TRANSVERSE SPIN STRUCTURE OF BARYONS USING LATTICE QCD

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CONTENT

• Motivation

• Spin Density

• Solving Three-point functions

• Form Factors

• Flavour Symmetry Breaking Expansion
MOTIVATION

• To look at the internal structure of baryons and view the positional density of the quarks depending on the spin polarisation of the quarks and nucleon in the infinite momentum frame, where the only remaining momentum is in the transverse x-y plane.
SPIN DENSITY
TRANSVERSE SPIN DENSITY

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2} \{ A_{10}(b_\perp^2) + s^i S^i_\perp \left( A_{T10}(b_\perp^2) - \frac{1}{4m^2} \Delta b_\perp \tilde{A}_{T10}(b_\perp^2) \right) + \frac{b^i_\perp \epsilon^{iji}}{m} \left( S^i_\perp B'_{10}(b_\perp^2) + s^i_\perp B'_{T10}(b_\perp^2) \right) + s^i_\perp \left( 2b^i_\perp b_\perp^j - b^2_\perp \delta^{ij} \right) S^j_\perp \frac{1}{m^2} \tilde{A}^{''}_{T10}(b_\perp^2) \}$$

- Where $b_\perp$ is the distance from the centre of momentum
- $s_\perp$ is the transverse spin of the quark
- $S_\perp$ is the transverse spin of the nucleon
TRANSVERSE SPIN DENSITY

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

$$
\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dxx^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)
$$

$$
= \frac{1}{2} \{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) 
+ \frac{b_\perp^i \epsilon^{ij}}{m} \left( S_\perp^i B_{n0}(b_\perp^2) + s_\perp^i \tilde{B}_{Tn0}(b_\perp^2) \right) + s_\perp^i \left( 2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij} \right) S_\perp^j \frac{1}{m^2} \tilde{A}_{Tn0}''(b_\perp^2) \} \}
$$

Unpolarised
TRANSVERSE SPIN DENSITY

In order to determine the spin density, we require each of the following form factors in terms of \( b_\perp \)

\[
\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dxx^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)
\]

\[
= \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp S_\perp \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right\}
\]

\[
+ \frac{b_\perp^i \epsilon^{ii}}{m} \left( S_\perp B_{n0}'(b_\perp^2) + s_{\perp} \tilde{B}_{Tn0}'(b_\perp^2) \right) + s_\perp \left( 2b_\perp b_\perp^i - b_\perp^2 \delta^{ij} \right) S_\perp \frac{1}{m^2} \tilde{A}_{Tn0}''(b_\perp^2) \}
\]

Hadron Spin Polarisation
TRANSVERSE SPIN DENSITY

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

$$
\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)
$$

$$
= \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp s^i_\perp \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right\}
$$

$$
+ \frac{b^i_\perp \epsilon^{ji}}{m} \left( S^i_\perp B'_{n0}(b_\perp^2) + s^i_\perp \overline{B}'_{Tn0}(b_\perp^2) \right) + s_\perp \left( 2b^i_\perp b^j_\perp - b_\perp^2 \delta^{ij} \right) S^i_\perp \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\}
$$

Quark Spin Polarisation
EM FORM FACTORS

$A_{n0}$ and $B_{n0}$ are the general forms of the Electromagnetic Form Factors given by the equation

$$\langle B(p', s') \mid j_\mu(q) \mid B(p, s) \rangle = \bar{u}(p', s') \left[ \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B} F_2(Q^2) \right] u(p, s)$$

and to solve the left hand side of the equation, we use lattice QCD to calculate three point and two point functions and equate the pre-factors on the right hand side to give us $F_1$ and $F_2$ for varying $Q^2$.
THREE-POINT FUNCTIONS
3-POINT FUNCTION

Create a state at time $t = 0$

$$\overline{\chi_\beta}(0)] \mid \Omega \rangle$$
3-POINT FUNCTION

Insert an operator with momentum $\vec{q}$ at some time $\tau$

$$O(\tau, \vec{x}_1)\overline{\chi}_\beta(0) \right| \Omega \rangle$$
3-POINT FUNCTION

Annihilate the state at a final time $t$

$$\langle \Omega | T \left[ \chi_\alpha(t, \vec{x}_2) O(\tau, \vec{x}_1) \bar{\chi}_\beta(0) \right] | \Omega \rangle$$

By performing all possible wick contractions
3-POINT FUNCTION

We can construct a ratio of 2-point and 3-point functions

\[ R(t, \tau; \vec{p}, \vec{p}) = \frac{C_{3pt}(t, \tau; \vec{p}, \vec{p})}{C_{2pt}(t, \tau; \vec{p}, \vec{p})} \times \left[ \frac{C_{2pt}(\tau, \vec{p}''t)C_{2pt}(t, \vec{p}'t)C_{2pt}(t-\tau, \vec{p})}{C_{2pt}(\tau, \vec{p})C_{2pt}(t, \vec{p})C_{2pt}(t-\tau, \vec{p}'t)} \right]^{\frac{1}{2}} \]

To solve the left hand side of our Electromagnetic Form Factor equation

\[ \langle B(p', s') | j_\mu(q) | B(p, s) \rangle = \bar{u}(p', s') \left[ \gamma_\mu F_1(Q^2) + \frac{i \sigma_{\mu\nu} q^\nu}{2m_B} F_2(Q^2) \right] u(p, s) \]
BARYON OCTET

Using $n_f = 2 + 1$ flavour configurations, allows us to simulate the outer octet baryons, with doubly and singly represented light and heavy quarks.
LATTICE QCD SET-UP

- $N_f = 2 + 1 \ O(a)$—improved Clover Fermions

- Novel method for tuning the quark masses
  - Keep the singlet quark mass fixed
    \[
    \overline{m}^R = \frac{1}{3} (2m_l^R + m_s^R)
    \]
  - At its physical value $\overline{m}^{R*}$

- Use multiple Lattice sizes including
  \[24^3 \times 48, \ 32^3 \times 64, \ 48^3 \times 96\]
E-M FORM FACTORS
The Electromagnetic form factors are in the form of the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$, and are obtained from the composition of the matrix elements from the electromagnetic current $j_\mu$ where

$$\langle B(p', s') | j_\mu(q) | B(p, s) \rangle = \bar{u}(p', s') \left[ \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B} F_2(Q^2) \right] u(p, s)$$

and $u(p, s)$ are Dirac spinors with momentum $p$ and spin polarisation $s$, the transfer momentum $q = p' - p$ and $Q^2 = -q^2$ and the mass of the baryon $m_B$. After solving for $F_1$ and $F_2$ we then fit them against a dipole fit.
Dirac and Pauli Form Factors

\[ F_1 = A_{10} \text{ Dirac Form Factor} \quad m_\pi = 465 \text{MeV} \]
Dirac and Pauli Form Factors

$m_\pi = 465 MeV$

$F_1 = A_{10}$ Dirac Form Factor

$F_2 = B_{10}$ Pauli Form Factor

Doubly-represented quark contributions

Singly-represented quark contributions
TENSOR FORM FACTORS
TENSOR FORM FACTORS

Similar to the Electromagnetic form factors, we calculate the tensor form factors using

\[ \langle P', \Lambda' | \bar{\psi}(0)i\sigma^{\mu\nu}\psi(0) | P\Lambda \rangle = \bar{U}(P', \Lambda') \left\{ i\sigma^{\mu\nu}A_{T10}(t) + \frac{\bar{P}[\mu\Delta^{\nu}]}{m^2}\tilde{A}_{T10}(t) + \frac{\gamma[\mu\bar{P}^{\nu}]}{2m}B_{T10}(t) \right\} U(P, \Lambda) \]

where \( \gamma[\mu\bar{P}^{\nu}] = \gamma^{\mu}\bar{P}^{\nu} - \gamma^{\nu}\bar{P}^{\mu} \), \( \Delta = P' - P \), \( \bar{P} = \frac{P' + P}{2} \) and \( i\sigma^{\mu\nu} = i\gamma^{\mu}\gamma^{\nu} \)

Where we again equate the left hand side of the equation with a ratio of two-point and three-point functions.
Tensor Form Factors

\[ A_{T10}(Q^2) = g_T(Q^2) \]

\[ m_\pi = 360 \text{MeV} \]
Tensor Form Factors

\[ \tilde{A}_{T10} \]

\[ m_\pi = 360 \text{MeV} \]

Doubly-represented quark Contribution to the Second Tensor FF \( \tilde{A}^{U}_{T10} \)

Singly-represented quark Contribution to the Second Tensor FF \( \tilde{A}^{D}_{T10} \)
Tensor Form Factors

$$\bar{B}_{T10}(Q^2) = B_{T10}(Q^2) + 2\tilde{A}_{T10} \quad m_\pi = 360 \text{MeV}$$
Now that we have all of the EM and tensor form factors, we use a Fourier transform to change from transverse momentum dependance $Q^2$ to distance $b_\perp$ from the Centre-of-momentum.

$$f(b_\perp) \equiv \int \frac{d^2q}{(2\pi)^2} e^{-ib_\perp \cdot q} f(-q^2),$$

where $Q^2 = -q^2$ is the transverse momentum transferred to the nucleon.

Finally calculating the x-moments of transverse quark spin density.
UNPOLARISED SPIN DENSITY

To remind you of the equation

\[
\rho_n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)
\]

\[
= \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp S_\perp \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right. \\
+ \frac{b_\perp^i \varepsilon^{ji}}{m} \left( S_\perp^i B'_{n0}(b_\perp^2) + s_\perp \overline{B}'_{Tn0}(b_\perp^2) \right) + s_\perp \left( 2b_\perp b_\perp^i - b_\perp^2 \delta^{ij} \right) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\}
\]

Unpolarised
UNPOLARISED

As we expect from unpolarised protons, the up and down quarks are centred around the centre-of-momentum, and are the same due to their $F_1$ dependance.

$u$ - quark

$\ d$ - quark
\[
\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)
\]

\[
= \frac{1}{2} \{ A_{n0}(b_\perp^2) + s_\perp S_\perp \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right)
\]

\[
+ \frac{b_j^i \epsilon^{ji}}{m} \left( S_\perp B'_{n0}(b_\perp^2) + s_\perp \bar{B}'_{Tn0}(b_\perp^2) \right) + s_\perp \left( 2b_j^i b_\perp^j - b_\perp^2 \delta^{ij} \right) S_\perp \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \}\]

Hadron Spin Polarisation
PROTON SPIN POLARISATION

We can see here the results of the density of the up and down quarks in a proton, where the proton has spin polarisation due to the $F_2$ dependance

$u$ - quark

d - quark
\[ \rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) \]

\[ = \frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp S_\perp \right\} \left( A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \]

\[ + \frac{b_j^i \epsilon_j^i}{m} \left( S_\perp B_{n0}(b_\perp^2) + s_\perp \tilde{B}_{Tn0}(b_\perp^2) \right) + s_\perp \left( 2b_j^i b_j^i - b_\perp^2 \delta^{ij} \right) S_\perp \frac{1}{m^2} \tilde{A}_{Tn0}''(b_\perp^2) \right\} \]

Quark Spin Polarisation
By varying the quark spin polarisation, we see that between the up and down quark there is only a relative magnitude difference in their position due to the form of $B_{T10}$.

$u$ - quark

$d$ - quark
SU(3) FLAVOUR SYMMETRY BREAKING EXPANSION

F-fan plots and expansion to physical mass
F-Fan Plot of generalised tensor form factor $A_{T10}(Q^2)$

- Scale X-axis by $X^2_{\pi} = (2M^2_K + M^2_{\pi})/3$
- Scale Y-axis by flavour singlet quantity $X_T$
• Physical point is normalised by $X_T$
• Extrapolate using the lattice values for constant along SU(3) flavour singlets
• Little dependence on the SU(3) flavour breaking and so mild slope is observed
We repeat this method by first binning our results across all our ensembles into six $t = Q^2$ bins and performing the $SU(3)$ flavour breaking expansion of the form factors for each of these bins.
PHYSICAL EXPANSION

Below is the expansion to the physical masses of the doubly represented quark contributions to the first tensor form factor $A_{T10}$ with a dipole fit.
HYPERON SPIN

Using the form factors that have been expanded to the physical mass with nucleon spin polarisation

Proton

Sigma
HYPERON SPIN

Using the form factors that have been expanded to the physical mass with both quark spin and nucleon spin polarisation

Proton

Sigma
IN CONCLUSION

- Electromagnetic and tensor form factors
- SU(3) flavour breaking expansion to physical mass
- Transverse spin densities