QED effects in lattice hadron spectroscopy

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July 12, 2017

Motivation

- In 1951 the Delta resonance was discovered by Fermi at the new Chicago cyclotron.
- Until 1964 many particles of the hadron spectrum were discovered.
- In 1974 charmed particles were discovered.
- These discoveries would eventually lead to the quark model.
- Experiments in hadron spectroscopy continue today.

However...

- If we look at the Particle Data Groups manual, there are gaps where isospin and charge need to be considered. For example the Δ^-.
- Recent surprising experimental result by SELEX, $\Xi_{cc}^{++} - \Xi_{cc}^{+} \approx 20 MeV$ $(u \leftrightarrow d = 20 MeV?)$
- Our aim is to explore these using lattice QCD+QED.

Isosplitting in the mass spectrum

J	$c = 0$	$c = 1$	$c = 2$	$c = 3$
$\frac{+3}{2}$	Δ^{-} Δ^{0} Δ^{+} Δ^{++} Σ^{-} Σ^{0} Σ^{+} Ξ^{-} Ξ^{0} Ω^{-}	Σ_c^{0} Σ_c^{-} Σ_c^{++} Ξ_c^{0} Ξ_c^{+} Ω_c^{0}	Ξ_{cc}^{+} Ξ_{cc}^{++} Ω_{cc}^{+}	Ω_{ccc}^{++}
sym. $\frac{+1}{2}$	Λ^{0}	Λ_c^{+} Ξ_c^{0} Ξ_c^{+}		
$\frac{+1}{2}$	n p Σ^{-} Σ^{0} Λ^{0} Σ^{+} Ξ^{-} Ξ^{0}	Σ_c^{0} Σ_c^{+} Λ_c^{+} Σ_c^{++} $\Xi_c'^{0}$ $\Xi_c'^{+}$ Ξ_c^{0} Ξ_c^{+} Ω_c^{0}	Ξ_{cc}^{+} Ξ_{cc}^{++} Ω_{cc}^{+}	

Isosplitting in the mass spectrum

J	$c = 0$	$c = 1$	$c = 2$	$c = 3$
$\frac{+3}{2}$	Δ^- Δ^0 Δ^+ Δ^{++} Σ^- Σ^0 Σ^+ Ξ^- Ξ^0 Ω^-	Σ_c^0 Σ_c^- Σ_c^{++} Ξ_c^0 Ξ_c^+ Ω_c^0	Ξ_{cc}^+ Ξ_{cc}^{++} Ω_{cc}^+	Ω_{ccc}^{++}
sym. $\frac{+1}{2}$	Λ^0	Λ_c^+ Ξ_c^0 Ξ_c^+		
$\frac{+1}{2}$	n p Σ^- Σ^0 Λ^0 Σ^+ Ξ^- Ξ^0	Σ_c^0 Σ_c^+ Λ_c^+ Σ_c^{++} $\Xi_c'^0$ $\Xi_c'^+$ Ξ_c^0 Ξ_c^+ Ω_c^0	Ξ_{cc}^+ Ξ_{cc}^{++} Ω_{cc}^+	

Lattice QCD+QED

The action we are using is

$$S = S_G + S_A + S_F^u + S_F^d + S_F^s$$

- where S_G is the tree-level Symanzik improved SU(3) gauge action
- S_A is the non compact U(1) gauge action of the photon
- and S_F^q is the fermion action for each flavour, using a single iterated mild stout smeared link.
- Volumes used are $32^3 \times 64$ and $48^3 \times 96$
- Larger than physical $\alpha_{QED} \approx 10 \times \alpha_{QED}^*$ (* means physical value)

Finite volume corrections

- The periodic boundary conditions modify the energy of the state.
- Though we expect the strong interaction effects to be sub dominant($\exp(-m_\pi L)$), the electromagnetic finite volume (FV) corrections are large enough within our precision to require correcting $(1/L)$.
- The equation used to correct the state energy is [2],

$$M_{corr} = M_{lat}\left[1 + \frac{q^2\alpha_{lat}k}{2M_{lat}L}\left(1 + \frac{2}{M_{lat}L}\left(1 - \frac{\pi T}{2kL}\right)\right)\right]$$

- $k = 2.837297$, α_{lat} is the fine structure constant on the lattice, q is the electron charge on the particle, L is the spatial lattice length and T is the temporal lattice length.

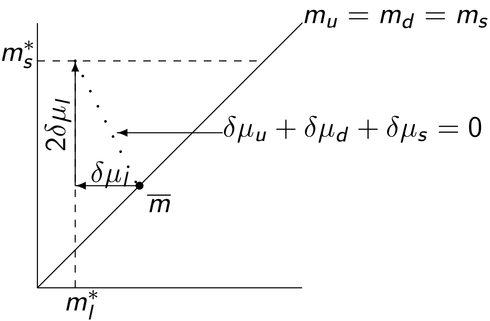
Extrapolating to physical quark mass

Our aim is to determine observables (mass) at the physical quark mass and charge. But...

- Our bare quark masses don't end up the same as our lattice quark masses.
- We don't know what bare quark mass we need to get the right physical mass.
- The physical quark mass is scale dependent, so we can only match to it through observables.
- Even if we did know exactly what the correct bare masses were, light quarks are expensive.

Ultimately, we will have to pick some bare quark masses and extrapolate to the physical point.

Choosing quark masses



This is a picture of the parameter space for observables $M(m_l, m_s)$.

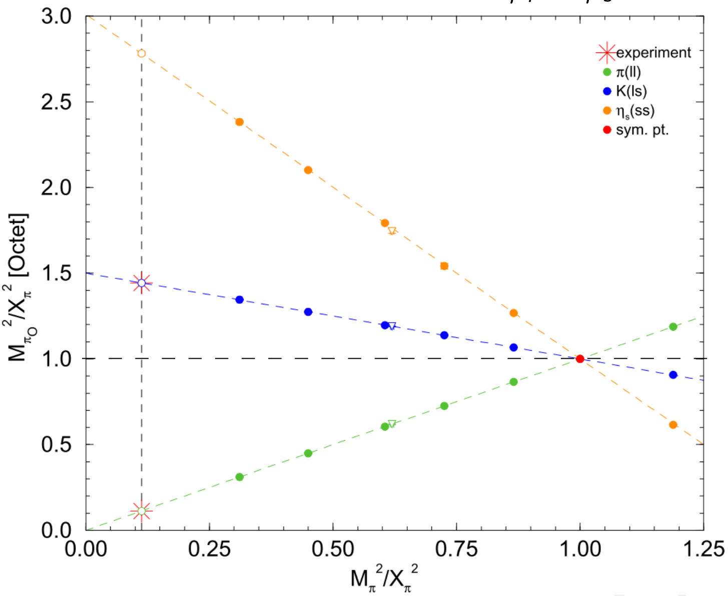
- Typically people start with $m_u = m_d = m_s$ (SU(3)) and work from there.

- In the past people have used $M^2_{s\bar{s}} = 2M_K^{*2} - M_\pi^{*2}$ for $m_s = m_u = m_d$, and reduced the light quark mass, keeping $M^2_{s\bar{s}}$ constant. They would then extrapolate the observable, using a linear fit (assumes analytic).
- However it is better to approach from the direction $\delta\mu_u + \delta\mu_d + \delta\mu_s = 0$ starting at $\overline{m}$.
- I will motivate this by explaining how we extrapolate to the physical point.

Notice symmetries

$$2\delta\mu_l + \delta\mu_s = 0$$

[1]



Analytic expansions

Extrapolate by assuming the function describing an observable is analytic,

$$M_{a\bar{b}}(\delta\mu_a, \delta\mu_b) = M_0^{ab} + \sum_{i=1}^{\infty} \alpha_i^{ab} \delta\mu_a^i + \beta_i^{ab} \delta\mu_b^i + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \gamma_{ij}^{ab} \delta\mu_a^i \delta\mu_b^j$$

- $\delta\mu_q = m_q - \overline{m}$
- If we expand around a point $m_u = m_d = m_s$ in pure QCD, the lattice mass spectrum will satisfy an S_3 symmetry under the flavour labels.
- To see this consider that at that point the quarks are indistinguishable, or consider that the lattice spectrum will satisfy $M_{a\bar{b}}(\delta\mu_1, \delta\mu_2) = M_{c\bar{e}}(\delta\mu_1, \delta\mu_2)$ for $a, b, c, e \in \{u, d, s\}$ and $\delta\mu_1, \delta\mu_2 \in \mathbb{R}$.

Using the symmetry the expansion can be simplified,

$$M_{a\bar{b}} = M_0 + \sum_{i=1}^{\infty} \alpha_i (\delta\mu_a^i + \delta\mu_b^i) + \sum_{i=1}^{\infty} \sum_{j=1}^{i-1} \gamma_{ij} (\delta\mu_a^i \delta\mu_b^j + \delta\mu_b^i \delta\mu_a^j)$$

Notice the coefficients are independent of flavour!

Why simplify the analytic expansions?

- Enforcing these constrains on the expansion reduces the number of coefficients.
- When working to a certain order in the expansion this makes it easier to fit the data and more accurate[1].
- We require less independent data points, and more of the data is used to improve statistics on coefficients.

Extending for charge parameters

The case is no different when we add additional dimensions. The analytic expansion is given by,

$$M_{a\bar{b}}(\delta\mu_a, \delta\mu_b, \delta e_a, \delta e_b) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty} \beta_{ijkm}^{ab}\delta\mu_a^i\delta\mu_b^j\delta e_a^k\delta e_b^m$$

If we expand about a point $m_u = m_d = m_s$ and $e_u = e_d = e_s$, the lattice states satisfy an S_3 symmetry when expanding, and $SU(3)$ at the point.

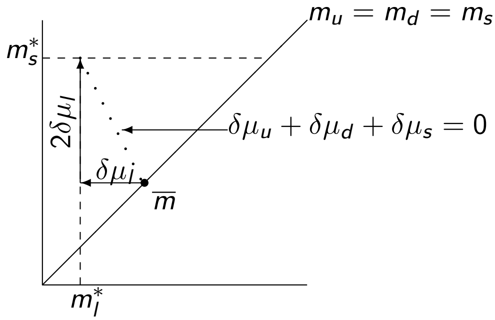
Mass expansions

Applying symmetries and choosing $e_u = e_d = e_s = 0$ as the expansion point. To second order we get $(a, b, c \in \{u, d, s\})$

$$M_{a\bar{b}} \approx M_0 + \alpha_1(\delta\mu_a + \delta\mu_b) + \alpha_2(\delta\mu_a^2 + \delta\mu_b^2) + \gamma_3(\delta\mu_a\delta\mu_b) + \beta_1(e_a^2 + e_b^2) + \beta_2(e_a e_b)$$

$$M_{abc} \approx M_0 + a_1(\delta\mu_a + \delta\mu_b + \delta\mu_c) + a_2(\delta\mu_a^2 + \delta\mu_b^2 + \delta\mu_c^2) + a_3(\delta\mu_a\delta\mu_b + \delta\mu_b\delta\mu_c + \delta\mu_a\delta\mu_c) + b_1(e_a^2 + e_b^2 + e_c^2) + b_2(e_a e_b + e_b e_c + e_a e_c)$$

Why choose $\overline{m}$ as the SU(3) point?

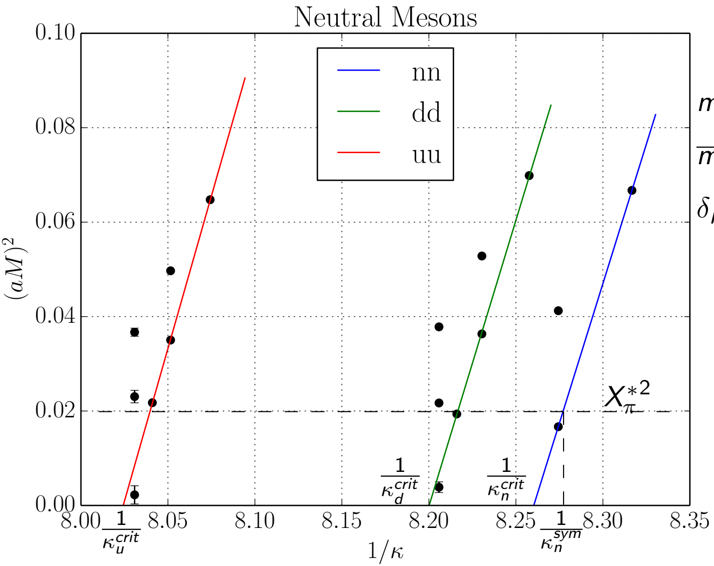


- There is no perfect $SU(3)$ point, however we want to minimise how far we extrapolate. Cancellation is optimal if at physical quark mass $\delta\mu_u + \delta\mu_d + \delta\mu_s = 0$.
- The accuracy of the extrapolation is increased because the linear terms in the expansions are reduced, and these are the most important terms.
- Additionally, singlets remain constant to first order.

Finding the SU(3) point $\overline{m}$

- If we let $m_u = m_d = m_s = (m_u^* + m_d^* + m_s^*)/3$, then we will satisfy $\delta\mu_u + \delta\mu_d + \delta\mu_s = 0$ when approaching the physical point, but we don't know the physical quark mass on the lattice.
- Instead we define it as $m_u = m_d = m_s = m_n$, so that $M_{n\bar{n}}^2 = X_\pi^{*2}$, where n is a quark with no charge, $X_\pi^{*2} = (M_{\pi^+}^{2*} + M_{K^0}^{2*} + M_{K^-}^{2*})/3$
- Think of X_π^* as the flavourless average meson at the physical point and $M_{n\bar{n}}$ as the flavourless (average because its at SU(3)) meson at SU(3).
- Assumed X_π is independent of second order effects (constant along $\delta\mu_u + \delta\mu_d + \delta\mu_s = 0$ path), and charge effects can be ignored.

Finding the SU(3) point $\overline{m}$



$$m_q = \frac{1}{2\kappa_q} - \frac{1}{2\kappa_q^{crit}}$$

$$\overline{m} = \frac{1}{2\kappa_n^{sym}} - \frac{1}{2\kappa_n^{crit}}$$

$$\delta\mu_q = m_q - \overline{m}$$

Extrapolating observables to the physical point

$$M_{a\bar{b}} \approx M_0 + \alpha_1(\delta\mu_a + \delta\mu_b) + \alpha_2(\delta\mu_a^2 + \delta\mu_b^2) + \alpha_3(\delta\mu_a\delta\mu_b) + \alpha_4(e_a^2 + e_b^2) + \alpha_5(e_a e_b)$$

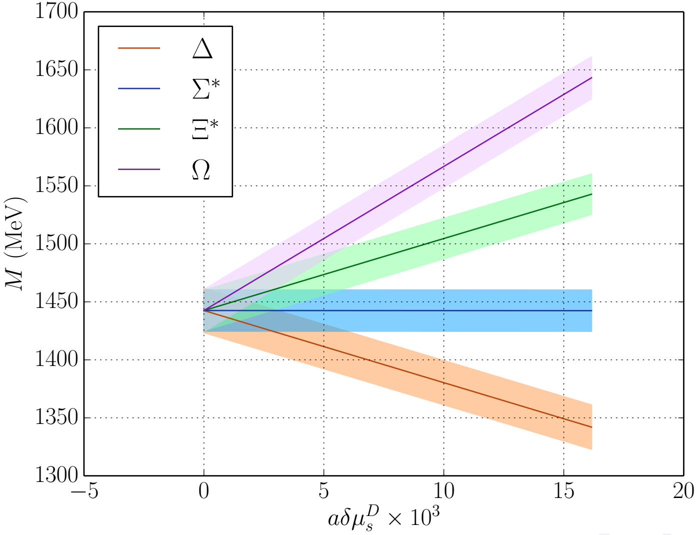
$$\delta\mu_q = m_q - \overline{m} = \frac{1}{2\kappa_q} - \frac{1}{2\kappa_q^{sym}}$$

$$\frac{1}{\kappa_q^{sym}} = \overline{m} + \frac{1}{\kappa_q^{crit}}$$

1. For mesons and baryons pick $\delta\mu$'s and use normal e's $\rightarrow$ plug into lattice to get $M(\delta\mu)$
2. Know $M, \delta\mu \rightarrow$ find α's.
3. Correct EM alpha's for greater than physical α_{QED}
4. Know M^* for mesons, α's $\rightarrow$ find $\delta\mu^*$ which satisfy.
5. Know $\delta\mu^*$ and α's $\rightarrow$ find estimates for baryon M^*.

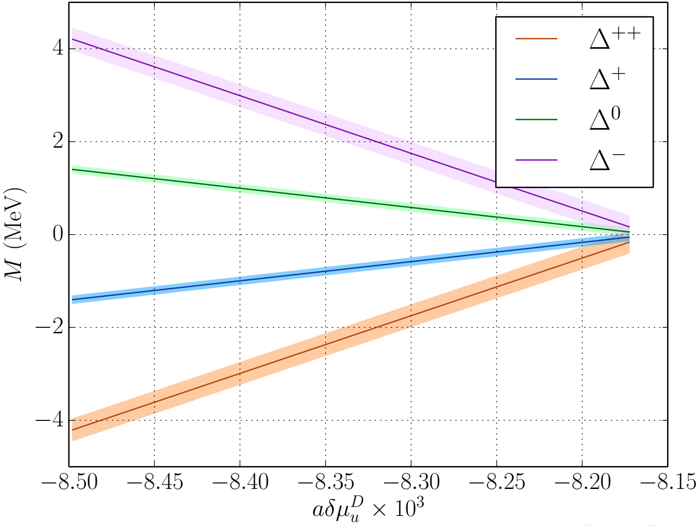
Decuplet (charge removed)

$2\delta\mu_l + \delta\mu_s = 0$



Decuplet result (charge removed)

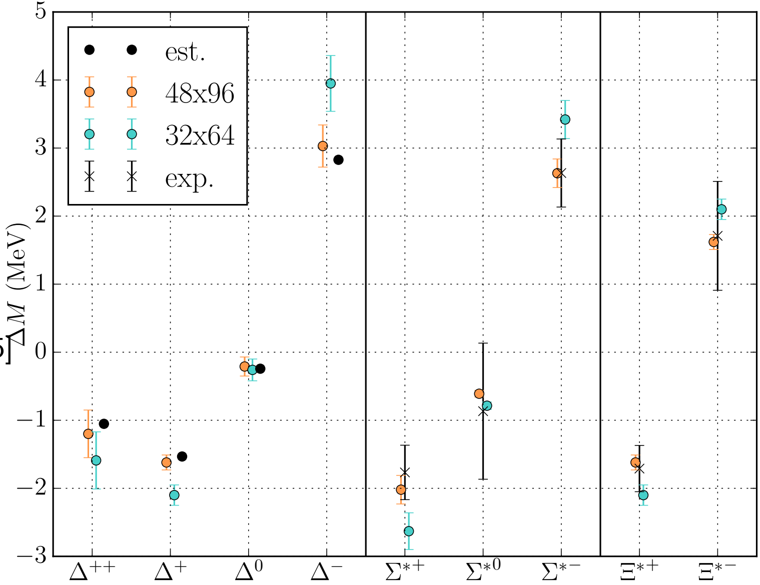
$\delta\mu_u + \delta\mu_d = 0$, at m_l^*, m_s^*



Decuplet result

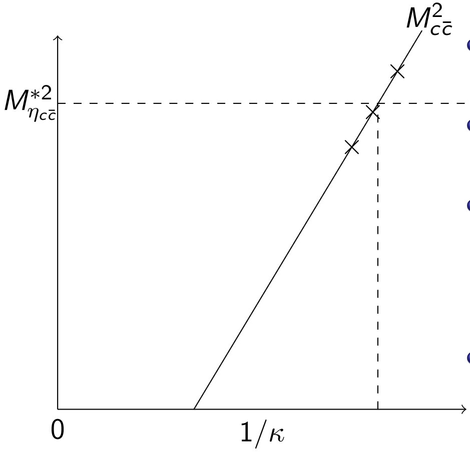
- The next slide shows our key result, however the plot needs to be explained. I do this here.
- Each multiplet has had the average mass of the multiplet removed.
- eg $\Delta M_{\Delta^+} = M_{\Delta^+} - M_{\Delta}$ where $M_{\Delta} = \frac{M_{\Delta^{++}}+M_{\Delta^+}+M_{\Delta^0}+M_{\Delta^-}}{4}$
- This is true for each set of data, meaning the M_{Ξ^0} $48^3 x96$ lattice data has had M_{Ξ} from the $48^3 x96$ data subtracted while the experimental M_{Ξ^0} data has had the experimental M_{Ξ} subtracted.

Decuplet result (charge included)



[3] [5]

Charmed sector extension



- A similar analysis can be done on the charm sector.
- The charm is not included in the sea.
- The lattice simulation is calculated at physical charm quark mass and physical charm quark charge.
- The charm quark mass is determined so that the $\eta_{c\bar{c}}$ has the correct physical mass.

Charmed sector expansions

- It is not necessary to expand around the charm mass as we are already at it's physical point. Due to the symmetries imposed on charge, we do need to include charm charge in the expansion.
- The expansions are similar to those used before

In summary, to second order we get (c denotes charm quark)

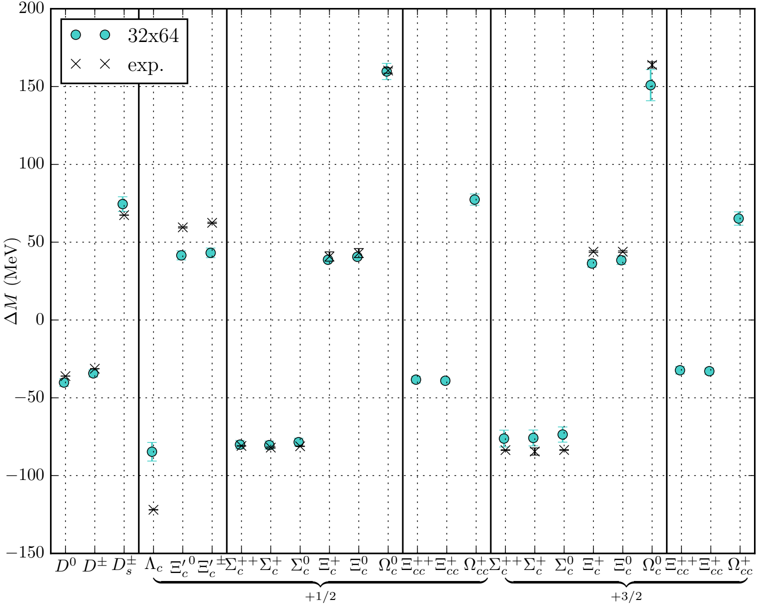
$$\begin{aligned} M_{a\bar{c}} \approx M_0^c + \alpha_1^c(\delta\mu_a) + \alpha_2^c(\delta\mu_a^2) \\ + \beta_1^c(e_a^2 + e_c^2) + \beta_2^c(e_a e_c) \end{aligned}$$

$$\begin{aligned} M_{abc} \approx M_0^c + a_1^c(\delta\mu_a + \delta\mu_b) + a_2^c(\delta\mu_a^2 + \delta\mu_b^2) \\ + a_3^c(\delta\mu_a\delta\mu_b) + b_1^c(e_a^2 + e_b^2 + e_c^2) \\ + b_2^c(e_a e_b + e_b e_c + e_a e_c) \end{aligned}$$

$$\begin{aligned} M_{acc} \approx M_0^{cc} + a_1^{cc}(\delta\mu_a) + a_2^{cc}(\delta\mu_a^2) \\ + b_1^{cc}(e_a^2 + 2e_c^2) + b_2^{cc}(2e_a e_c + e_c e_c) \end{aligned}$$

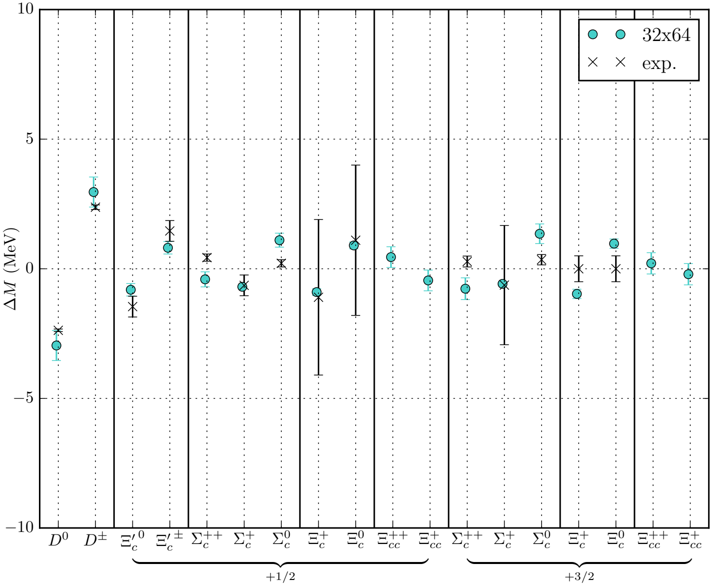
Charmed sector

[5]



Charmed sector (isosplitting)

[5]



Summary

- Motivated by isospin and charge splitting of Δ and Ξ_{cc}
- Extrapolating to the physical point
- Why pick $\overline{m}$ as the expansion point.
- Extension for charmed particles

Key results:

1. Produced estimate for decuplet splitting.
2. Produced estimate for Ξ_{cc} splitting.
3. Showed that we do not agree with SELEX 20MeV prediction.

References

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Lattice QCD+QED

The photon action is,

$$S_A = \frac{1}{2e^2} \sum_{x,\mu<\nu} (A_\mu(x) + A_\nu(x+\mu) - A_\mu(x+\nu) - A_\nu(x))^2$$

The fermion action for each flavour is,

$$S_F^q = \sum_x \Bigg\{ \frac{1}{2} \sum_\mu \Big[\bar{q}(x)(\gamma_\mu - 1) e^{-ie_q A_\mu(x)} \tilde{U}_\mu(x) q(x+\hat{\mu}) - \bar{q}(x)(\gamma_\mu + 1) e^{ie_q A_\mu(x)} \tilde{U}_\mu^\dagger(x-\hat{\mu}) q(x-\hat{\mu}) \Big] + \frac{1}{2\kappa_q} \bar{q}(x) q(x) - \frac{1}{4} c_{SW} \sum_{\mu\nu} \bar{q}(x) \sigma_{\mu\nu} F_{\mu\nu} q(x) \Bigg\}$$

where $\tilde{U}_\mu$ is a single iterated mild stout smeared link. The clover coefficient c_{SW} has been computed non-perturbatively for pure QCD.

Finite volume corrections

- Due to periodic boundary conditions, the average EM field may not be zero over the volume.
- The zero mode of the field can be zeroed on each time slice. Alternatively, the observable can be corrected after calculation.
- For the energy of the state, the correction can be applied as below [4],

$$M_{corr}^2 = M_{lat}^2 - q^2 \overline{B^2}$$

- $\overline{B^2}$ is the zero mode electromagnetic field squared averaged over lattice ensembles and q is the charge of the state in units of electron charge.
- For the octet and decuplet, we corrected the observables at the end. For the charmed particles we removed the zero mode on each time slice.