Implications of Charge Symmetry Violation in the Determination of Strangeness Form Factors

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Parity-Violating Electron Scattering (PVES)

\[ \gamma e^- P e^- P \sim 2 \text{Re}(M^*_Z M_{\gamma}) / |M_{\gamma}|^2 \]

- \[ A^{PV} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \], Parity violating (PV) asymmetry for the scattering of a polarised electron form unpolarised target defined as the deference over the sum of the cross sections. The signs indicate positive and negative electron helicity.

- The PV asymmetry is determined by the interaction of electromagnetic and neutral weak amplitudes.
The asymmetry arises from terms involving vector coupling of the Z to the electron and axial coupling to the proton, and vice-versa:

\[ A_{PV}^p = \left[ \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} \right] \cdot (A_E + A_M + A_A) \]

\[ A_E = \frac{\epsilon G_E^{\gamma,p} G_E^{Z,p}}{\epsilon(G_E^{\gamma,p})^2 + \tau(G_M^{\gamma,p})^2}, \quad A_M = \frac{\tau G_M^{\gamma,p} G_M^{Z,p}}{\epsilon(G_E^{\gamma,p})^2 + \tau(G_M^{\gamma,p})^2}, \quad A_A = \frac{-\epsilon'(1 - 4\sin^2 \hat{\theta}_W) G_M^{\gamma,p} G_A^{e,p}}{\epsilon(G_E^{\gamma,p})^2 + \tau(G_M^{\gamma,p})^2} \]

where \( \tau = \frac{Q^2}{4M_p^2}, \quad \epsilon = \frac{1}{1 + 2(1 + \tau)\tan^2 \frac{\theta}{2}}, \quad \epsilon' = \sqrt{\tau(1 + \tau)(1 - \epsilon^2)}. \)

For forward angle, \( \theta \to 0 \), the asymmetry \( A_{PV}^p \) is sensitive to \( A_E + A_M \), while at backwards angle, \( \theta \to \pi \), the asymmetry \( A_{PV}^p \) is sensitive to \( A_M + A_A \).
Isolating the Strangeness Vector

The EM form factors, can be written in terms of quarks flavor as:

\[
G_{E,M}^{\gamma,p} = \frac{2}{3} G_{E,M}^{p,u} - \frac{1}{3} \left( G_{E,M}^{p,d} + G_{E,M}^{p,s} \right), \\
G_{E,M}^{\gamma,n} = \frac{2}{3} G_{E,M}^{n,u} - \frac{1}{3} \left( G_{E,M}^{n,d} + G_{E,M}^{n,s} \right).
\]

Neutral weak form factor:

\[
G_{E,M}^{Z,p} = \left( 1 - \frac{8}{3} \sin^2 \theta_w \right) G_{E,M}^{u} - \left( 1 - \frac{4}{3} \sin^2 \theta_w \right) \left( G_{E,M}^{d} + G_{E,M}^{s} \right),
\]

→ Assuming charge symmetry (CS):

\[
G_{E,M}^{p,u} = G_{E,M}^{n,d} = G_{E,M}^{u}, \\
G_{E,M}^{p,d} = G_{E,M}^{n,u} = G_{E,M}^{d},
\]

\[
G_{E,M}^{\gamma,p} = \frac{2}{3} G_{E,M}^{u} - \frac{1}{3} G_{E,M}^{d} - \frac{1}{3} G_{E,M}^{s}, \\
G_{E,M}^{\gamma,n} = \frac{2}{3} G_{E,M}^{d} - \frac{1}{3} G_{E,M}^{u} - \frac{1}{3} G_{E,M}^{s},
\]

\[\Rightarrow G_{E,M}^{Z,p} = (1 - 4 \sin^2 \hat{\theta}_w) G_{E,M}^{\gamma,p} - G_{E,M}^{\gamma,n} - G_{E,M}^{s}.\]
Isolating the strangeness vector

The neutral weak form factor, in terms of the measured electromagnetic form factors $G_{E,M}^{\gamma,p(n)}$ including electroweak radiative correction, is defined as:

$$G_{E,M}^{Z,p(n)} = (1 - 4 \sin^2 \hat{\theta}_w)(1 + R_p^p)G_{E,M}^{\gamma,p(n)} - (1 + R_n^p)G_{E,M}^{\gamma,n(p)} - (1 + R_0)G_{E,M}^s,$$

the values $R_p^p$, $R_n^p$ and $R_0$ can be obtained using the SM predictions for the effective electron-quark couplings $C_{1q}$. $\sin^2 \hat{\theta}_w(M_Z)$ is the weak mixing angle in the $\overline{\text{MS}}$ renormalization scheme at $Z$-pole.
Using $G_{E,M}^Z, p \rightarrow A_{PV}^p = \left[ -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right] \cdot (A_V^p + A_s^p + A_A^p)$

where

$A_V^p = (1 - 4 \sin^2 \hat{\theta}_W)(1 + R_V^p) - (1 + R_V^n) \frac{\epsilon G_{E}^{\gamma,p} G_{E}^{\gamma,n} + \tau G_{M}^{\gamma,p} G_{M}^{\gamma,n}}{\epsilon (G_{E}^{\gamma,p})^2 + \tau (G_{M}^{\gamma,p})^2}$

$A_s^p = -(1 + R_V^{(0)}) \frac{\epsilon G_{E}^{\gamma,p} G_{E}^{\gamma,M} + \tau G_{M}^{\gamma,p} G_{M}^{\gamma,M}}{\epsilon (G_{E}^{\gamma,p})^2 + \tau (G_{M}^{\gamma,p})^2}$

$A_A^p = \frac{-\epsilon'(1-4 \sin^2 \hat{\theta}_W) G_{M}^{\gamma,p} \tilde{G}_{A}^{e,p}}{\epsilon (G_{E}^{\gamma,p})^2 + \tau (G_{M}^{\gamma,p})^2}$,

$G_{A}^{e,p} \rightarrow \tilde{G}_{A}^{e,p}$: where the entire contributions from axial radiative and anapole corrections are to be fit to data:
• Helium-4 PV asymmetry:
  The $^4\text{He}$ scattering is isoscalar transition (no magnetic and axial contributions) so only we have a strange electric contribution,
  
  \[ A_{PV}^{He} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ (1 - 4\sin^2\theta_w)(1 + R_p^V) - (1 + R_n^V) + 2 \frac{(1+R_{V}^{(0)}) G_{E}}{G_{E}^{\gamma,p} + G_{E}^{\gamma,n}} \right] \]

  • Deuteron PV Asymmetry:
    It is, in static approximation, the cross section weighted averaged the PV asymmetries of the proton and neutron, $A_{PV}^d = \frac{\sigma_p A_{PV}^p + \sigma_n A_{PV}^n}{\sigma_p + \sigma_n}$.
    
    In reality the calculation has included nuclear correction (realistic deuteron wave function, rescattering effect, etc ... )
Analysis and Results

- Dataset: We consider a set of all available PV asymmetries data from these experiments \textbf{SAMPLE, PVA4, HAPPEEx, G0, and Q}_{\text{weak}} with varying kinematics and targets (Proton, \text{^4}He, Deuteron ), so we have (35 data points)

- The PV asymmetry that will be fitted to this combined data rewritten in this form: [Kelly parametrisation for EM form factors]

$$A^{PV} = \left[ \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} \right]
\cdot (\eta_0 + \eta_A^p \tilde{G}_A^{e,p} + \eta_A^n \tilde{G}_A^{e,n} + \eta_E G_E^s + \eta_M G_M^s),$$

where \(\eta_0\) is the zero strangeness vector, \(\eta_A^p(n) = \frac{-\epsilon'(1-4\sin^2\hat{\theta}_W)G_M^{\gamma,p(n)}}{\epsilon(G_E^{\gamma,p(n)})^2+\tau(G_M^{\gamma,p(n)})^2}\),

\(\eta_E = -(1 + R_V^{(0)}) \frac{\epsilon G_E^{\gamma}}{\epsilon(G_E^{\gamma})^2}\), \(\eta_M = -(1 + R_V^{(0)}) \frac{\tau G_M^{\gamma}}{\tau(G_M^{\gamma})^2}\) and

$$\tilde{G}_A^N = \tilde{g}_A^N \left(1 + \frac{Q^2}{M_A^2}\right),$$

with the axial dipole mass \(M_A = 1.026\text{ GeV}\).
1- Taylor expansion:
Strange form factors $Q^2$-dependence can be parameterized as $Q^2$ expansion:

$$G_E^s = \rho_s \ Q^2 + \rho'_s \ Q^4$$
$$G_M^s = \mu_s + \mu'_s \ Q^2,$$

where $\rho_s$ is the strange electric radius, $\mu_s \equiv G_M^s(0)$ is the strange magnetic moment, $\mu'_s$ is strange magnetic radius.

Using this definition: 

\[ G_{E,M}^{s, Z-\exp} = \sum_{k=0}^{k_{max}} a_k z^k, \]

so that we can write the strange form factors:

\[ G_E^s = \rho_{s,z} z + \rho'_{s,z} z^2, \]
\[ G_M^s = \mu_{s,z} + \mu'_{s,z} z, \]

where \( z = \frac{\sqrt{t_{\text{cut}}+Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}}+Q^2} + \sqrt{t_{\text{cut}}}} \), with \( t_{\text{cut}} = (2m_K)^2 \). We calculate the strange electric radius from the fitted slope \( \rho_{s,z} \) using

\[ \rho_s = \frac{dG_E^s}{dQ^2} \bigg|_{Q^2=0}. \]
Our analysis divided into:

- **Leading order fit (LO):** 4 parameters ($\tilde{g}_A^p$, $\tilde{g}_A^n$, $\rho_s$ and $\mu_s$)

- **Next-leading order fit (NLO):** 6 parameters ($\tilde{g}_A^p$, $\tilde{g}_A^n$, $\rho_s$, $\mu_s$, $\rho'_s$ and $\mu'_s$)
The extracted strange electric and magnetic form factors from the global fit up to $Q^2 \sim 1$ GeV$^2$. The black (red) solid curve shows the LO fit (NLO fit) and 1-$\sigma$ bound shown by the dotted curves.

- $G_M^s$ has a similar behavior in both expansions.
- $G_E^s$ has a different behavior and the magnitude has been reduced in Z-expansion.
The 95% and 68% confidence level ellipse in the \((G_M^s, G_E^s)\) plane at \(Q^2 = 0.1 \text{ GeV}^2\) for LO fit (solid) and (NLO) fit (dot-dashed) ellipse for both Taylor and Z-expansions strangeness parameterisations. This work seems to favor positive values of strange electric radius and negative values of the strange magnetic moment.
The red ellipse (19 data, $Q^2 \sim 0.3 \text{ GeV}^2$), nucl-ex/0604010 [Young et. al. 2006]) and black (this work) contours display the the 68% and 95% confidence intervals in $(G^s_M, G^s_E)$ plane at $Q^2 = 0.1 \text{ GeV}^2$. The difference between the areas explains the the importance of including more data.
Comparing with some theoretical lattice results: $Q^2 = 0.1$ GeV$^2$

We see that the very recent lattice results (Red and Green cross) are inside 95% ellipses from NLO fti.
Investigation of the stability of the fits to truncation of the data set at a maximum $Q^2$:

- The goodness of the fit $\chi^2$ values are acceptable at different $Q_{max}^2$.
- Significant reduction of the uncertainty with increasing $Q_{max}^2$.

Behavior of $\mu_s$ and $\rho_s$ with increasing $Q^2$ and the number of data points for LO and NLO fit using Taylor expansion.
Charge symmetry is slightly broken in nature by the u and d quark mass difference and by electromagnetic effects,

\[
G_{E,M}^{p,u} \neq G_{E,M}^{n,d} \\
G_{E,M}^{p,d} \neq G_{E,M}^{n,u}
\]

CSV form factor: 

\[
G_{E,M}^{CSV} = \frac{2}{3}(G_{E,M}^{p,d} - G_{E,M}^{n,u}) - \frac{1}{3}(G_{E,M}^{p,u} - G_{E,M}^{n,d})
\]
In order to explore the CSV’s effect we need to modify the above $G_{Z,p}^{E,M}$ as

$$G_{E,M}^{Z,p(n)} = (1 - 4 \sin^2 \theta_w)(1 + R_p^E)G_{E,M}^{\gamma,p(n)}$$

$$- (1 + R_p^n)G_{E,M}^{\gamma,n(p)} - (1 + R_p^{(0)})G_{E,M}^s - G_{E,M}^{CSV},$$

where we define $G_{E,M}^{CSV}$ as:

$$G_{E,M}^{CSV}(Q^2) = G_{E,M}^{CSV}(0) - \rho_{E,M}^{CSV} Q^2 + \mathcal{O}(Q^4), \text{ with } G_{E}^{CSV}(0) = 0$$

Using $G_{E,M}^{Z,p} \to A_{PV}^p = \left[ \frac{-G_F Q^2}{4\sqrt{2}\pi\alpha} \right] (A_{V}^{p,CSV} + A_s^p + A_A^p)$

where $A_{V}^{p,CSV} =$

$$ (1 - 4 \sin^2 \hat{\theta_w})(1 + R_p^E) - (1 + R_p^n) \frac{\epsilon G_{E}^{\gamma,p} G_{E}^{\gamma,n} + \epsilon G_{E}^{\gamma,p} G_{E}^{CSV} + \tau G_{M}^{\gamma,p} G_{M}^{\gamma,n} + \tau G_{M}^{\gamma,p} G_{M}^{CSV}}{\epsilon (G_{E}^{\gamma,p})^2 + \tau (G_{M}^{\gamma,p})^2}. $$

$$A_{PV}^{He} = \left[ \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \right] \left[ (1 - 4 \sin^2 \theta_w)(1 + R_p^E) - (1 + R_p^n) + 2 \frac{-1 + R_p^{(0)}}{G_E^p + G_E^n} \right].$$
Theoretical CSV evaluation: 
\[ G_{E,M}^{CSV}(Q^2) = G_{E,M}^{CSV}(0) - \rho_{E,M}^{CSV} Q^2 + \mathcal{O}(Q^4). \]

- **Kubis & Lewis (KL) [arXiv:nucl-th/0605006]:**
  - Resonance saturation technique to estimate the contact term in a resonance exchange model.
  - CSV is driven by $\rho - \omega$ mixing.
  - Calculations in HB$\chi$PT and infrared regularized baryon chiral perturbation theory $\rightarrow G_{M}^{CSV}(0)$ of 0.025 $\pm$ 0.02 including resonance parameter uncertainty.
  - The source of the large uncertainty of that estimation is arises from using a large $\omega$-nucleon coupling constant $g_\omega \sim 42$ taken from dispersion analysis.

- **Wagman & Miller (WM) [arXiv:1402.7169 [nucl-th]]:**
  - Relativistic chiral perturbation.
  - More realistic $\omega$-nucleon coupling i.e. $g_\omega \sim 10$.

- **P. E. Shanahan et al, (Lattice CSV) , [arXiv:1503.01142 [hep-lat]]:**
  - they extracted the quantities of CSV based on an analysis of lattice QCD data.
No CSV

CSV K&L

CSV W&M

LO Fit, z.exps
NLO Fit, z.exps
LO Fit, Tay.exps
NLO Fit, Tay.exps

ρs[GeV$^{-2}$]
A complete global analysis of all available PV asymmetry data has been done.

Two models of strangeness parametrization have been used:
- Behavior of $G^s_E(Q^2)$ depends on the chosen strange electric from factor description.

CSV impacts:
- The central values of the extracted quantities ($\mu_s$ and $\rho_s$) shifted slightly.
- No significant impact on the uncertainties of extracted parameters except from CSV calculated by K&L where they used a large coupling value $g_\omega \sim 42$.

CSV not a problem for future!!