Landau mode quark operators for the nucleon in a magnetic field

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Collaborators
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Energy Shift in a Magnetic Field

- For small magnetic fields, the shift in energy for a baryon is

\[ E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|q B|}{2M} - \frac{4\pi}{2} \beta B^2 + \mathcal{O}(B^3) \]
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- Magnetic moment \( \mu \) and magnetic polarisability \( \beta \),

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Dirac Particle in a Magnetic Field

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• For a constant background magnetic field $\vec{B} = \nabla \times \vec{A}$,

$$\left( D^2 + q \begin{bmatrix} \vec{\sigma} \cdot \vec{B} & 0 \\ 0 & \vec{\sigma} \cdot \vec{B} \end{bmatrix} + m^2 \right) \psi = 0$$
• Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$(D^2 + \alpha qB + m^2) \psi_j = 0.$$
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H\psi = E\psi, \quad H = \frac{1}{2m} \vec{\Pi}^2 = \frac{1}{2m}(\vec{p} - q\vec{A})^2.
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• Kinetic momenta have a constant commutator,

\[
[\Pi_x, \Pi_y] = [p_x - qA_x, p_y - qA_y] = iqB.
\]
• Define creation and annihilation operators

\[ a = \sqrt{\frac{1}{2qB}}(\Pi_x + i\Pi_y), \quad a^\dagger = \sqrt{\frac{1}{2qB}}(\Pi_x - i\Pi_y) \]

such that \([a, a^\dagger] = 1\).
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- Energy spectrum is equivalent to a harmonic oscillator,

\[ H = \hbar \omega (a^\dagger a + \frac{1}{2}), \quad E = \hbar \omega (n + \frac{1}{2}), \]

where \(\omega = |qB|/m\) is the classical cyclotron frequency.

- Tower of Landau levels

\[ E = \frac{1}{2} \hbar \omega, \quad \frac{3}{2} \hbar \omega, \quad \frac{5}{2} \hbar \omega, \ldots \]

- Each (continuum) Landau level is infinitely degenerate.
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Hadronic Landau Levels

• Canonical momenta $p_x, p_y$ do not commute with $H$. 

• Consider the momentum-projected two-point correlator,
  
$$G(t, \vec{p}) = \sum \vec{x} e^{-i\vec{p} \cdot \vec{x}} \langle \Omega | T\{\chi(t, \vec{x}) \bar{\chi}(0)\} | \Omega \rangle.$$ 

• A charged hadron (such as the proton) in a magnetic field is not an eigenstate of $p_x, p_y$.

• Instead, can project onto the lowest Landau level,
  
$$G(t, \vec{B}, p_z) = \sum \vec{x} \Psi_{\vec{B}}(x, y) e^{-ip_z z} \langle \Omega | T\{\chi(t, \vec{x}) \bar{\chi}(0)\} | \Omega \rangle.$$ 

• Lowest (continuum) Landau level $\Psi_{\vec{B}}(x, y) \sim e^{-|qB|(x^2 + y^2)/4}$.
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Lattice Landau Levels

- Uniform magnetic field (in the z direction) on the lattice,

\[ U_1(x, y) = \exp(-iBy), \]
\[ U_2(x, y) = \begin{cases} 
1, & y < n_y - 1, \\
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- Periodic boundary conditions impose quantisation condition,
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- Degeneracy of lattice Landau modes is field-strength dependent.
- Lowest level has degeneracy equal to the flux quanta \( |k| \).
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Lattice Landau mode, $k = 1$
Lattice Landau modes, $k = 2$
• A hadron of (integer) charge $q$ experiences a field strength 3 times that of the $d$ quark.

• Neutron is charge-neutral, $q = 0$.

• Neutron does not have Landau levels.

• Neutron is a composite object!
Hadronic Landau Levels

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- Project onto the space spanned by the modes associated with the lowest lattice Landau level,

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• Define projection operator onto lowest $n$ eigenmodes

$$P_n = \sum_{i=1}^{n} |\Psi_{i,\vec{B}}\rangle \langle \Psi_{i,\vec{B}}|$$
Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.

- Gaussian-smeared quark source, tuned for $B = 0$ nucleon.

- Landau-projected quark sink.

- Sink projection is applied to quark propagator,
  \[ S'(x, y) = \sum_z P_n(x, z) S(z, y) \]

- $n = |q^f k d|$ modes for lowest Landau level.

- Fix to Landau gauge for QED-only eigenmodes.
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Neutron energy shifts (QED-only projection)

\[ \delta E(B) = -\frac{4\pi}{2} \beta B^2 + \mathcal{O}(B^4) \]
Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.

\[ \sum_{i} |\Psi_i\rangle \langle \Psi_i| \]

This has a similar effect to performing (2D) smearing.

i.e. filtering out high frequency modes.
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\[
\left( G_{\downarrow}(B, t) + G_{\uparrow}(B, t) \right) \\
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\[
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- e.g. spin-averaged energy shift,

$$
\left( \frac{G_{\downarrow}(B+, t) + G_{\uparrow}(B-, t)}{G_{\downarrow}(0, t) + G_{\uparrow}(0, t)} \right) \times \left( \frac{G_{\downarrow}(B-, t) + G_{\uparrow}(B+, t)}{G_{\downarrow}(0, t) + G_{\uparrow}(0, t)} \right)
$$
Proton energy shifts (QED+QCD projection)

\[ \delta E(B) = \frac{|q B|}{2M} - \frac{4\pi}{2} \beta B^2 + \mathcal{O}(B^4) \]
QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!

- Why?
  - Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.
    $\Rightarrow$ Confined quarks cannot have individual Landau levels.
  - Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
  - The addition of background magnetic field alters the physics.
  - 3D spatial symmetry is broken by the $\vec{B}$ field!
  - Competing QED and QCD interactions.
  - Dynamical fermions, but electro-quenched.
  - Magnetic field + topology = chiral magnetic effect?
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  - Captures important physics that standard operators don’t.
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