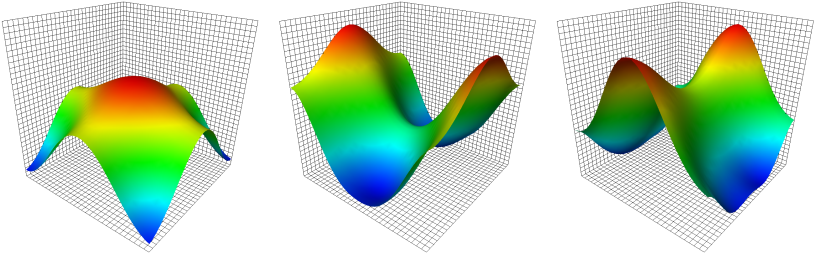
Landau mode quark operators for the nucleon in a magnetic field

Waseem Kamleh

Collaborators
Ryan Bignell, Derek Leinweber, Matthias Burkardt



QCD Down Under
Cairns, Australia, July 10-14, 2017

Energy Shift in a Magnetic Field

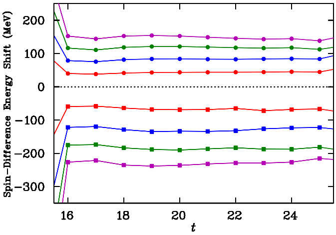
- For small magnetic fields, the shift in energy for a baryon is

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|q\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

Energy Shift in a Magnetic Field

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|q\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

- Magnetic moment μ and magnetic polarisability β,

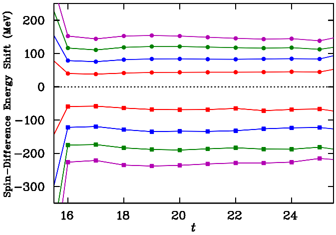


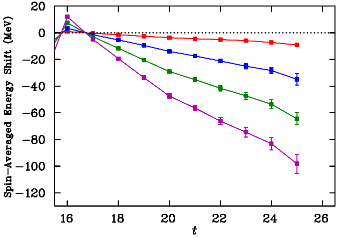
T. Primer, WK, D. Leinweber, M. Burkardt, *Phys.Rev. D89 (2014) no.3, 034508*

Energy Shift in a Magnetic Field

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|q\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

- Magnetic moment μ and magnetic polarisability β,



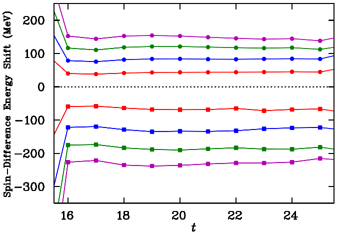


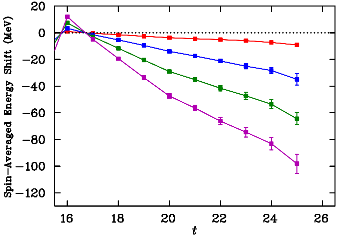
T. Primer, WK, D. Leinweber, M. Burkardt, *Phys.Rev. D89 (2014) no.3, 034508*

Energy Shift in a Magnetic Field

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|q\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

- Magnetic moment μ and magnetic polarisability β,





T. Primer, WK, D. Leinweber, M. Burkardt, *Phys.Rev. D89 (2014) no.3, 034508*

Dirac Particle in a Magnetic Field

- The Dirac operator coupled to electromagnetism is given by

$$\not{D} = \gamma^\mu(\partial_\mu + iqA_\mu)$$

Dirac Particle in a Magnetic Field

- The Dirac operator coupled to electromagnetism is given by

$$\not{D} = \gamma^\mu(\partial_\mu + iqA_\mu)$$

- Yields a second order equation

$$(D^2 + \frac{1}{2}q\sigma^{\mu\nu}F_{\mu\nu} + m^2)\psi = 0$$

Dirac Particle in a Magnetic Field

- The Dirac operator coupled to electromagnetism is given by

$$\not{D} = \gamma^{\mu}(\partial_{\mu} + iqA_{\mu})$$

- Yields a second order equation

$$(D^2 + \frac{1}{2} q\sigma^{\mu\nu}F_{\mu\nu} + m^2)\psi = 0$$

- For a constant background magnetic field $\vec{B} = \vec{\nabla} \times \vec{A}$,

$$\left(D^2 + q\begin{bmatrix} \vec{\sigma}\cdot\vec{B} & 0 \\ 0 & \vec{\sigma}\cdot\vec{B} \end{bmatrix} + m^2\right)\psi = 0$$

Dirac Particle in a Magnetic Field

- Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$\left(D^2 + \alpha q B + m^2\right)\psi_j = 0.$$

Dirac Particle in a Magnetic Field

- Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$\left(D^2 + \alpha q B + m^2\right)\psi_j = 0.$$

- Non-relativistic limit $\psi \sim e^{-imt} \to$ Schrödinger-like equation,

$$H\psi = E\psi, \quad H = \tfrac{1}{2m}\vec{\Pi}^2 = \tfrac{1}{2m}(\vec{p} - q\vec{A})^2.$$

Dirac Particle in a Magnetic Field

- Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$\left(D^2 + \alpha q B + m^2\right)\psi_j = 0.$$

- Non-relativistic limit $\psi \sim e^{-imt} \rightarrow$ Schrödinger-like equation,

$$H\psi = E\psi, \quad H = \tfrac{1}{2m}\vec{\Pi}^2 = \tfrac{1}{2m}(\vec{p} - q\vec{A})^2.$$

    - Spin components related by a linear shift.

Dirac Particle in a Magnetic Field

- Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$\left(D^2 + \alpha q B + m^2\right) \psi_j = 0.$$

- Non-relativistic limit $\psi \sim e^{-imt} \rightarrow$ Schrödinger-like equation,

$$H\psi = E\psi, \quad H = \frac{1}{2m}\vec{\Pi}^2 = \frac{1}{2m}(\vec{p} - q\vec{A})^2.$$

  - Spin components related by a linear shift.

$\Rightarrow$ Eigenfunctions do not depend on spin factor.

Dirac Particle in a Magnetic Field

- Choose $\vec{B} = B\hat{z}$ in the z direction, spin factor $\alpha = \pm 1$,

$$\left(D^2 + \alpha q B + m^2\right) \psi_j = 0.$$

- Non-relativistic limit $\psi \sim e^{-imt} \to$ Schrödinger-like equation,

$$H\psi = E\psi, \quad H = \frac{1}{2m}\vec{\Pi}^2 = \frac{1}{2m}(\vec{p} - q\vec{A})^2.$$

  - Spin components related by a linear shift.

  $\Rightarrow$ Eigenfunctions do not depend on spin factor.

- Kinetic momenta have a constant commutator,

$$[\Pi_x, \Pi_y] = [p_x - qA_x, p_y - qA_y] = iqB.$$

Landau Levels

- Define creation and annhilation operators

$$a = \sqrt{\tfrac{1}{2qB}}(\Pi_x + i\Pi_y), \quad a^\dagger = \sqrt{\tfrac{1}{2qB}}(\Pi_x - i\Pi_y)$$

such that $[a, a^\dagger] = 1$.

Landau Levels

- Define creation and annhilation operators

$$a = \sqrt{\tfrac{1}{2qB}}(\Pi_x + i\Pi_y), \quad a^\dagger = \sqrt{\tfrac{1}{2qB}}(\Pi_x - i\Pi_y)$$

such that $[a, a^\dagger] = 1$.

- Energy spectrum is equivalent to a harmonic oscillator,

$$H = \hbar\omega(a^\dagger a + \tfrac{1}{2}), \quad E = \hbar\omega(n + \tfrac{1}{2}),$$

where $\omega = |qB|/m$ is the classical cyclotron frequency.

Landau Levels

- Define creation and annhilation operators

$$a = \sqrt{\frac{1}{2qB}}(\Pi_x + i\Pi_y), \quad a^\dagger = \sqrt{\frac{1}{2qB}}(\Pi_x - i\Pi_y)$$

  such that $[a, a^\dagger] = 1$.
- Energy spectrum is equivalent to a harmonic oscillator,

$$H = \hbar\omega(a^\dagger a + \tfrac{1}{2}), \quad E = \hbar\omega(n + \tfrac{1}{2}),$$

  where $\omega = |qB|/m$ is the classical cyclotron frequency.
- Tower of Landau levels $E = \frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \ldots$

Landau Levels

- Define creation and annhilation operators

$$a = \sqrt{\tfrac{1}{2qB}}(\Pi_x + i\Pi_y), \quad a^\dagger = \sqrt{\tfrac{1}{2qB}}(\Pi_x - i\Pi_y)$$

  such that $[a, a^\dagger] = 1$.

- Energy spectrum is equivalent to a harmonic oscillator,

$$H = \hbar\omega(a^\dagger a + \tfrac{1}{2}), \quad E = \hbar\omega(n + \tfrac{1}{2}),$$

  where $\omega = |qB|/m$ is the classical cyclotron frequency.

- Tower of Landau levels $E = \frac{1}{2}\hbar\omega, \frac{3}{2}\hbar\omega, \frac{5}{2}\hbar\omega, \ldots$
- Each (continuum) Landau level is infinitely degenerate.

Hadronic Landau Levels

- Canonical momenta p_x, p_y do not commute with H.

Hadronic Landau Levels

- Canonical momenta p_x, p_y do not commute with H.
- Consider the momentum-projected two-point correlator,

$$G(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \Omega | T\{\chi(t, \vec{x})\bar{\chi}(0)\} | \Omega \rangle.$$

Hadronic Landau Levels

- Canonical momenta p_x, p_y do not commute with H.
- Consider the momentum-projected two-point correlator,

$$G(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \Omega | T\{\chi(t, \vec{x})\bar{\chi}(0)\} | \Omega \rangle.$$

- A charged hadron (such as the proton) in a magnetic field is not an eigenstate of p_x, p_y.

Hadronic Landau Levels

- Canonical momenta p_x, p_y do not commute with H.
- Consider the momentum-projected two-point correlator,

$$G(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle\Omega| T\{\chi(t,\vec{x})\bar{\chi}(0)\}|\Omega\rangle.$$

- A charged hadron (such as the proton) in a magnetic field is not an eigenstate of p_x, p_y.
- Instead, can project onto the lowest Landau level,

$$G(t,\vec{B},p_z) = \sum_{\vec{x}} \Psi_{\vec{B}}(x,y) e^{-ip_z z} \langle\Omega| T\{\chi(t,\vec{x})\bar{\chi}(0)\}|\Omega\rangle.$$

Hadronic Landau Levels

- Canonical momenta p_x, p_y do not commute with H.
- Consider the momentum-projected two-point correlator,

$$G(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle \Omega | T\{\chi(t, \vec{x}) \bar{\chi}(0)\} | \Omega \rangle.$$

- A charged hadron (such as the proton) in a magnetic field is not an eigenstate of p_x, p_y.
- Instead, can project onto the lowest Landau level,

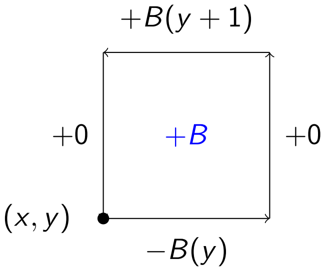
$$G(t, \vec{B}, p_z) = \sum_{\vec{x}} \Psi_{\vec{B}}(x, y) e^{-ip_z z} \langle \Omega | T\{\chi(t, \vec{x}) \bar{\chi}(0)\} | \Omega \rangle.$$

- Lowest (continuum) Landau level $\Psi_{\vec{B}}(x, y) \sim e^{-|qB|(x^2+y^2)/4}$

Lattice Landau Levels

- Uniform magnetic field (in the z direction) on the lattice,

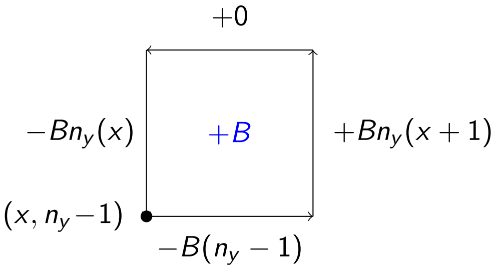
$$U_1(x,y) = \exp(-iBy),$$
$$U_2(x,y) = \begin{cases} 1, & y < n_y - 1, \\ \exp(+iBn_y x), & y = n_y - 1. \end{cases}$$



Lattice Landau Levels

- Uniform magnetic field (in the z direction) on the lattice,

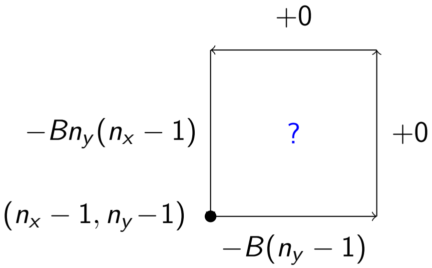
$$U_1(x,y) = \exp(-iBy),$$
$$U_2(x,y) = \begin{cases} 1, & y < n_y - 1, \\ \exp(+iBn_y x), & y = n_y - 1. \end{cases}$$



Lattice Landau Levels

- Uniform magnetic field (in the z direction) on the lattice,

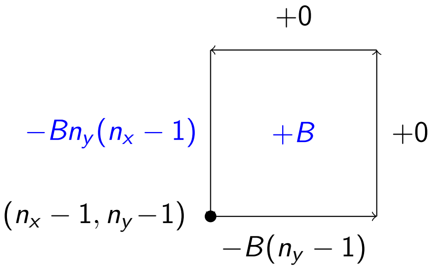
$$U_1(x,y) = \exp(-iBy),$$
$$U_2(x,y) = \begin{cases} 1, & y < n_y - 1, \\ \exp(+iBn_y x), & y = n_y - 1. \end{cases}$$



Lattice Landau Levels

- Uniform magnetic field (in the z direction) on the lattice,

$$U_1(x,y) = \exp(-iBy),$$
$$U_2(x,y) = \begin{cases} 1, & y < n_y - 1, \\ \exp(+iBn_y x), & y = n_y - 1. \end{cases}$$



Lattice Landau Levels

- Periodic boundary conditions impose quantisation condition,

$$q\,B\,a^2 = \frac{2\,\pi\,k}{n_x\,n_y}.$$

Lattice Landau Levels

- Periodic boundary conditions impose quantisation condition,

$$q\,B\,a^2 = \frac{2\,\pi\,k}{n_x\,n_y}.$$

- Lattice Landau levels are eigenmodes of the 2D Laplacian,

$$\Delta_{\vec{x},\vec{x}'} = 4\delta_{\vec{x},\vec{x}'} - \sum_{\mu=1,2} \left(U_\mu(\vec{x})\delta_{\vec{x}+\hat{\mu},\vec{x}'} + U_\mu^\dagger(\vec{x}-\hat{\mu})\delta_{\vec{x}-\hat{\mu},\vec{x}'} \right)$$

Lattice Landau Levels

- Periodic boundary conditions impose quantisation condition,

$$q\,B\,a^2 = \frac{2\,\pi\,k}{n_x\,n_y}.$$

- Lattice Landau levels are eigenmodes of the 2D Laplacian,

$$\Delta_{\vec{x},\vec{x}'} = 4\delta_{\vec{x},\vec{x}'} - \sum_{\mu=1,2} \left(U_\mu(\vec{x})\delta_{\vec{x}+\hat{\mu},\vec{x}'} + U_\mu^\dagger(\vec{x}-\hat{\mu})\delta_{\vec{x}-\hat{\mu},\vec{x}'} \right)$$

- Degeneracy of lattice Landau modes is field-strength dependent.

Lattice Landau Levels

- Periodic boundary conditions impose quantisation condition,

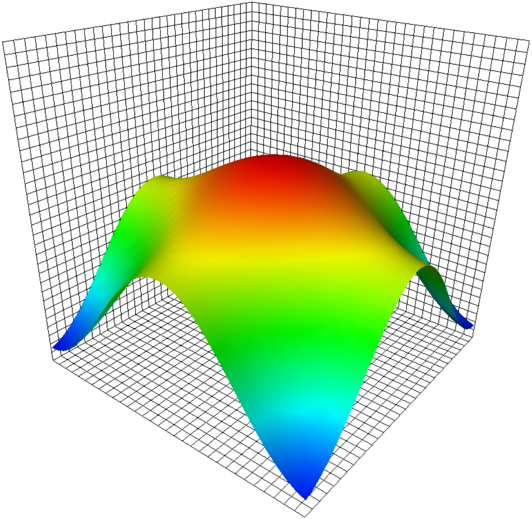
$$q\,B\,a^2 = \frac{2\,\pi\,k}{n_x\,n_y}.$$

- Lattice Landau levels are eigenmodes of the 2D Laplacian,

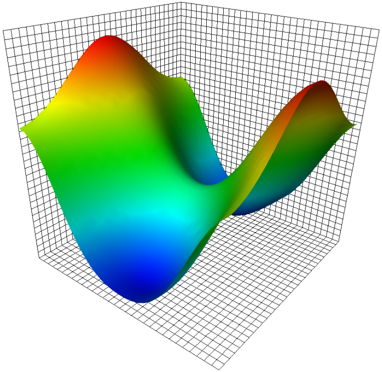
$$\Delta_{\vec{x},\vec{x}'} = 4\delta_{\vec{x},\vec{x}'} - \sum_{\mu=1,2}\left(U_\mu(\vec{x})\delta_{\vec{x}+\hat{\mu},\vec{x}'} + U^\dagger_\mu(\vec{x}-\hat{\mu})\delta_{\vec{x}-\hat{\mu},\vec{x}'}\right)$$

- Degeneracy of lattice Landau modes is field-strength dependent.
- Lowest level has degeneracy equal to the flux quanta $|k|$.

Lattice Landau mode, $k = 1$



Lattice Landau modes, $k = 2$



Hadronic Landau Levels

- A hadron of (integer) charge q experiences a field strength 3 times that of the d quark.

Hadronic Landau Levels

- A hadron of (integer) charge q experiences a field strength 3 times that of the d quark.
- Project onto the space spanned by the modes associated with the lowest lattice Landau level,

$$G(t, k_d, p_z) = \sum_{\vec{x}} \sum_{j=1}^{|3qk_d|} \Psi_{j,\vec{B}}(x, y) e^{-ip_z z} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle.$$

Hadronic Landau Levels

- A hadron of (integer) charge q experiences a field strength 3 times that of the d quark.
- Project onto the space spanned by the modes associated with the lowest lattice Landau level,

$$G(t, k_d, p_z) = \sum_{\vec{x}} \sum_{j=1}^{|3qk_d|} \Psi_{j,\vec{B}}(x, y) e^{-ip_z z} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle.$$

- Neutron is charge-neutral, $q = 0$.

Hadronic Landau Levels

- A hadron of (integer) charge q experiences a field strength 3 times that of the d quark.
- Project onto the space spanned by the modes associated with the lowest lattice Landau level,

$$G(t, k_d, p_z) = \sum_{\vec{x}} \sum_{j=1}^{|3qk_d|} \Psi_{j,\vec{B}}(x,y) e^{-ip_z z} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle.$$

- Neutron is charge-neutral, $q = 0$.
  - Neutron does not have Landau levels.

Hadronic Landau Levels

- A hadron of (integer) charge q experiences a field strength 3 times that of the d quark.
- Project onto the space spanned by the modes associated with the lowest lattice Landau level,

$$G(t, k_d, p_z) = \sum_{\vec{x}} \sum_{j=1}^{|3qk_d|} \Psi_{j,\vec{B}}(x, y) e^{-ip_z z} \langle \Omega | T\{\chi(x)\bar{\chi}(0)\} | \Omega \rangle.$$

- Neutron is charge-neutral, $q = 0$.
  - Neutron does not have Landau levels.
- Neutron is a composite object!

Quark Landau Levels

- In the absence of QCD, quarks have individual Landau levels.

Quark Landau Levels

- In the absence of QCD, quarks have individual Landau levels.
  - At small field strengths, Landau levels are closely spaced.

Quark Landau Levels

- In the absence of QCD, quarks have individual Landau levels.
  - At small field strengths, Landau levels are closely spaced.
  - Takes longer in Euclidean time for levels above ground state to be exponentially suppressed.

Quark Landau Levels

- In the absence of QCD, quarks have individual Landau levels.
  - At small field strengths, Landau levels are closely spaced.
  - Takes longer in Euclidean time for levels above ground state to be exponentially suppressed.
- To what extent do Landau mode effects remain in the presence of QCD?

Quark Landau Levels

- In the absence of QCD, quarks have individual Landau levels.
  - At small field strengths, Landau levels are closely spaced.
  - Takes longer in Euclidean time for levels above ground state to be exponentially suppressed.
- To what extent do Landau mode effects remain in the presence of QCD?
- Define projection operator onto lowest n eigenmodes

$$P_n = \sum_{i=1}^{n} \left|\Psi_{i,\vec{B}}\right\rangle \left\langle\Psi_{i,\vec{B}}\right|$$

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.
    - Gaussian-smeared quark source, tuned for $B = 0$ nucleon.

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.
  - Gaussian-smeared quark source, tuned for $B = 0$ nucleon.
  - Landau-projected quark sink.

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.
  - Gaussian-smeared quark source, tuned for $B = 0$ nucleon.
  - Landau-projected quark sink.
- Sink projection is applied to quark propagator,

$$S'(x, y) = \sum_z P_n(x, z)\, S(z, y)$$

Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.
  - Gaussian-smeared quark source, tuned for $B = 0$ nucleon.
  - Landau-projected quark sink.
- Sink projection is applied to quark propagator,

$$S'(x, y) = \sum_z P_n(x, z)\, S(z, y)$$

- $n = |3\, q_f\, k_d|$ modes for lowest Landau level.

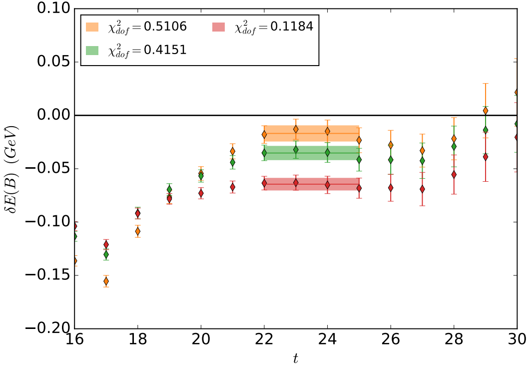
Landau mode projection (QED-only)

- Standard quark operator uses Gaussian spatial smearing.
- Try to capture balance between competing QED and QCD effects.
  - Gaussian-smeared quark source, tuned for $B = 0$ nucleon.
  - Landau-projected quark sink.
- Sink projection is applied to quark propagator,

$$S'(x, y) = \sum_{z} P_n(x, z)\, S(z, y)$$

- $n = |3\, q_f\, k_d|$ modes for lowest Landau level.
- Fix to Landau gauge for QED-only eigenmodes.

Neutron energy shifts (QED-only projection)



$$\delta E(B) = -\frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^4\right)$$

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.
- Add QCD gauge fields to the 2D Laplacian.

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.
- Add QCD gauge fields to the 2D Laplacian.
  - Eigenmodes are QCD covariant (no need to gauge fix!)

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.
- Add QCD gauge fields to the 2D Laplacian.
  - Eigenmodes are QCD covariant (no need to gauge fix!)
- Projector P_n is then a truncation of the completeness relation,

$$1 = \sum_i |\Psi_i\rangle\langle\Psi_i|$$

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.
- Add QCD gauge fields to the 2D Laplacian.
  - Eigenmodes are QCD covariant (no need to gauge fix!)
- Projector P_n is then a truncation of the completeness relation,

$$1 = \sum_i |\Psi_i\rangle\langle\Psi_i|$$

- This has a similar effect to performing (2D) smearing.

Laplacian mode projection (QED+QCD)

- Hadron correlation function is gauge invariant.
  - Gauge fixing can only affect overlap with ground state.
- Add QCD gauge fields to the 2D Laplacian.
  - Eigenmodes are QCD covariant (no need to gauge fix!)
- Projector P_n is then a truncation of the completeness relation,

$$1 = \sum_i |\Psi_i\rangle\langle\Psi_i|$$

- This has a similar effect to performing (2D) smearing.
  - i.e. filtering out high frequency modes.

Laplacian mode projection (QED+QCD)

- QCD mixes modes between the different Landau levels.

Laplacian mode projection (QED+QCD)

- QCD mixes modes between the different Landau levels.
    - Can no longer identify lowest Landau level modes.

Laplacian mode projection (QED+QCD)

- QCD mixes modes between the different Landau levels.
  - Can no longer identify lowest Landau level modes.
- Choose $n > qk$ modes to project.

Laplacian mode projection (QED+QCD)

- QCD mixes modes between the different Landau levels.
  - Can no longer identify lowest Landau level modes.
- Choose $n > qk$ modes to project.
- Laplacian projector is well defined at $B = 0$.

Laplacian mode projection (QED+QCD)

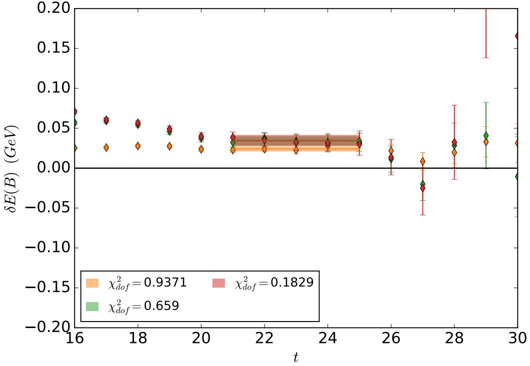
- QCD mixes modes between the different Landau levels.
  - Can no longer identify lowest Landau level modes.
- Choose $n > qk$ modes to project.
- Laplacian projector is well defined at $B = 0$.
- Ratio of correlated fluctuations provides improved signal.

Laplacian mode projection (QED+QCD)

- QCD mixes modes between the different Landau levels.
    - Can no longer identify lowest Landau level modes.
- Choose $n > qk$ modes to project.
- Laplacian projector is well defined at $B = 0$.
- Ratio of correlated fluctuations provides improved signal.
- e.g. spin-averaged energy shift,

$$\left(\frac{G_{\downarrow}(B+,t) + G_{\uparrow}(B-,t)}{G_{\downarrow}(0,t) + G_{\uparrow}(0,t)} \right) \times \left(\frac{G_{\downarrow}(B-,t) + G_{\uparrow}(B+,t)}{G_{\downarrow}(0,t) + G_{\uparrow}(0,t)} \right)$$

Proton energy shifts (QED+QCD projection)



$$\delta E(B) = \frac{|q\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^4\right)$$

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.

  $\Rightarrow$ Confined quarks cannot have individual Landau levels.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.
  - $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.
  - $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
- The addition of background magnetic field alters the physics.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.

  $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
- The addition of background magnetic field alters the physics.
  - 3D spatial symmetry is broken by the $\vec{B}$ field!

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.
  - $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
- The addition of background magnetic field alters the physics.
  - 3D spatial symmetry is broken by the $\vec{B}$ field!
  - Competing QED and QCD interactions.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.

  $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
- The addition of background magnetic field alters the physics.
  - 3D spatial symmetry is broken by the $\vec{B}$ field!
  - Competing QED and QCD interactions.
  - Dynamical fermions, but electro-quenched.

QED vs QCD

- Laplacian (QED/QED+QCD) eigenmode projectors work!
  - Why?
- Want small $\vec{B}$ field strengths for perturbative energy expansion to be valid.
  - In confined phase of QCD.

  $\Rightarrow$ Confined quarks cannot have individual Landau levels.
- Nonetheless, effects of $\vec{B}$ field on quark distribution in the nucleon appear to be significant.
- The addition of background magnetic field alters the physics.
  - 3D spatial symmetry is broken by the $\vec{B}$ field!
  - Competing QED and QCD interactions.
  - Dynamical fermions, but electro-quenched.
  - Magnetic field + topology = chiral magnetic effect?

Summary

- Defined three different lattice Landau mode projectors:

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.
  - Quark propagator QED Landau mode projection.

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.
  - Quark propagator QED Landau mode projection.
  - Quark propagator QED+QCD Laplacian mode projection.

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.
  - Quark propagator QED Landau mode projection.
  - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.
  - Quark propagator QED Landau mode projection.
  - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:
  - Enables formation of plateaus in energy shift ratios.

Summary

- Defined three different lattice Landau mode projectors:
  - Hadronic correlator Landau mode projection.
  - Quark propagator QED Landau mode projection.
  - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:
  - Enables formation of plateaus in energy shift ratios.
  - Captures important physics that standard operators don't.

Summary

- Defined three different lattice Landau mode projectors:
    - Hadronic correlator Landau mode projection.
    - Quark propagator QED Landau mode projection.
    - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:
    - Enables formation of plateaus in energy shift ratios.
    - Captures important physics that standard operators don't.
    - Cannot rely on Euclidean time evolution alone for baryonic physics.

Summary

- Defined three different lattice Landau mode projectors:
    - Hadronic correlator Landau mode projection.
    - Quark propagator QED Landau mode projection.
    - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:
    - Enables formation of plateaus in energy shift ratios.
    - Captures important physics that standard operators don't.
    - Cannot rely on Euclidean time evolution alone for baryonic physics.
- For the first time, we can provide precision determinations of magnetic polarisabilities!

Summary

- Defined three different lattice Landau mode projectors:
    - Hadronic correlator Landau mode projection.
    - Quark propagator QED Landau mode projection.
    - Quark propagator QED+QCD Laplacian mode projection.
- Quark level sink projection plus Gaussian-smeared source:
    - Enables formation of plateaus in energy shift ratios.
    - Captures important physics that standard operators don't.
    - Cannot rely on Euclidean time evolution alone for baryonic physics.
- For the first time, we can provide precision determinations of magnetic polarisabilities!
    - See next talk by Ryan Bignell.