPE; no us $J=0$ subtraction; improvement through increased statistics
    - *Parasitic on lattice a_μ effort (a major effort in the lattice community)*
  - Spectral integrals:
    - Theory errors still small for weights strongly suppressing higher multiplicity contributions
    - Strong K, $K\pi$ dominance of spectral integral
    - Significant experimental improvements possible through just improved $K\pi$ BFs, distributions

BACKUP SLIDES

- FB FESR w-, s_0-stability tests



  - ∘ Left panel: conventional implementation $w_\tau(y)$, $\hat{w}(y) = (1-y)^3$ results (re $D = 6, 8$ assumptions)

  - ∘ Right panel: conventional (solid) c.f. new (dashed) implementation results, $w_N(y)$ FESRs

- Finite t behavior

  - Current-current two-point function

$$C^{\mu\nu}_{V/A}(t) = \sum_{\vec{x}} \langle J^{\nu}_{V/A}(\vec{x},t)(J^{\mu}_{V/A}(0,0))^{\dagger}\rangle$$

  - $J = 0,\, 1$ components: $C^{(1)}_{V/A}(t) = \frac{1}{3}\sum_{k=x,y,z} C^{kk}_{V/A}(t)$, $C^{(0)}_{V/A}(t) = C^{tt}_{V/A}(t)$

  - $Q^2 = 0$-subtracted $J = 0,\, 1$ polarizations

[Bernecker, Meyer Eur. Phys. J. A47 (2011) 47]

$$\Pi^{(J)}_{V/A}(Q^2) - \Pi^{(J)}_{V/A}(0) = \sum_{t} K(Q,t)\, C^{(J)}_{V/A}(t)$$

$$K(Q,t) = \frac{\cos \hat{Q}t - 1}{\hat{Q}^2} + \frac{1}{2}t^2$$

- Resulting $J = 0$, 1, V, A contributions to $\tilde{F}_{w_N}$

$$\tilde{F}^{(J)}_{V/A;w_N} = \lim_{t\to\infty} L^{(J)}_{V/A;w_N}(t)$$

where

$$L^{(J)}_{V/A;\omega_N}(t) = \sum_{l=-t}^{t} w^{(J)}_N(l) C^{(J)}_{V/A}(l)$$

with

$$w^{(1)}_N = \sum_{k=1}^{N} K\left(\sqrt{Q_k^2}, t\right)\left(1 - \frac{2Q_k^2}{m_\tau^2}\right) \mathrm{Res}\left[w_N(s)\right]_{s=-Q_k^2}$$

$$w^{(0)}_N = \sum_{k=1}^{N} K\left(\sqrt{Q_k^2}, t\right) \mathrm{Res}\left[w_N(s)\right]_{s=-Q_k^2}$$

◦ Large t convergence behavior