TOWARDS A COMPETITIVE $|V_{us}|$
DETERMINATION FROM STRANGE HADRONIC $	au$ DECAY DATA AND LATTICE HVPs

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QCD Down Under III

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OUTLINE

• Background/motivation
  
  o *The long-standing FB FESR $\tau V_{us}$ puzzle*
  
  o *Systematic issues in the conventional implementation, a new implementation (and solution) + current practical limitations to this new approach*

• *A new lattice-inclusive $us V+A \tau$ data strategy: dispersive formulation, results, and future prospects*
• The puzzle: $\tau$ vs. non-$\tau$ determinations

| $|V_{us}|$          | Source                                      |
|-------------------|---------------------------------------------|
| 0.2258(9)(?)      | 3-family unitarity, HT14 $|V_{ud}|$        |
| 0.2231(4)$_{exp}(7)_{latt}$ | $K_\ell^3$, 2+1+1 lattice $f_+(0)$            |
| 0.2253(4)$_{exp}(6)_{latt}$ | $\Gamma[K\mu_2]/\Gamma[\pi\mu_2]$, lattice $f_K/f_\pi$ |
| 0.2186(18)$_{exp}(10?)_{th}$ | $\tau$ FB FESRs (HFAG17)                   |

• $\tau$ result is from restrictive “conventional implementation” of more general inclusive FB FESR framework
• Interesting (if true) given other hints of lepton universality breakdown [see e.g. Nature 546 (2017) 227]

  ○ Combined Belle, BaBar, LHCb results for

  \[ R[D^{(*)}] \equiv B[\bar{B}^0 \to D^{(*)+}\tau^-\nu_{\tau}] / B[\bar{B}^0 \to D^{(*)+}\mu^-\nu_{\mu}] \]

  (2nd, 3rd generation quarks; 3rd vs 2nd generation leptons) combined \( \sim 3.9\sigma \) above SM expectations

  ○ LHCb results for

  \[ R_{K^{(*)}} \equiv B[B^+ \to K^{(*)+}\mu^+\mu^-] / B[B^+ \to K^{(*)+}e^+e^-] \]

  (2nd, 3rd generation quarks; 2nd vs 1st generation leptons) \( \sim 2.5\sigma \) below SM expectations

• C.f. non-strange, strange \( \tau \) decays: 3rd generation lepton; 1st, 2nd generation quarks
BASICS: HADRONIC $\tau$ DECAYS IN THE SM

- $R_{ij;V/A} \equiv \Gamma[\tau \rightarrow \nu_\tau \text{hadrons}_{ij;V/A}(\gamma)]/\Gamma[\tau^- \rightarrow \nu_\tau e^-\bar{\nu}_e(\gamma)]$

- With $y_\tau \equiv s/m_\tau^2$, flavor $ij$ decays in SM [Tsai PRD4 (1971) 2821]

$$\frac{dR_{ij;V+A}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[1 - y_\tau\right]^2 \tilde{\rho}_{ij;V+A}(s)$$

$$\tilde{\rho}_{ij;V+A}(s) \equiv \left[(1 + 2y_\tau) \rho_{ij;V+A}^{(J=1)}(s) + \rho_{ij;V+A}^{(J=0)}(s)\right]$$

kinematic weight: $w_{\tau}(y) = (1 - y)^2(1 + 2y)$
THE INCLUSIVE FB $\tau |V_{us}|$ DETERMINATION

- **FESRs for $\Pi = \Pi_{ud-us;V+A}(Q^2)$, $\rho = \rho_{ud-us;V+A}(s)$**: 
  \[
  \int_{s_{th}}^{s_0} ds \, w(s) \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds \, w(s) \Pi(s) \]

- **Experiment**: 
  \[|V_{ij}|^2 \rho_{ij;V/A}^{(0+1)}(s)\text{ from } dR_{ij;V/A}/ds\]

- **$R_{ij;V/A}^w(s_0)$**: re-weighted $R_{ij;V/A}$ analogue 
  \[R_{ij;V/A}^w(s_0) \sim \int_{th}^{s_0} ds \frac{dR_{ij;V/A}}{ds} \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}\]

- **FB differences $\delta R^w(s_0)$** \[\equiv \frac{R_{ud;V+A}^w(s_0)}{|V_{ud}|^2} - \frac{R_{us;V+A}^w(s_0)}{|V_{us}|^2}\]
• FESR, OPE for $\delta R^w(s_0)$, input $|V_{ud}| \Rightarrow$

$$|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{R^w_{ud;V+A}(s_0)} - \left[\delta R^w(s_0)\right]^{OPE}}$$

Self-consistency: $|V_{us}|$ independent of $s_0$, $w$

• **The conventional implementation** [Gamiz et al. JHEP03(2003)060]

  ○ $s_0 = m_\tau^2$, $w = w_\tau$ only [spectral integrals from inclusive $ud$, $us$ BF, but no self-consistency tests]

  ○ $w_\tau$ degree 3 $\Rightarrow$ OPE to $D = 8$

  ○ Strong assumptions re $D = 6$ (VSA), $D = 8$ ($\sim 0$)
- $|V_{us}|$ results from extended variable-$s_0$, -$w$ FB FESR analyses with same $D = 6, 8$ assumptions show [see KM, JZ et al arXiv:1702.01767]

- strong unphysical $s_0$ dependence

- strong unphysical $w$ dependence even at $s_0 = m_T^2$

- apparent convergence toward common value for different $w$ beyond $s_0 = m_T^2$

- Observed unphysical behavior strongly suggests breakdown of $D = 6, 8$ OPE assumptions
AN ALTERNATE FB FESR IMPLEMENTATION

- Theory side

  - No $D > 4$ assumptions: effective condensates $C_{D>4}$ from fits to data *(N.B. requires variable $s_0$)*

  - 3-loop-truncated FOPT $D = 2$, standard $D = 2 + 4$ error estimates [from comparison to lattice data]

  - $C_{2N+2}$ ($D = 2N + 2$ condensate), $|V_{us}|$ both from $w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ FESR ($y = s/s_0$)

  - $|V_{us}|$ from different $w_N$ as self-consistency check
• Experimental input (also for new lattice approach)

  ○ Updated/corrected 2013 ALEPH for $ud$ V+A

  ○ $us$ V+A from sum over exclusive modes

    * $K$ from $K_{\mu 2}$ or $B[\tau \rightarrow K\nu\tau]$

    * $K\pi$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$: BaBar, Belle unit-normalized distributions, BFs

      [Note: HFAG $B[K^-\pi^0] + B[\bar{K}^0\pi^-] = 0.01273(21)$ c.f. 0.01327(48) with additional dispersive $K\pi$ ff constraints [ACL13, JHEP 1310 (2013) 070]]

    * Remaining ("residual modes") from 1999 ALEPH (note: $\sim$ 25% errors, some MC)
NEW FB FESR RESULTS

- With fitted $D > 4$ condensates
  - unphysical $s_0$, $w(y)$-dependence problems resolved
  - $|V_{us}|$ increased by $\sim 0.0020$ ($0.2186(21) \rightarrow 0.2209(23)$)
  - Non-trivial $K\pi$ normalization uncertainty: with alternate ACLP choice, $0.2209(23) \rightarrow 0.2233(28)$ (c.f. $0.2207(27)$ from conventional implementation)

- Very favorable ($\sim 0.0005$) theory error situation

- $us$ spectral integral uncertainty dominates current error
• *Theory error* ⇒ *competitive with* $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$
  with sufficient $u_s$ experimental error improvement

• $u_s$ experimental uncertainties currently BF dominated

• BF's more easily improved experimentally than exclusive mode $dR/ds$ distribution contributions

• Near-term low-multiplicity mode progress likely [combined BaBar, Belle (+Belle II) effort on spectral functions from existing B-factory data under way]

• **However** sub-0.5% $|V_{us}|$ needs sub-% $R_{us;V+A}$ error
- Exclusive us mode $w_N$ spectral integral contributions

Relative exclusive mode $R^w_{us:V+A}$ contributions

<table>
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<tr>
<th>$W_t$</th>
<th>$s_0$ [GeV$^2$]</th>
<th>$K$</th>
<th>$K\pi$</th>
<th>$K\pi\pi$ (B-factory)</th>
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<tr>
<td>$w_2$</td>
<td>2.15</td>
<td>0.496</td>
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- "Other": 1999 ALEPH data/MC, $\sim$ 25% error

$\Rightarrow$ "sufficient improvement" includes experimentally (much) more challenging higher-multiplicity modes
A LATTICE $τ$-BASED ALTERNATIVE


- Basic idea: generalized dispersion relations for products of combination $\tilde{\Pi}$ of $J = 0, 1$ $us$ V+A polarizations with weights having poles at Euclidean $Q^2$
  
  - $\tilde{\Pi}(Q^2)$: polarization sum with spectral function $\tilde{\rho}(s)$ (experimental $dR_{us;V+A}/ds$)
  
  - Theory: Lattice $us$ 2-point function data (no OPE)

  - Weights tunable, allow suppression of larger-error, higher-multiplicity $us$ spectral contributions
More on the lattice-inclusive $us \tau$ approach

- $|V_{us}|^2 \tilde{\rho}_{us;V+A}(s)$ from experimental $dR_{us;V+A}/ds$

$$\tilde{\rho}_{us;V+A}(s) \equiv \left( 1 + 2 \frac{s}{m^2_\tau} \right) \rho_{us;V+A}^{(J=1)}(s) + \rho_{us;V+A}^{(J=0)}(s)$$

(no continuum $us \ J = 0$ subtraction required)

- Associated (kinematic-singularity-free) polarization

$$\tilde{\Pi}_{us;V+A}(Q^2) \equiv \left( 1 - 2 \frac{Q^2}{m^2_\tau} \right) \Pi_{us;V+A}^{(J=1)}(Q^2) + \Pi_{us;V+A}^{(J=0)}(Q^2)$$

- $\tilde{\rho}_{us;V+A}(s) \sim s$ as $s \to \infty$
• For weights \( w_N(s) \equiv \frac{1}{\prod_{k=1}^{N}(s+Q_k^2)} \), \( N \geq 3 \), obtain convergent, unsubtracted 'dispersion relation'

\[
\int_{th}^{\infty} ds \, w_N(s) \tilde{\rho}_{us;V+A}(s) = \sum_{k=1}^{N} \frac{\tilde{\Pi}_{us;V+A}(Q_k^2)}{\Pi_{j \neq k}(Q_j^2 - Q_k^2)} \equiv \tilde{F}_w N
\]

○ Lattice data for \( \tilde{\Pi}_{us;V+A}(Q_k^2) \) on RHS

○ LHS from experimental \( dR_{us;V+A}/ds \), up to \( |V_{us}|^2 \)

○ \( w_N(s) \): rapid fall-off if all \( Q_k^2 < 1 \text{ GeV}^2 \)

\[ \Rightarrow K, K\pi \text{ dominate LHS, near-endpoint multi-particle, } s > m_T^2 \text{ contributions strongly suppressed} \]

○ Optimization: increasing \( \{Q_k^2\} \) decreases RHS lattice error, increases LHS experimental error
• A few details:

  ○ Near-physical-point RBC/UKQCD \( n_f = 2 + 1 \) DWF ensembles

    * \( 48^3 \times 96, \ 1/a = 1.73 \text{ GeV}, \ m_\pi = 0.139 \text{ GeV}, \ m_K = 0.499 \text{ GeV} \)

    * \( 64^3 \times 128, \ 1/a = 2.36 \text{ GeV}, \ m_\pi = 0.139 \text{ GeV}, \ m_K = 0.508 \text{ GeV} \)

    * (Small) retuning to physical valence masses [PQ]

  ○ Time-momentum representation for polarizations in \( \tilde{F}_{w_N} \) sum (back-up slides)
\begin{itemize}
  \item Weighted spectral integrals (include \(|V_{us}|^2\) factor)

  \[
  \tilde{R}_{wN} \equiv \int_0^{m_T^2} \frac{m_T^2}{12\pi^2 S_{SW}(1 - y_T)^2} \frac{dR_{us;V + A(s)}}{ds} w_N(s)ds
  \]

  \[
  \tilde{F}^{pQCD}_{wN} \equiv \int_{m_T^2}^{\infty} [\tilde{\rho}_{us}(s)]^{pQCD} w_N(s)ds
  \]

  \[
  |V_{us}| = \sqrt{\tilde{R}_{us;wN} / [\tilde{F}_{wN} - \tilde{F}^{pQCD}_{wN}]}
  \]

  \item Expect continuum spectral \(A, J = 0\) channel contributions negligible for \(w_N\) employed (confirmed by lattice data), hence exclusive \(A, J = 0\) analysis also possible (using only \(K\) spectral contribution)

  \item \(w_N\) below: uniform pole spacing \(\Delta\), centroid \(C\)
\end{itemize}
Reweighting of exclusive mode distributions

Left panel: un-re-weighted experimental data; right panel: $N = 4$, $C = 0.5 \text{ GeV}^2$ re-weighted version

\[ N = 4, \ C = 0.5 \text{ GeV}^2 \text{ reweighting} \]
High-$s$ spectral suppression, $N = 4$ example

Left panel: relative exclusive mode spectral contributions
Right panel: relative lattice residue contributions
Lattice error breakdown vs. $C$, $N = 4$
Exclusive $A$, $J = 0$ channel determination
Inclusive determinations

Left panel: HFAG $K\pi$ normalization; right panel: alternate, dispersively constrained ACLP $K\pi$ normalization
| $V_{us}$ error budget [%], HFAG $K \pi$ normalization |

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<td>1.43</td>
<td>0.65</td>
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| Combined total      |                  | 0.88    | 1.70  | 0.87    | 0.88    |
INCLUSIVE ANALYSIS RESULTS SUMMARY

• With $B[\tau \rightarrow K\nu_\tau]$ for $K$ pole contribution:
  
  ◦ HFAG $K\pi$ norm’n: $|V_{us}| = 0.2228(14)_{exp}(13)_{th}$
  
  ◦ ACLP $K\pi$ norm’n: $|V_{us}| = 0.2241(18)_{exp}(13)_{th}$

• With $\Gamma[K_{\mu 2}]$ for $K$ pole contribution:
  
  ◦ HFAG $K\pi$ norm’n: $|V_{us}| = 0.2245(10)_{exp}(13)_{th}$
  
  ◦ ACLP $K\pi$ norm’n: $|V_{us}| = 0.2258(15)_{exp}(13)_{th}$
Comparison to results from other methods

With HFAG $K\pi$ normalization

$|V_{us}|$ (HFAG 2016 $K\pi$) c.f. other sources

- $K_{3\pi}$, PDG 2016
- $\Gamma[K_{\mu2}]$
- 3-family unitarity, HT14 $|V_{ud}|$
- $\tau$ FB FESR, HFAG17 (problematic conventional implementation)
- $\tau$ FB FESR, HLMZ17 (new implementation)
- $\tau$, lattice $[N=3, C=0.3\text{ GeV}^2]$
- $\tau$, lattice $[N=4, C=0.7\text{ GeV}^2]$
- $\tau$, lattice $[N=5, C=0.9\text{ GeV}^2]$

This work (preliminary)
With alternate ACLP $K\pi$ normalization

$|V_{us}|$ (ACLP 2013 $K\pi$) c.f. other sources

- $K_{3\gamma}$, PDG 2016
- $\Gamma |K_{\mu2}|$
- 3-family unitarity, HT14 $|V_{ud}|$
- $\tau$ FB FESR, ACLP13 (problematic conventional implementation)
- $\tau$ FB FESR, HLMZ17 (new implementation)
- $\tau$, lattice [N=3, C=0.3 GeV$^2$]
- $\tau$, lattice [N=4, C=0.7 GeV$^2$]
- $\tau$, lattice [N=5, C=0.9 GeV$^2$]

This week (preliminary)
Comments/Prospects

• Lattice analysis confirms larger $|V_{us}|$ from $\tau$ decay data 
  [as per alternate HMLZ FB FESR implementation]

• Significant error reduction from lattice approach c.f. 
  FB FESR determination employing same data

• Theory uncertainty under better control for lattice than 
  for OPE

• Improved suppression of high-$s$, higher-error spectral 
  contributions in lattice approach without blowing up 
  theory errors
• Lattice thus superior to FB FESR approach and should replace it going forward

• Trend to $|V_{us}|$ fall-off for $N = 3$, larger $C$ compatible with missing high-$s$, higher-multiplicity spectral strength (larger impact on FB FESR than lattice results)

• Theory (lattice) errors straightforwardly reducible through improved statistics

• Significant experimental error reduction from improved $K, K\pi\tau$ BFs even without unit-normalized distribution improvements (e.g., Belle II)
SUMMARY

- Old $3\sigma$ low inclusive FB $\tau$ FESR $|V_{us}|$ problem resolved
  - Conventional FB FESR implementation using only inclusive BFs no longer tenable
    - Alternate, no-assumptions implementation: $|V_{us}|$ higher by $\sim 0.0020$, compatible with other determinations
    - Near-term improvements via $u_s$ exclusive BFs
    - Highly favorable theoretical error situation
    - However, competitive $|V_{us}|$ needs improvements to old ALEPH higher-multiplicity, low-statistics data
### Advantage of new lattice-inclusive $us V + A \tau$ approach

- **Theory:**
  * Lattice in place of OPE; no $us J = 0$ subtraction; improvement through increased statistics
  * *Parasitic on lattice $a_\mu$ effort* (a major effort in the lattice community)

- **Spectral integrals:**
  * Theory errors still small for weights strongly suppressing higher multiplicity contributions
  * Strong $K$, $K\pi$ dominance of spectral integral
  * Significant experimental improvements possible through just improved $K\pi$ BFs, distributions
• FB FESR $w$-, $s_0$-stability tests

○ Left panel: conventional implementation $w_T(y)$, $\hat{w}(y) = (1 - y)^3$ results (re $D = 6, 8$ assumptions)

○ Right panel: conventional (solid) c.f. new (dashed) implementation results, $w_N(y)$ FESRs
Finite $t$ behavior

- Current-current two-point function

$$C^{\mu\nu}_{V/A}(t) = \sum_{\vec{x}} \langle J^{\nu}_{V/A}(\vec{x}, t)(J^{\mu}_{V/A}(0, 0))^\dagger \rangle$$

- $J = 0$, 1 components:
  $$C^{(1)}_{V/A}(t) = \frac{1}{3} \sum_{k=x,y,z} C^{kk}_{V/A}(t),$$
  $$C^{(0)}_{V/A}(t) = C^{tt}_{V/A}(t)$$

- $Q^2 = 0$-subtracted $J = 0$, 1 polarizations


$$\Pi^{(J)}_{V/A}(Q^2) - \Pi^{(J)}_{V/A}(0) = \sum_{t} K(Q, t) C^{(J)}_{V/A}(t)$$

$$K(Q, t) = \frac{\cos \hat{Q}t - 1}{\hat{Q}^2} + \frac{1}{2} t^2$$
Resulting $J = 0, 1, V, A$ contributions to $\tilde{F}_{w_N}$

$$\tilde{F}^{(J)}_{V/A;w_N} = \lim_{t \to \infty} L^{(J)}_{V/A;w_N}(t)$$

where

$$L^{(J)}_{V/A;\omega_N}(t) = \sum_{l=-t}^{t} w^{(J)}_{N}(l) C^{(J)}_{V/A}(l)$$

with

$$w^{(1)}_{N} = \sum_{k=1}^{N} K \left( \sqrt{Q_k^2}, t \right) \left( 1 - \frac{2Q_k^2}{m_T^2} \right) \text{Res} \left[ w_N(s) \right]_{s=-Q_k^2}$$

$$w^{(0)}_{N} = \sum_{k=1}^{N} K \left( \sqrt{Q_k^2}, t \right) \text{Res} \left[ w_N(s) \right]_{s=-Q_k^2}$$
Large $t$ convergence behavior

48$^3 \times 96$ lattice data, N=4, C=0.5 [GeV$^2$]