Hadron Physics:
From Solitons, DSEs, to Light-Front QCD

Peter C. Tandy

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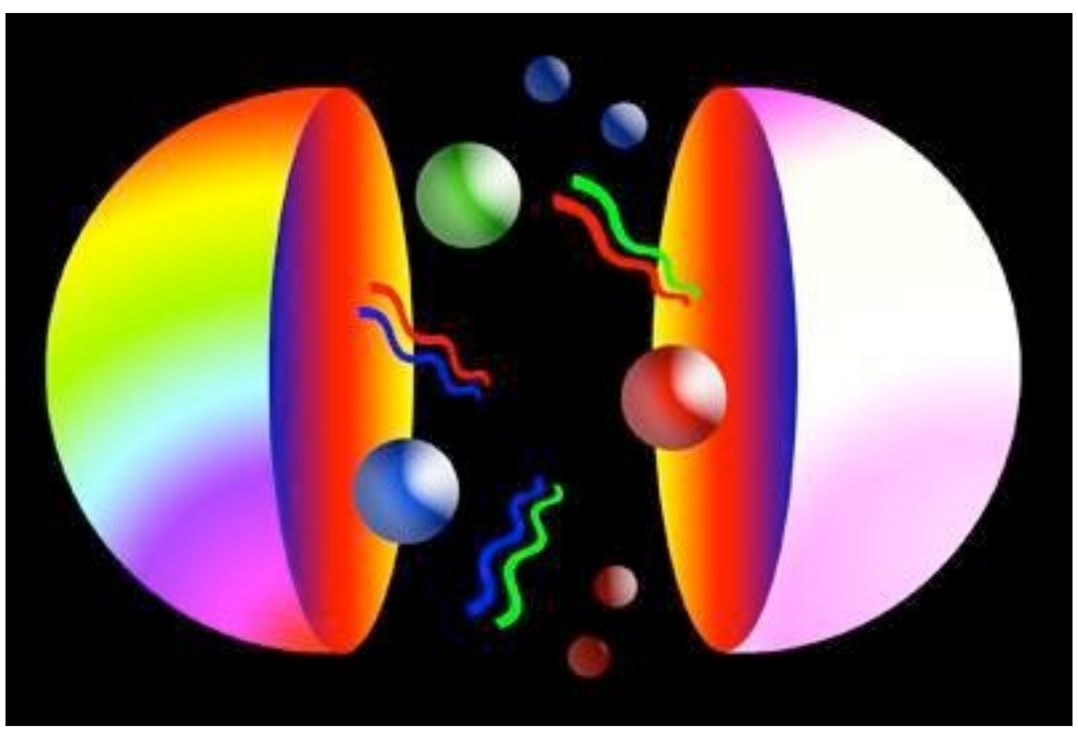
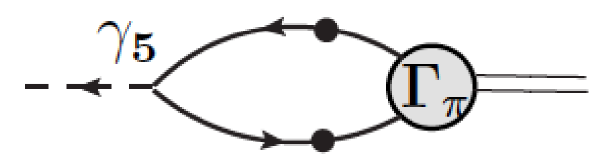
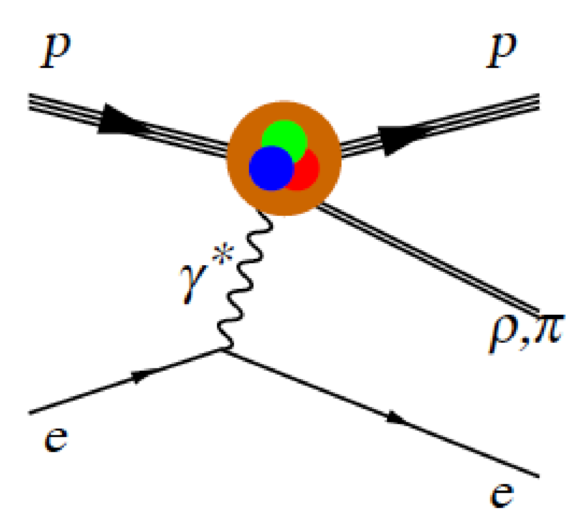






KENT STATE UNIVERSITY

Topics

- Things from the past reTony Williams: Solitons, Perturbation Integral Representation, LQCD propagators,….
- Pion and kaon Distribution Amplitudes—insight into DSE-QCD vs LQCD for hadron physics.
- PDFs including X. Ji's space-like correlator approximation for LQCD—a model investigation.
- Pion elastic and transition Form Factors

From the Past

Univ of Maryland, 1987-88…

Soliton Models

Volume 212, number 3 PHYSICS LETTERS B 29 September 1988

CONFINEMENT, CHIRAL SYMMETRY BREAKING, AND THE PION IN A CHROMO-DIELECTRIC MODEL OF QUANTUM CHROMODYNAMICS

G. KREIN [1], P. TANG, L. WILETS [2] and A.G. WILLIAMS
Institute for Nuclear Theory, Department of Physics, FM-15, University of Washington, Seattle, WA 98195, USA

Received 5 May 1988

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Nontopological solitons, Green's functions, and bound states

A. G. Williams and R. T. Cahill
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia, 5042, Australia
(Received 25 October 1983)

PHYSICAL REVIEW D VOLUME 28, NUMBER 8 15 OCTOBER 1983

Nontopological solitons from functional integrals. I

A. G. Williams and R. T. Cahill
School of Physical Sciences, The Flinders University of South Australia, Bedford Park, South Australia, 5042 Australia
(Received 3 December 1982)

From the Past

1991

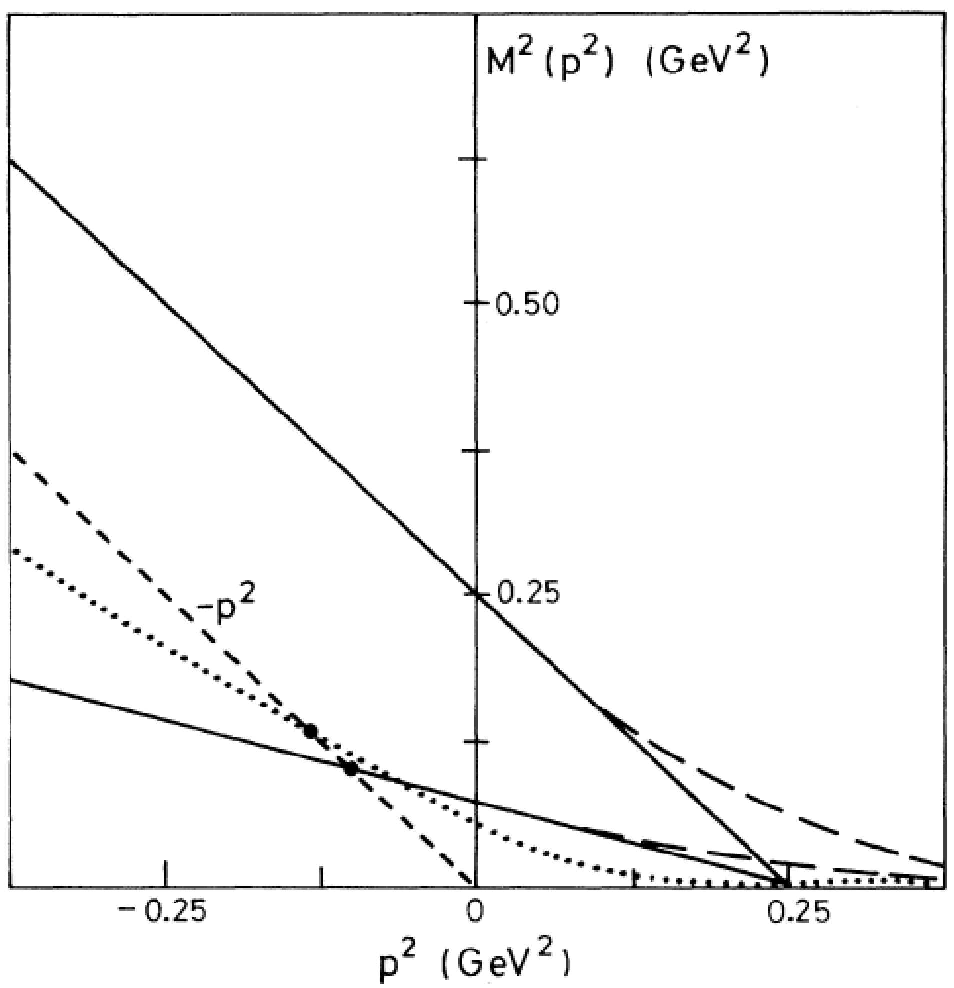
PHYSICAL REVIEW C VOLUME 46, NUMBER 1 JULY 1992

Confining quark condensate model of the nucleon

M. R. Frank
Department of Physics, Hampton University, Hampton, Virginia 23668
and Continuous Electron Beam Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606

P. C. Tandy
Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242
(Received 18 December 1991)

We obtain a mean-field solution for the nucleon as a quark-meson soliton obtained from the action of the global color-symmetry model of QCD. All dynamics is generated from an effective interaction of quark currents. At the quark-meson level there are two novel features: (1) absolute confinement is produced from the space-time structure of the dynamical self-energy in the vacuum quark propagator; and (2) the related scalar meson field is an extended $\bar{q}q$ composite that couples nonlocally to quarks. The influence of these features upon the nucleon mass contributions and other nucleon properties is presented.



The underlying action is taken to be the GCM [6] given in Euclidean space as

$$S[\bar{q},q]=\int d^4x\, d^4y \left[\bar{q}(x)(\gamma\cdot\partial+m)\delta(x-y)q(y) + \frac{g^2}{2} j_\mu^a(x) D_{\mu\nu}(x-y) j_\nu^a(y)\right], \quad (1)$$

[4] T. Eguchi, Phys. Rev. D **14**, 2755 (1976).
[5] R. T. Cahill and C. D. Roberts, Phys. Rev. D **32**, 2419 (1985).
[6] J. Praschifka, C. D. Roberts, and R. T. Cahill, Phys. Rev. D **36**, 209 (1987); J. Praschifka, R. T. Cahill, and C. D. Roberts, Int. J. Mod. Phys. A **4**, 4929 (1989).

realization of this is produced from use of the model gluon propagator [16]

$$D(p)=(2\pi)^4 \tfrac{3}{16}\alpha^2\delta^{(4)}(p)\,, \quad (25)$$

in the Schwinger-Dyson equation (6). The resulting self-energy amplitudes are

$$A(p^2)=\begin{cases} 2, & p^2\le\alpha^2/4 \\ \frac{1}{2}[1+(1+2\alpha^2/p^2)^{1/2}], & p^2\ge\alpha^2/4\,, \end{cases}$$
$$B(p^2)=\begin{cases} (\alpha^2-4p^2)^{1/2}, & p^2\le\alpha^2/4 \\ 0, & p^2\ge\alpha^2/4\,. \end{cases} \quad (26)$$

For $p^2<0$ this gives $M^2(p^2)=\alpha^2/4-p^2$. Several studies

Morelia May 2015

Hadron Physics & QCD

Pergamon

Prog. Part. Nucl. Phys., Vol. 33, pp. 477–575, 1994

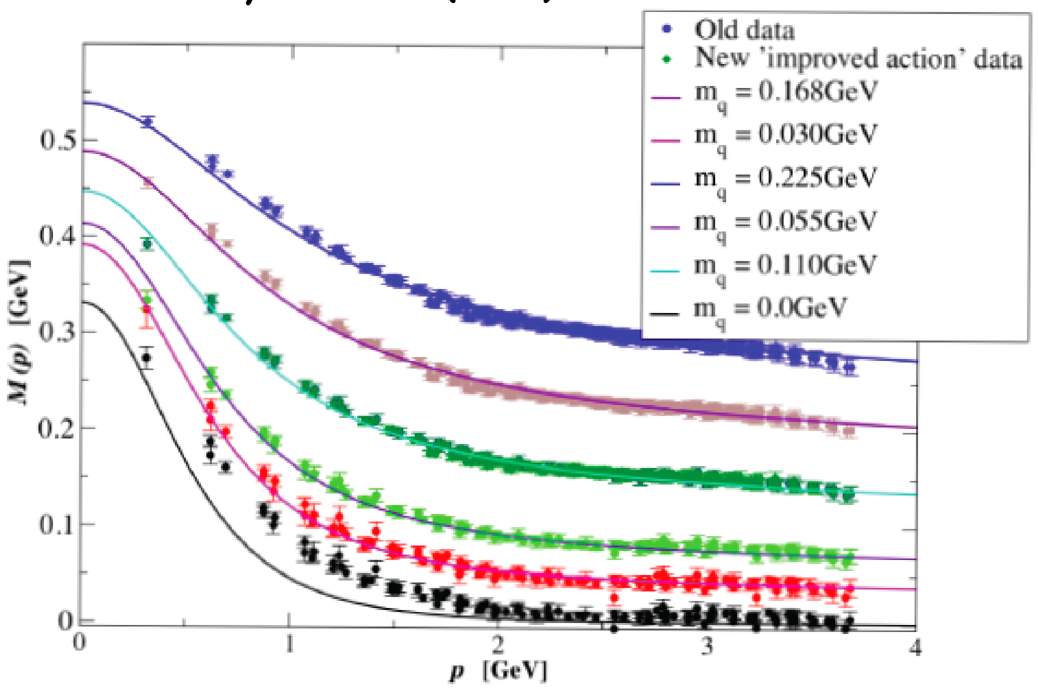
0146–6410/94 $26.00
0146-6410(94)E0006-P

Dyson–Schwinger Equations and their Application to Hadronic Physics

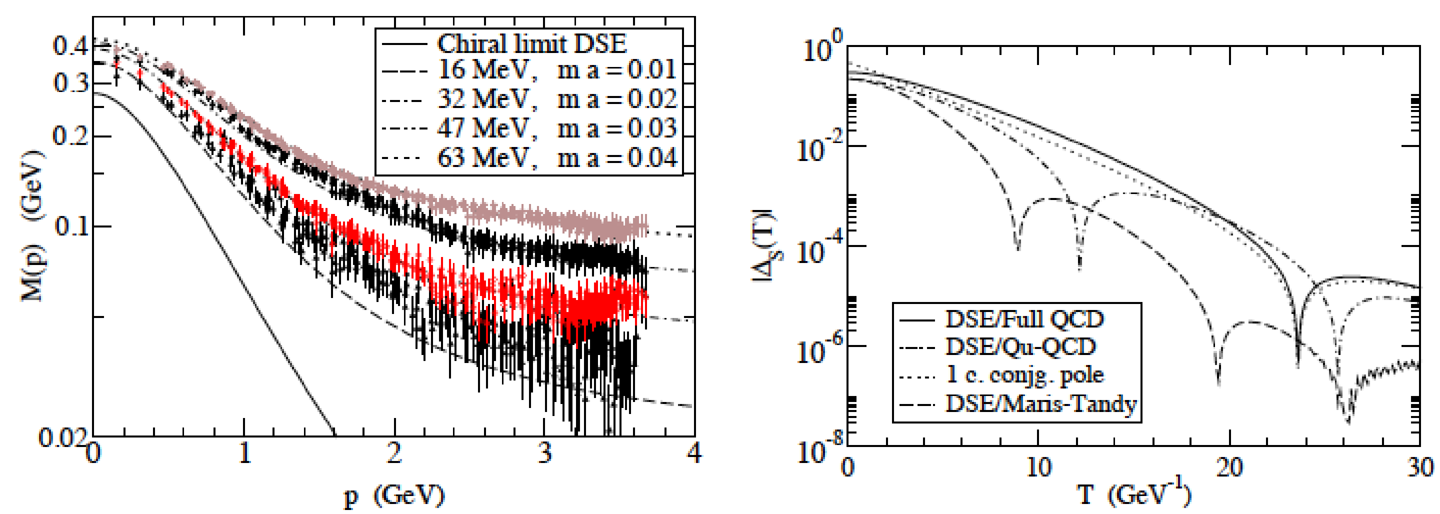
CRAIG D. ROBERTS* and ANTHONY G. WILLIAMS†,‡

* *Physics Division, Argonne National Laboratory, Argonne, IL 60439-4843, U.S.A.*
† *Department of Physics and Mathematical Physics, University of Adelaide, SA 5005, Australia*
‡ *Department of Physics and the Supercomputer Computations Research Institute, Florida State University, Tallahassee, FL 32306, U.S.A.*

Analysis of a quenched lattice QCD dressed quark propagator
M.S. Bhagwat, M.A. Pichowsky (Kent State U.),
C.D. Roberts (Argonne, PHY), P.C. Tandy (Kent State U.). Apr 2003. 9 pp.
Published in Phys.Rev. C68 (2003) 015203 e-Print: nucl-th/0304003 | PDF



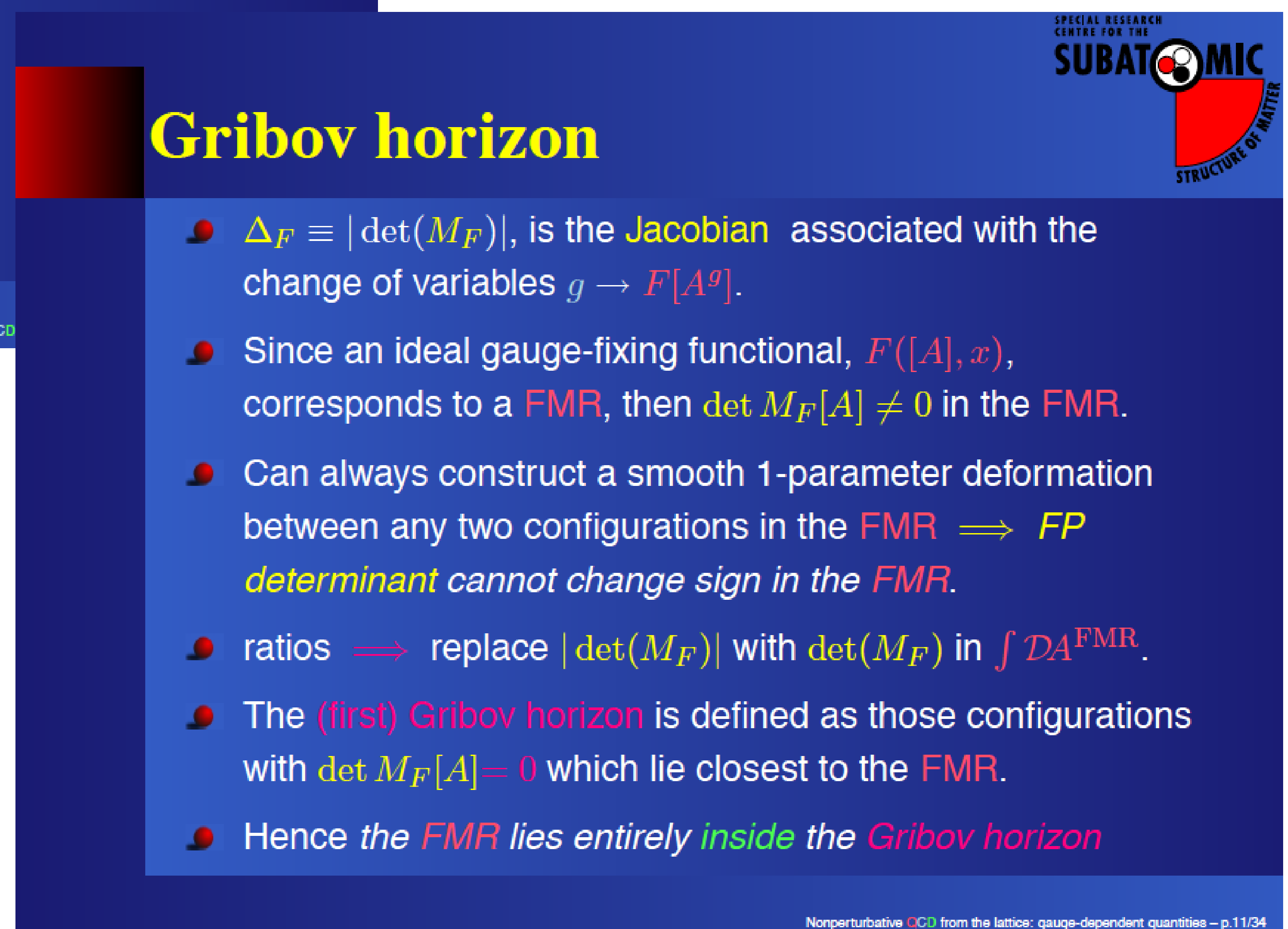
[13] P.O. Bowman, U.M. Heller, and A.G. Williams, Phys. Rev. D **66**, 014505 (2002).
[14] P.O. Bowman, U.M. Heller, D.B. Leinweber, and A.G. Williams, hep-lat/0209129.



Analysis of full-QCD and quenched-QCD lattice propagators
M.S. Bhagwat, P.C. Tandy (Kent State U.). Jan 2006. 3 pp.
AIP Conf. Proc. 842 (2006) 225-227 (PANIC05) e-Print: nucl-th/0601020 | PDF

Lattice QCD

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER
Nonperturbative QCD from the lattice: gauge-dependent quantities
Anthony G. Williams
at the Tokyo-Adelaide Joint Workshop on Quarks, Astrophysics, and Space Physics
CSSM, University of Adelaide
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER



SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER
Gribov horizon
$\Delta_F \equiv |\det(M_F)|$, is the Jacobian associated with the change of variables $g \to F[A^g]$.
Since an ideal gauge-fixing functional, $F([A], x)$, corresponds to a FMR, then $\det M_F[A] \neq 0$ in the FMR.
Can always construct a smooth 1-parameter deformation between any two configurations in the FMR $\Longrightarrow$ *FP determinant cannot change sign in the FMR*.
ratios $\Longrightarrow$ replace $|\det(M_F)|$ with $\det(M_F)$ in $\int \mathcal{D}A^{\mathrm{FMR}}$.
The (first) Gribov horizon is defined as those configurations with $\det M_F[A] = 0$ which lie closest to the FMR.
Hence *the FMR lies entirely inside the Gribov horizon*
Nonperturbative QCD from the lattice: gauge-dependent quantities – p.11/34

Perturbation Integral Representation

PHYSICAL REVIEW D VOLUME 51, NUMBER 12 15 JUNE 1995

Solving the Bethe-Salpeter equation for scalar theories in Minkowski space

Kensuke Kusaka* and Anthony G. Williams†
Department of Physics and Mathematical Physics, University of Adelaide, South Australia 5005, Australia
(Received 11 January 1995)

The Bethe-Salpeter (BS) equation for scalar-scalar bound states in scalar theories without a derivative coupling is formulated and solved in Minkowski space. This is achieved using the perturbation theory integral representation, which allows these amplitudes to be expressed as integrals over weight functions and known singularity structures and hence allows us to convert the BS equation into an integral equation involving weight functions. We obtain numerical solutions using this formalism for a number of scattering kernels to illustrate the generality of the approach. It applies even when the naive Wick rotation is invalid. As a check we verify, for example, that this method applied to the special case of the massive ladder exchange kernel reproduces the same results as are obtained by Wick rotation.

PHYSICAL REVIEW D VOLUME 56, NUMBER 8 15 OCTOBER 1997

Solving the Bethe-Salpeter equation for bound states of scalar theories in Minkowski space

Kensuke Kusaka*
Department of Physics, Tokyo Metropolitan University, Minami-Osawa 1-1, Hachioji-shi, Tokyo 192-03, Japan

Ken Simpson†
Department of Physics and Mathematical Physics, University of Adelaide, South Australia 5005, Australia

Anthony G. Williams‡
Department of Physics and Mathematical Physics, University of Adelaide, South Australia 5005, Australia and Special Research Centre for the Subatomic Structure of Matter, University of Adelaide, South Australia 5005, Australia
(Received 14 May 1997)

Fit numerical DSE-BSE solns to PTIRs (Nakanishi)

EG: $$\mathbf{\Gamma_\pi(q^2, q\cdot P) = \gamma_5 \left\{ E_\pi(q^2, q\cdot P) + \not{P}\, F_\pi(..) + \not{q}\, q\cdot P\; G_\pi(..) + \sigma : qP\; H_\pi(..) \right\}}$$

Use Nakanishi Repn (or PTIR) (1965) :- $\mathcal{F} = \mathbf{E},\ \mathbf{F},\ \mathbf{G},\ \text{or } \mathbf{H}$

$$\mathcal{F}(\mathbf{q^2; q\cdot P}) = \int_{-1}^{1} d\alpha \int_0^\infty d\Lambda\, \left\{ \frac{\rho_{\mathrm{IR}}(\alpha;\Lambda)}{(\mathbf{q^2} + \alpha \mathbf{q\cdot P} + \Lambda^2)^{\mathbf{m+n}}} + \frac{\rho_{\mathrm{UV}}(\alpha;\Lambda)}{(\mathbf{q^2} + \alpha \mathbf{q\cdot P} + \Lambda^2)^{\mathbf{n}}} \right\}$$

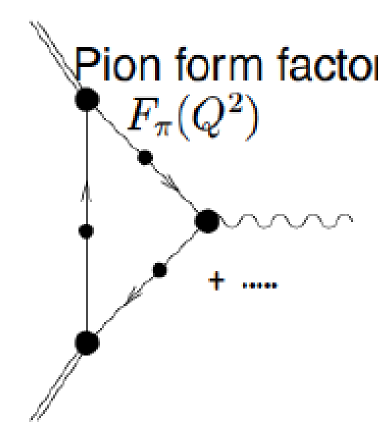
npQCD info is in the variables and constants that are not momenta
---Wick rotation is trivial as in pert thy.

$$\rho_{\mathrm{IR}}(\alpha;\Lambda) \to \rho_1(\alpha)\,\delta(\Lambda - \Lambda_{\mathrm{IR}_1}) \;+\; \cdots \mathbf{3}$$

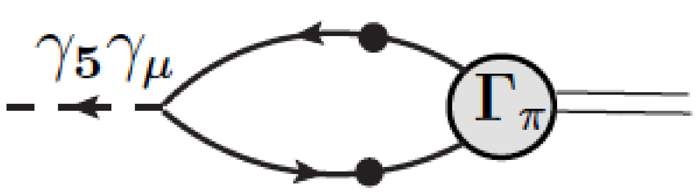
$$S(q) = \sum_{k=1}^{3} \left(\frac{z_k}{i\,\not{q} + m_k} + \frac{z_k^*}{i\,\not{q} + m_k^*} \right)$$

Works for u-, d-, s-, c-, b-quarks.
Also for lattice-QCD propagators.

N. Souchlas, PhD thesis KSU, (2009), J. Phys. G37, 115001 (2010)



EG: $$\mathbf{q_A(x) = i\, N_c\, \mathrm{tr} \int \frac{dk^+\, dk^-\, d^2k_\perp}{(2\pi)^4}\, \delta(k^+ - x\, P^+)\;\; \mathrm{tr}[\Gamma_\pi\; S\; (i\gamma^+)\; S\; \Gamma_\pi\; S]}$$



Spacelike Correlator Approximation for PDFs

To help lattice-QCD be more applicable to hadron PDFs and GPDs than just the first 3 moments ?

PRL 110, 262002 (2013) PHYSICAL REVIEW LETTERS week ending 28 JUNE 2013

Parton Physics on a Euclidean Lattice

Xiangdong Ji[1,2]

Standard light-cone correlator, leading twist: $\mathbf{x = k \cdot n / P \cdot n = k^+/P^+ \;\; \epsilon \;\; [0,1]}$

$$\mathbf{q_f(x) = \frac{1}{4\pi} \int d\lambda \; e^{-ix\,P\cdot n\,\lambda} \, \langle \pi(P) | \, \bar{\psi}_f(\lambda n) \, \not{n} \, \psi_f(0) \, | \pi(P) \rangle_c}$$

$$\mathbf{n^2 = 0 \;\; ; \;\; z^- = \lambda\, n \;\; ; \;\; z^+ = 0 = z_\perp}$$

Ji: Take large Pz limit of frame-dependent equal-time correlator: $\mathbf{x = kz/Pz \;\; \epsilon \;\; [-\infty, +\infty]}$

$$\mathbf{\tilde{q}_f(x; Pz) = \frac{1}{4\pi} \int dz \; e^{-ix\,P_z\,z} \, \langle \pi(P) | \, \bar{\psi}_f(z) \, \gamma_z \, \psi_f(0) \, | \pi(P) \rangle_c}$$

$$\mathbf{\to q_f(x) \text{ as } Pz \to \infty}$$

How fast?

Quark Distribution

§ Back to the continuum

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Finite $P_z \to \infty P_z$, perturbative matching

$$\tilde{q}(x,\mu,P_z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu)$$

$$Z(x,\mu/P_z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, 1310.7471 [hep-ph]; Y. Zhao, this wokshop

UNIVERSITY OF WASHINGTON

Huey-Wen Lin — Light Cone 2014

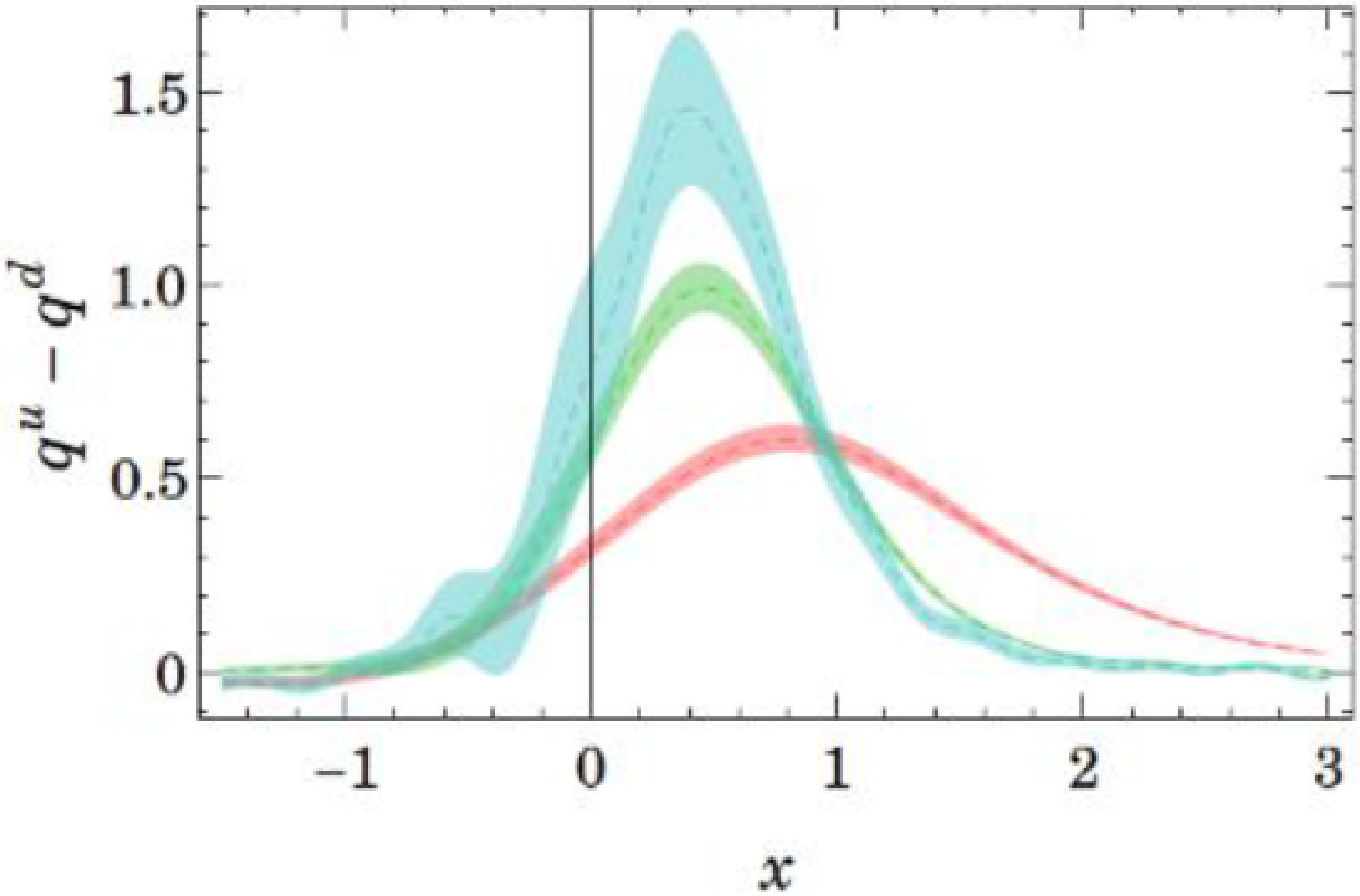
FIG. 1: One loop corrections to quasi quark distribution.

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\, \gamma_z \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

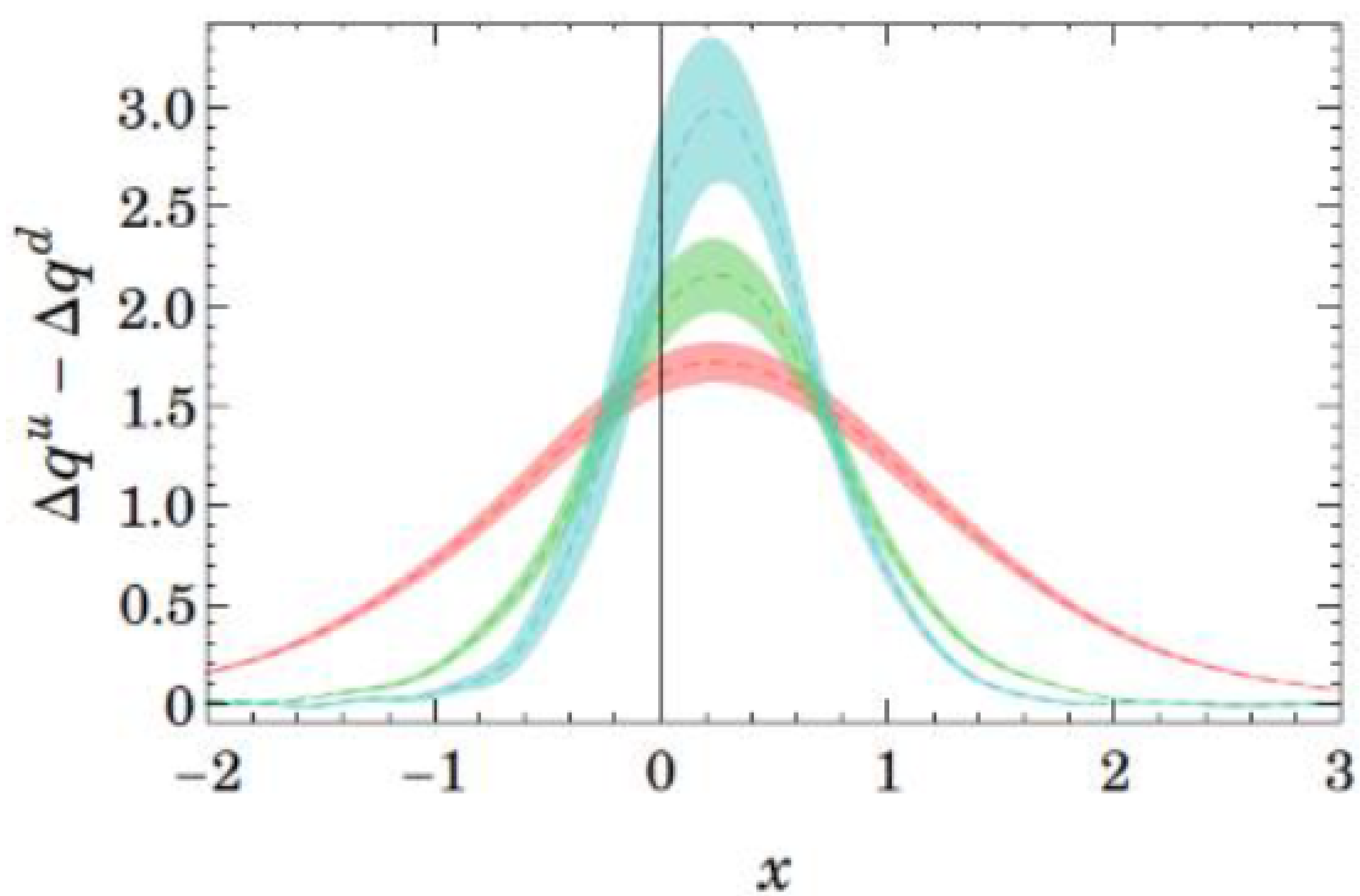
Improvement?

Work out leading-P_z corrections

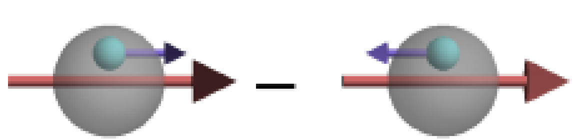
Helicity Distribution

§ Exploratory study

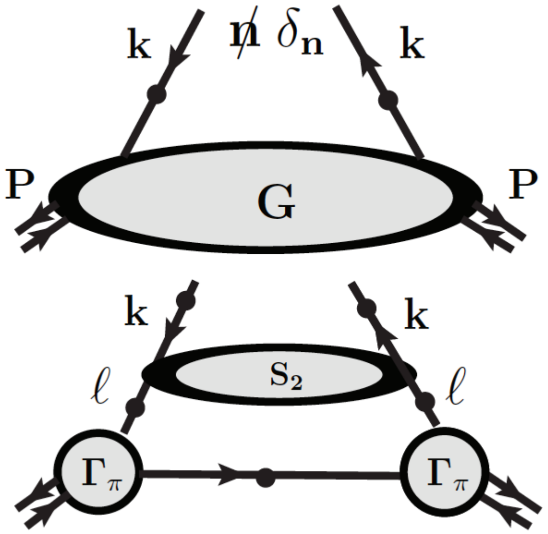
$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z\, \gamma_5 \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



Uncorrected bare lattice results



Simple model for pion PDF & Quasi-PDF



$$\mathbf{S(k) = 1/(i\not k + M)}\ , \quad \mathbf{M = 0.4\ GeV}$$

$$\mathbf{\Gamma_\pi(q,P) = \gamma_5 N_\pi \int_{-1}^{1} d\alpha\ \frac{\rho(\alpha)}{q^2 + \alpha q\cdot P + \Lambda^2}}\ , \quad \rho(\alpha) = \mathbf{even}$$

Euclidean to Minkowski:-

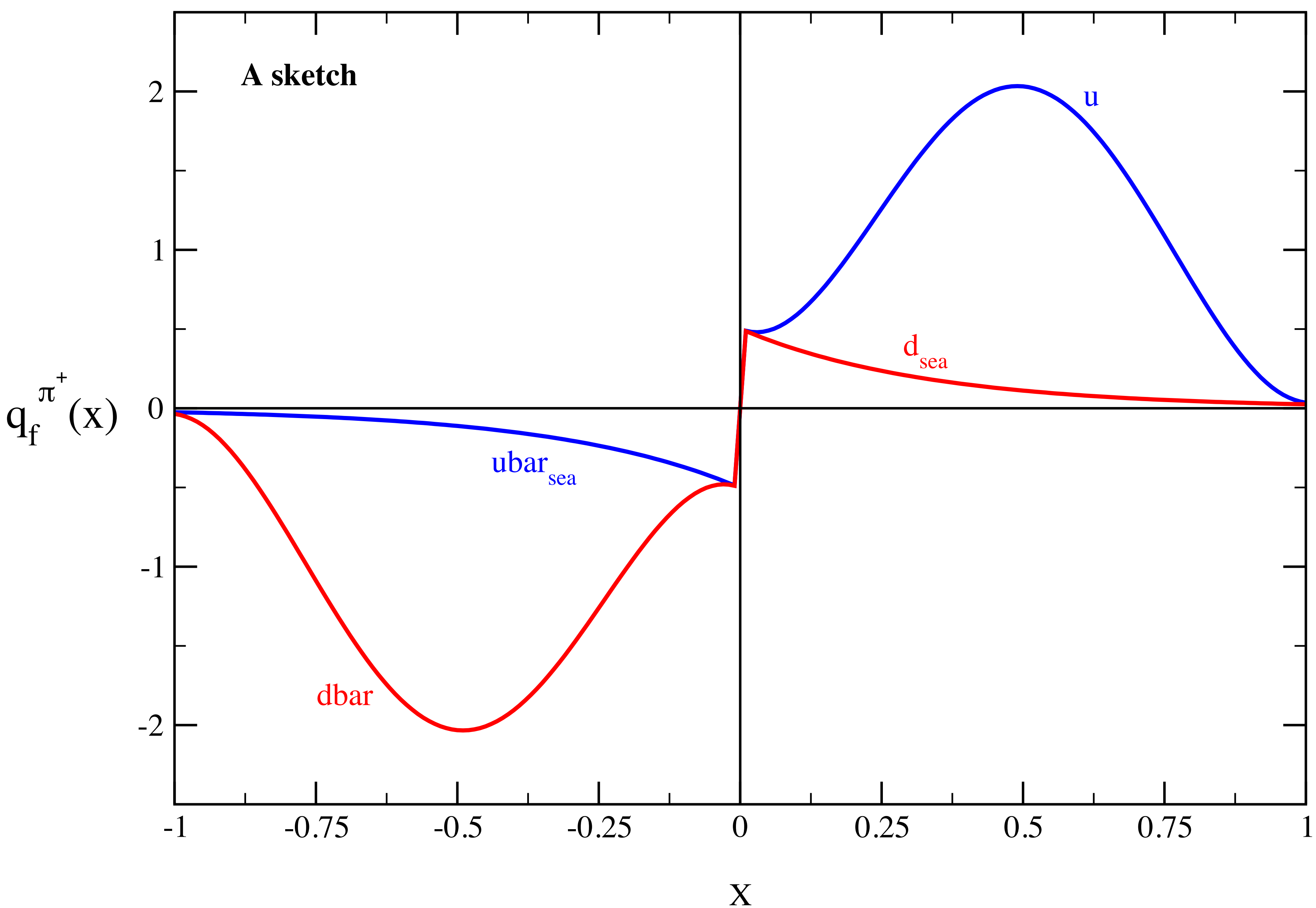
Evaluate $\mathbf{q(x)}$ directly using Cauchy Residue Thm for $\displaystyle\int_{-\infty}^{\infty} \mathbf{dk^-}$

$$\mathbf{q_A(x) = i\, N_c\, tr \int \frac{dk^+\, dk^-\, d^2k_\perp}{(2\pi)^4}\ \delta(k^+ - x\, P^+)\ \ tr[\Gamma_\pi\ S\ (i\gamma^+)\ S\ \Gamma_\pi\ S]}$$

Evaluate $\mathbf{\tilde{q}(x;P_z)}$ directly using Cauchy Residue Thm for $\displaystyle\int_{-\infty}^{\infty} \mathbf{dk^0}$

$$\mathbf{\tilde{q}_A(x) = i\, N_c\, tr \int \frac{dk^0\, dk_z\, d^2k_\perp}{(2\pi)^4}\ \delta(k_z - x\, P_z)\ \ tr[\Gamma_\pi\ S\ (i\gamma^z)\ S\ \Gamma_\pi\ S]}$$

Typical Hadron PDF q(x): a sketch for pion



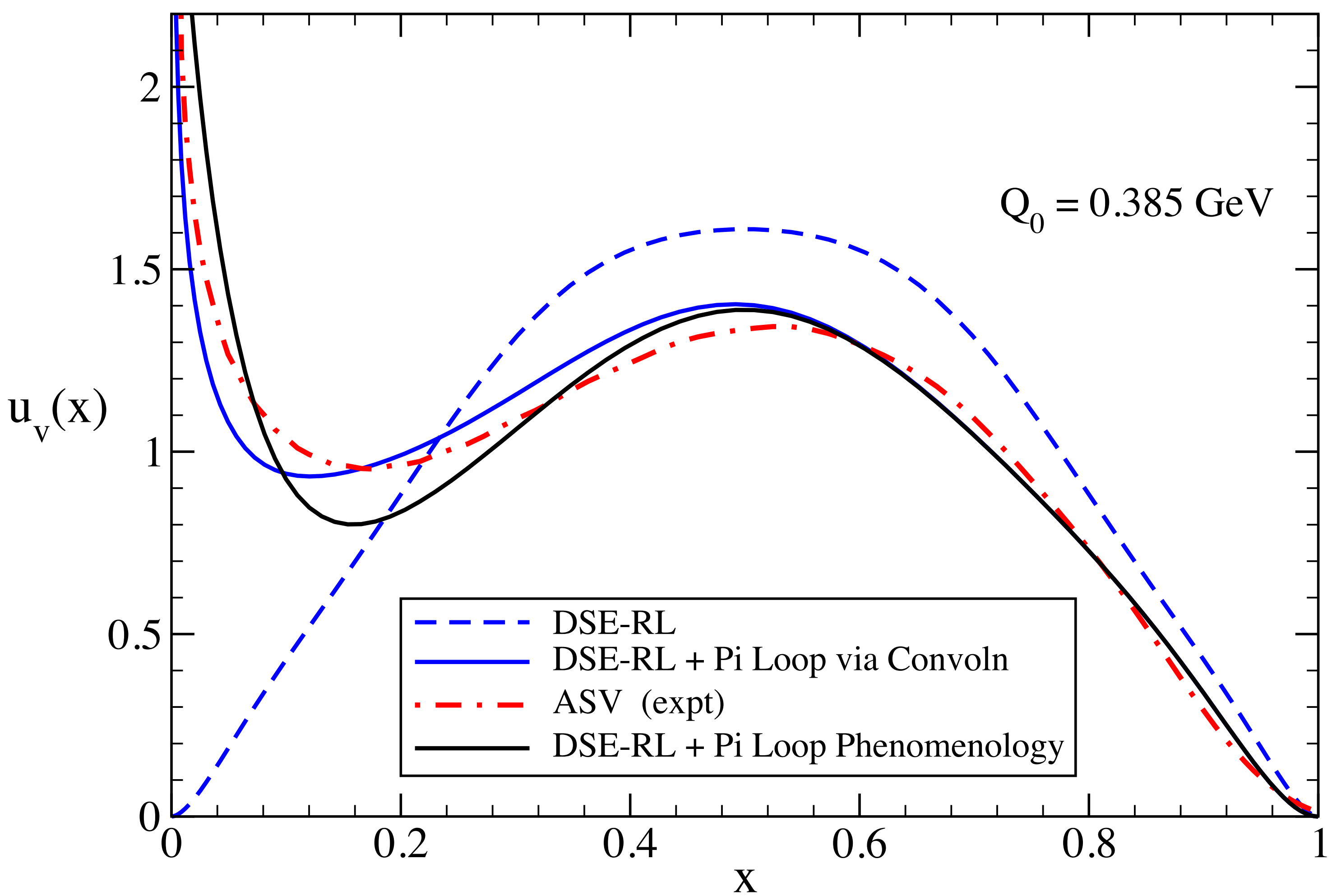
DIVERSION———A full DSE calculation of the true pion valence PDF

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

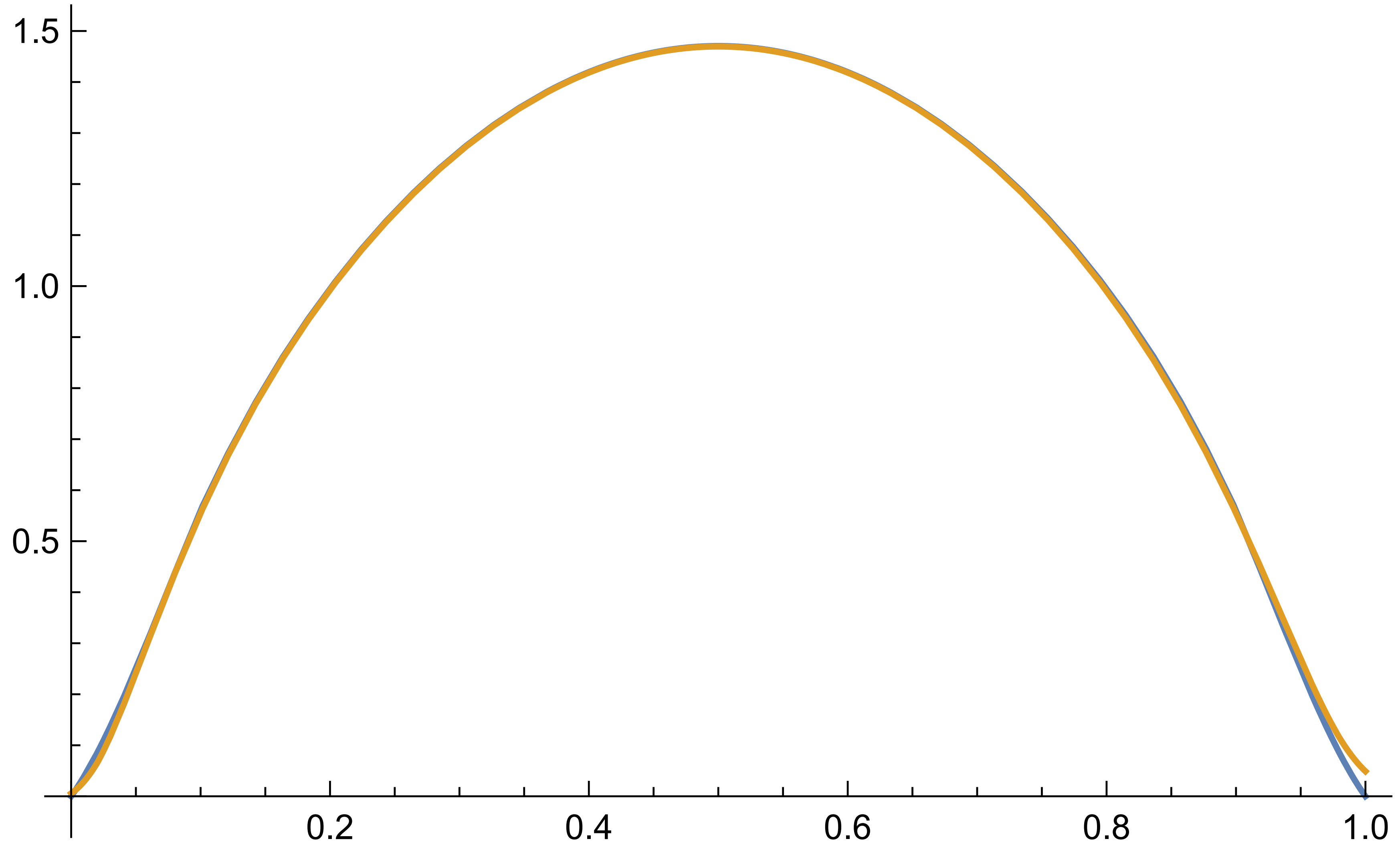
	$2\,q_{\rm val}^{\rm RL}$	$2\,q_{\rm val}^{\rm DSE}$	$4\,q_{\rm sea}^{\rm ASV}$	gluon	Total
$\langle x \rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

K. Khitrin, P. Tandy, in progress (2015)

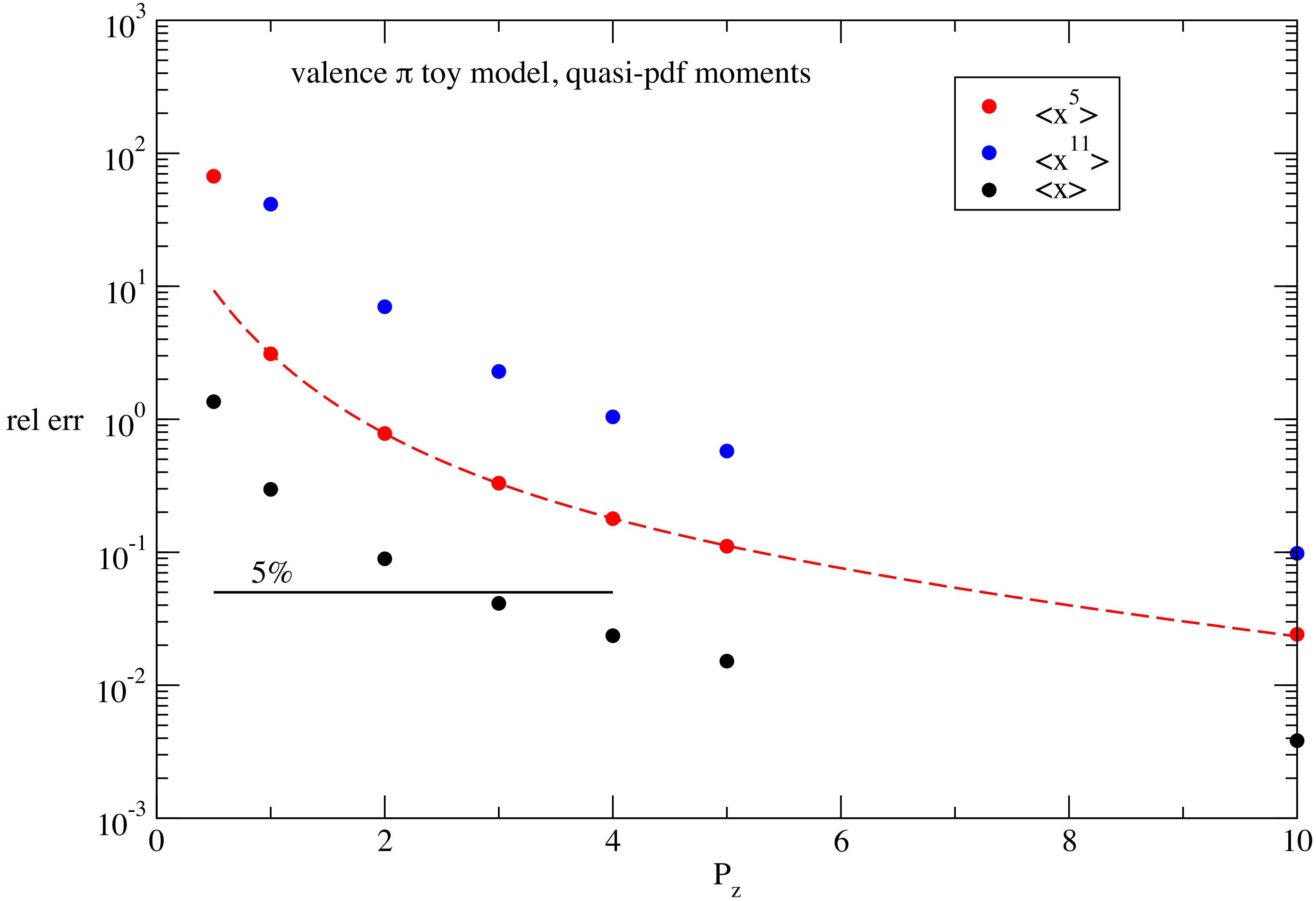
Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)



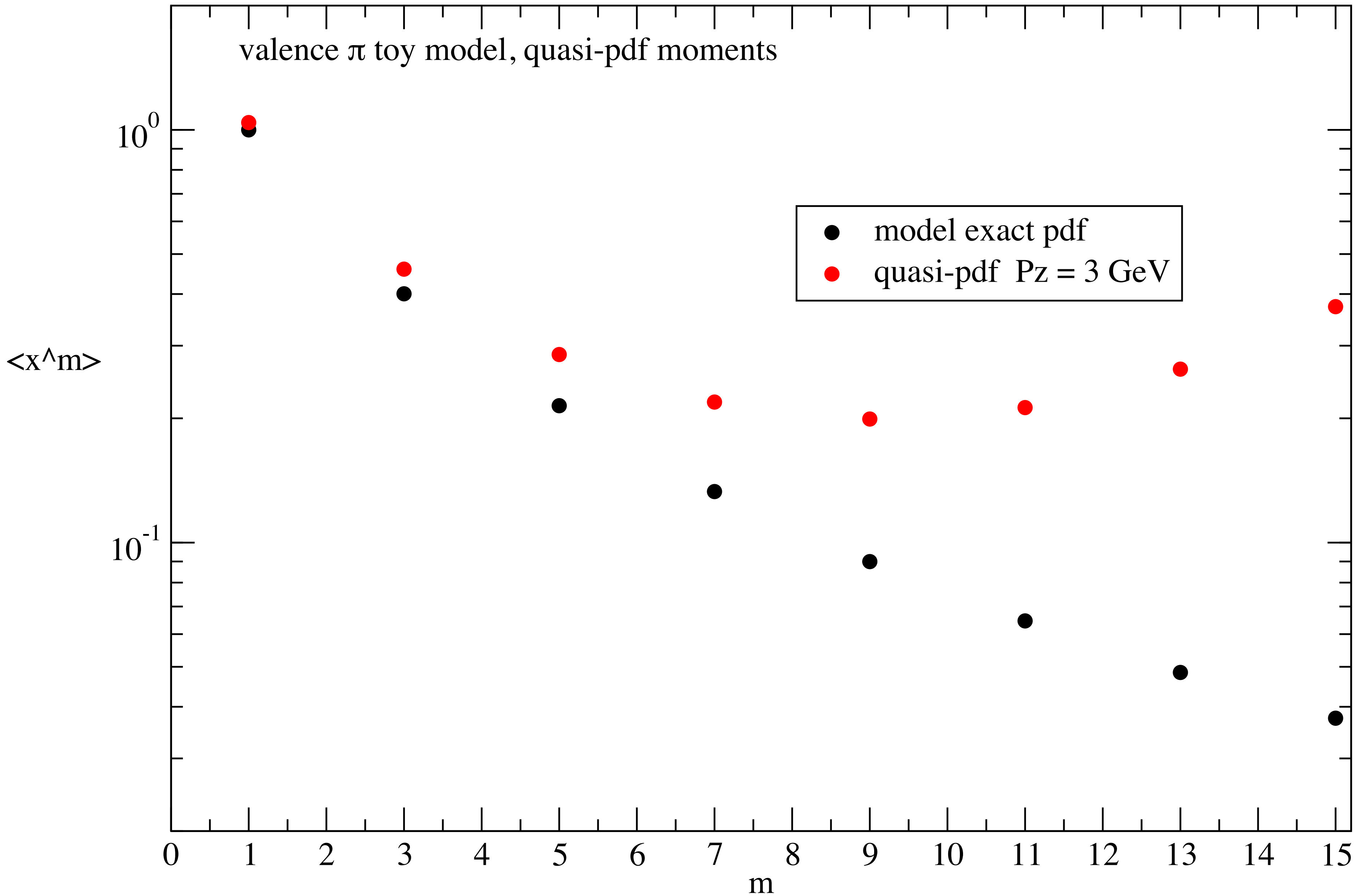
Model-exact PDF & Quasi-PDF @ Pz=10 GeV



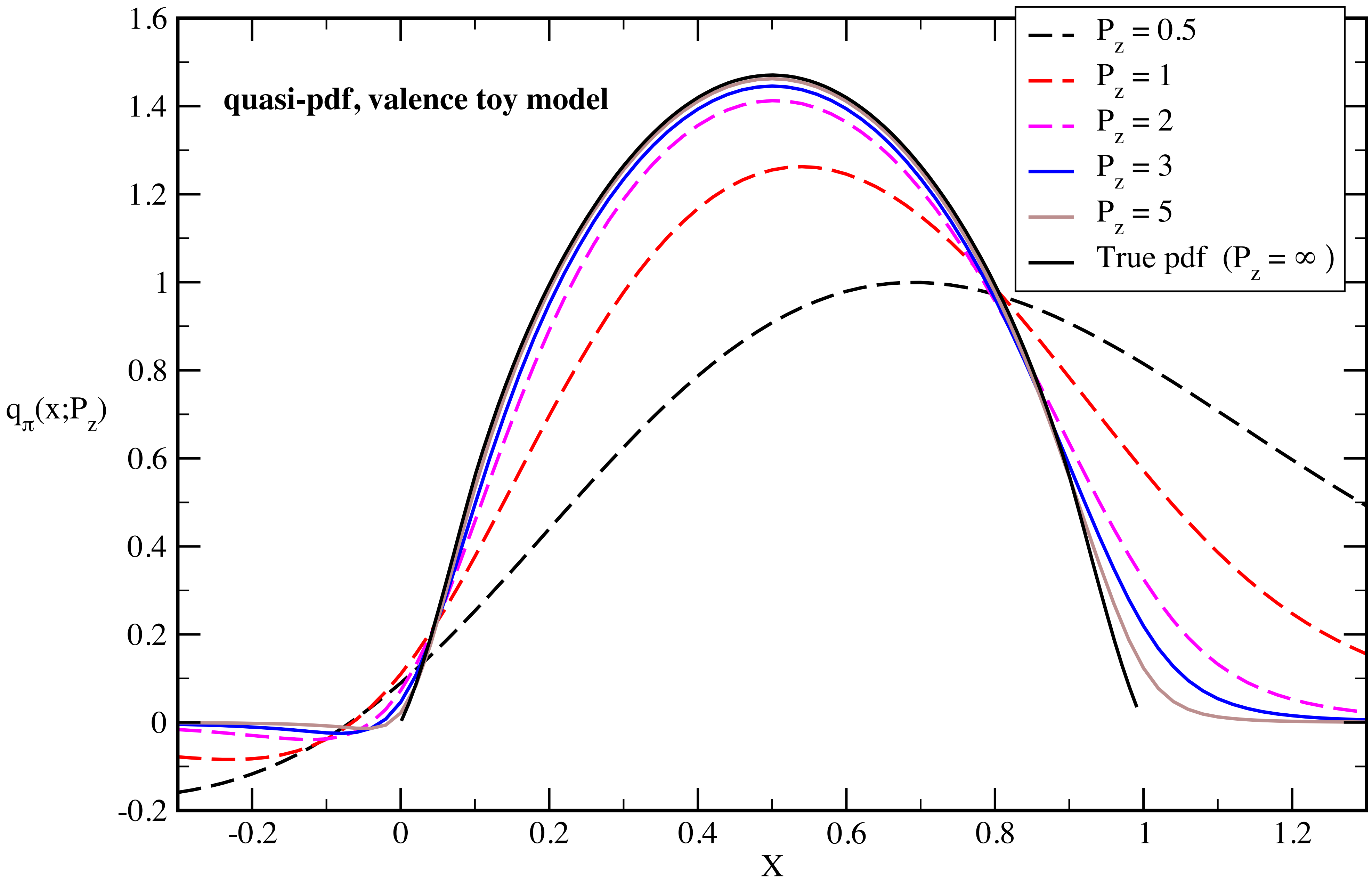
Pz Dependence of quasi-pdf of valence model pion



<x^m> for toy model pion at Pz = 3 GeV



Pz Dependence of quasi-pdf of u-ubar "pion"

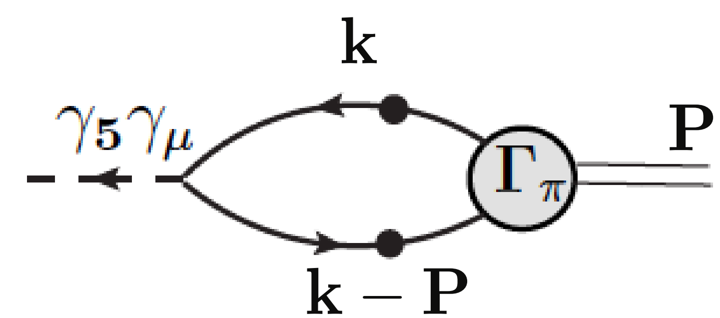


---I.Cloet, Lei Chang, PCT, in progress (2015).......

Parton Distribution Amplitudes of Mesons

Pion Distribution Amplitude (leading twist)

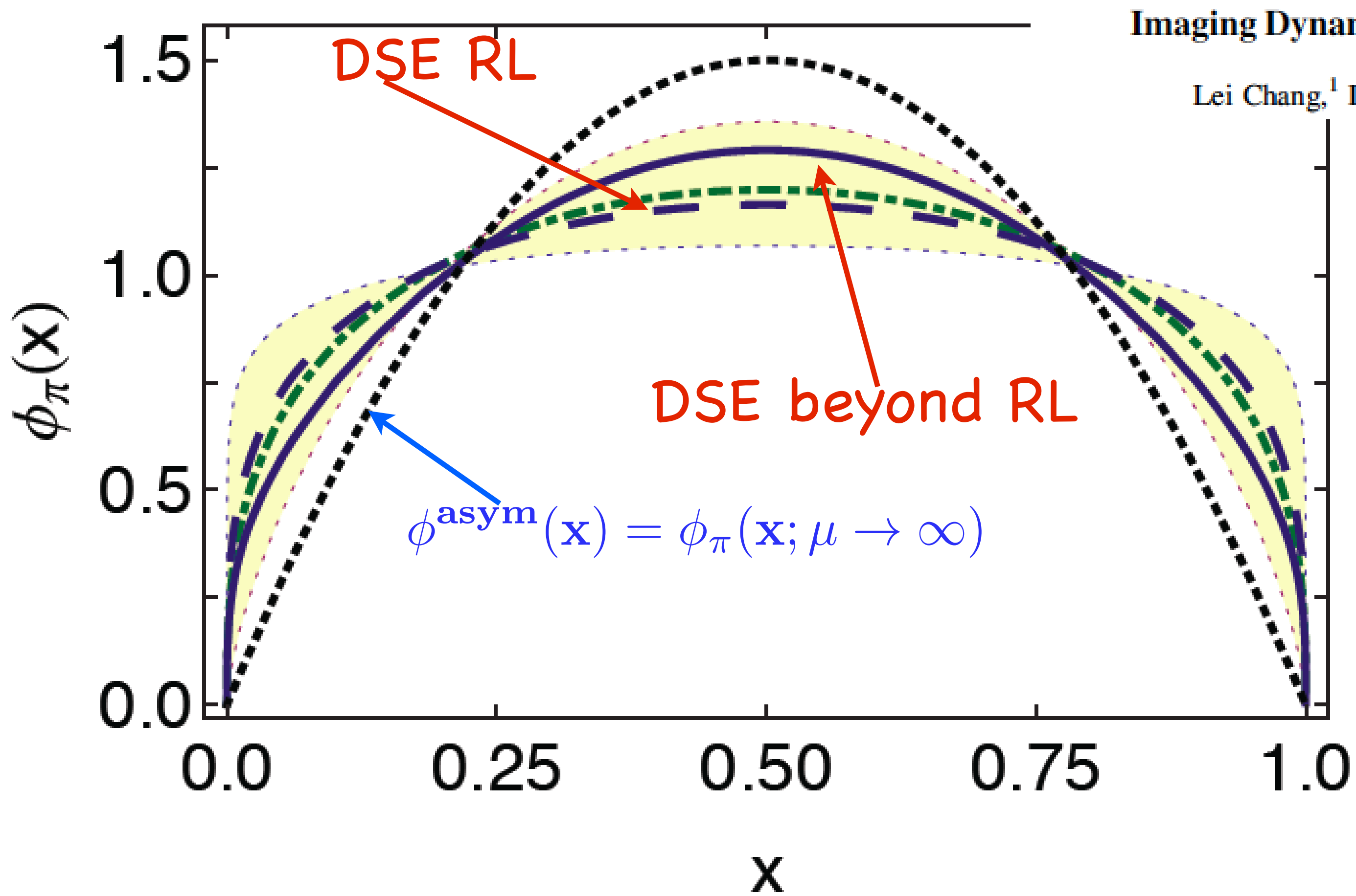
$$\mathbf{f_\pi\, \phi_\pi(x) = \int \frac{d\lambda}{2\pi}\, e^{-ixP.n\lambda}\, \langle 0|\, \bar{q}(0)\, \gamma_5 \not{n}\, q(\lambda n)\, |\pi(P)\rangle}$$



$$\mathbf{f_\pi\, \langle x^m\rangle_\phi = \frac{Z_2 N_c}{P\cdot n}\, \mathrm{tr} \int_k \left(\frac{k\cdot n}{P\cdot n}\right)^m \gamma_5 \not{n} \left[S(k)\, \Gamma_\pi(k - \frac{P}{2}; P)\, S(k-P) \right]}$$

BS Wavefn

$\mu = 2$ GeV



PRL 110, 132001 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

Imaging Dynamical Chiral-Symmetry Breaking: Pion Wave Function on the Light Front

Lei Chang,[1] I. C. Cloët,[2,3] J. J. Cobos-Martinez,[4,5] C. D. Roberts,[3,6] S. M. Schmidt,[7] and P. C. Tandy[4]

Broadening of PDA is an expression of DCSB ---long sought after in LF QFT

Pion Distribution Amplitude

ERBL (~1980): $$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\ \mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$$

$$\mathbf{a_n}(\mu) = \mathbf{a_n}(\mu_\mathbf{0})\left[\frac{\alpha_\mathbf{s}(\mu_\mathbf{0})}{\alpha_\mathbf{s}(\mu)}\right]^{\gamma_\mathbf{n}^{(\mathbf{0})}/\beta_\mathbf{0}}$$

Evolution to higher scales is
EXTREMELY SLOW
Not much change up to LHC energy

Conformal limit: $\mathbf{a_n}(\mu\to\infty) = \mathbf{0}$

Efficient representation of DSE results:

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{N}_\alpha\ \mathbf{x}^\alpha(\mathbf{1-x})^\alpha\ \{\,\mathbf{1} + \Sigma_{\mathbf{n=2}}^{\infty}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\,\mathbf{C_n}^{\alpha+\mathbf{1/2}}(\mathbf{2x-1})\,\}$$

$$\phi_\mathbf{K}(\mathbf{x};\mu) = \mathbf{N}_\alpha\ \mathbf{x}^\alpha(\mathbf{1-x})^\alpha\ \{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\ \mathbf{C_n}^{\alpha+\mathbf{1/2}}(\mathbf{2x-1})\,\}$$
$$+\ \mathbf{N}_\beta\ \mathbf{x}^\beta(\mathbf{1-x})^\beta\ \{\,\Sigma_{\mathbf{n=1,3\cdots}}\ \tilde{\mathbf{a}}_\mathbf{n}(\mu)\ \mathbf{C_n}^{\beta+\mathbf{1/2}}(\mathbf{2x-1})\,\}$$

Low Order Truncation of ERBL-Gegenbauer Expn of PDA

$$\phi_\pi(\mathbf{x};\mu) = \mathbf{6x(1-x)}\,\{\,\mathbf{1} + \Sigma_{\mathbf{n=2,4\cdots}}\ \mathbf{a_n}(\mu)\,\mathbf{C_n^{3/2}(2x-1)}\,\}$$

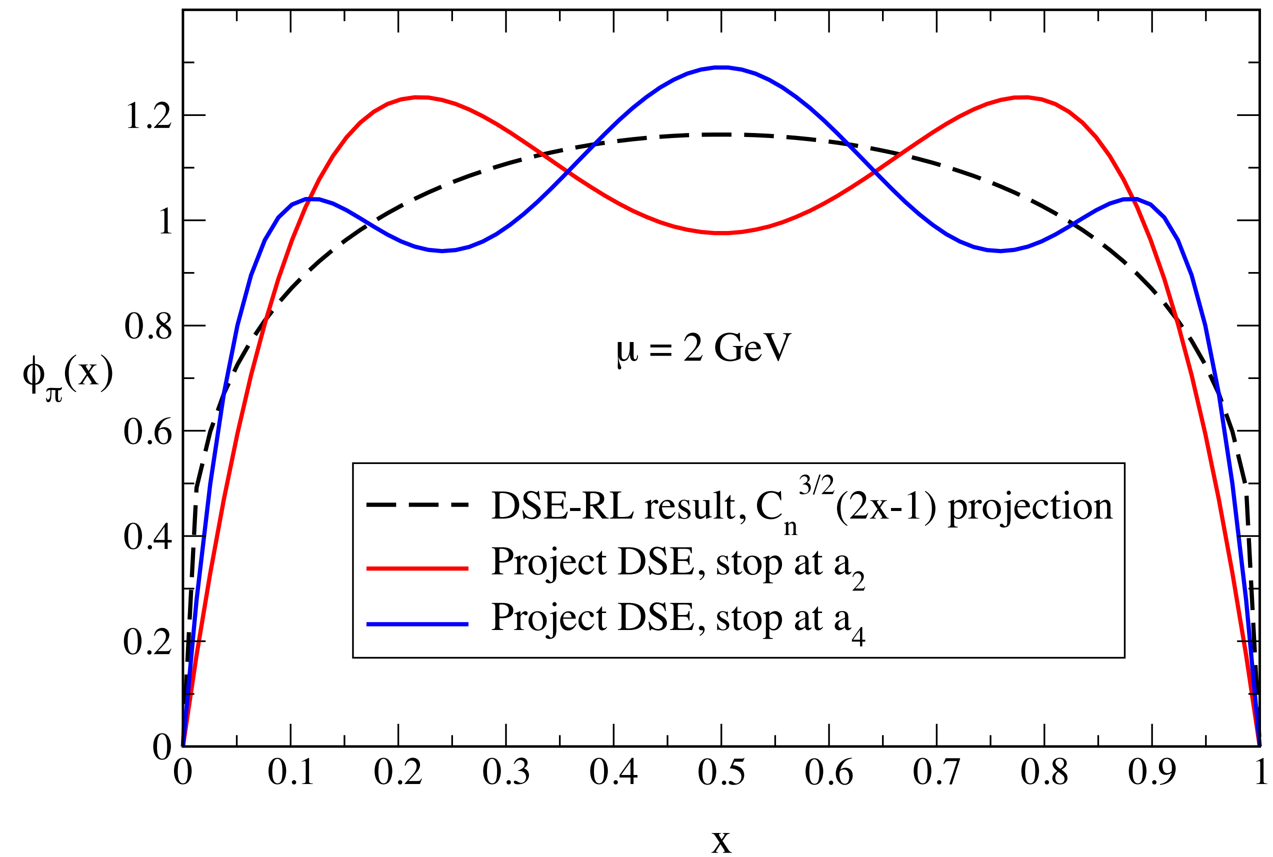
DSE soln

{{0, 1.}, {**2, 0.233104**}, {4,0.112135},

{6, 0.0683202}, {8, 0.0469145},
{10, 0.0346469}, {12, 0.0268732},
{**14, 0.0215933**}, {16, 0.0178199}, **10%**
{18, 0.0150159}, {20, 0.0128672},
{22, 0.0111788}, {24, 0.00982438},
{26, 0.00871886}, {28, 0.00780296},

{30, 0.00703438}, {32, 0.0063823},
{34, 0.00582279}, {36, 0.00534272},
{38, 0.00493277}, {**40, 0.00447911**}} +.....

2%



A double-humped PDA is almost ruled out by
V. Braun, I. Filyanov, Z. Phys. C44, 157 (1989)

$$\phi_\pi^{\mathbf{QCDSR}}(\mathbf{x}=\mathbf{1/2};\mu=\mathbf{2}) = \mathbf{1.2 \pm 0.3}$$

One Lattice-QCD Moment Almost Determines Pion DA

PRL 111, 092001 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Pion Distribution Amplitude from Lattice QCD

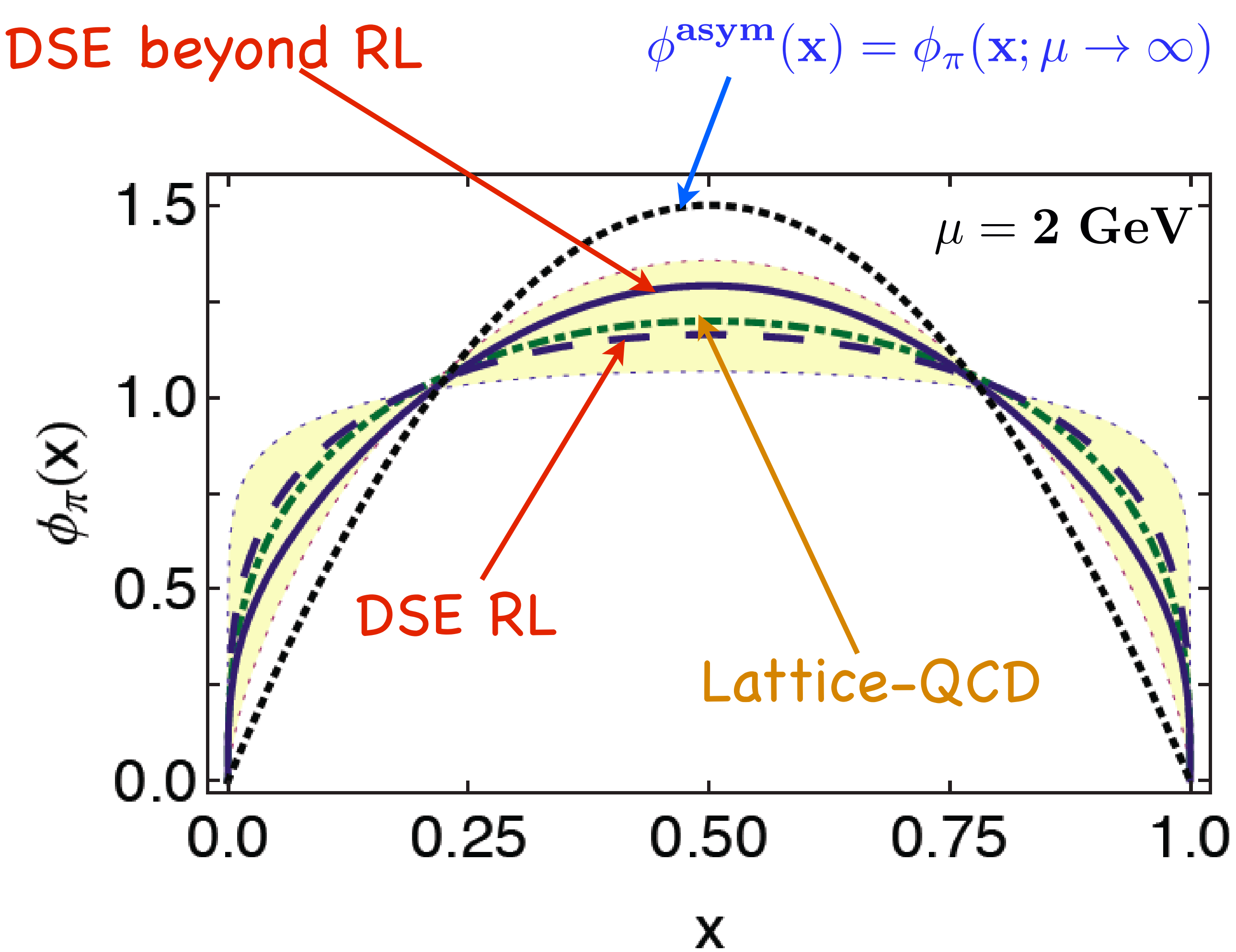
I. C. Cloët,[1] L. Chang,[2] C. D. Roberts,[1] S. M. Schmidt,[3] and P. C. Tandy[4]

$$\phi_\pi^{\mathbf{LQCD}}(\mathbf{x};\mu=\mathbf{2}) = \mathbf{N\,x^\alpha(1-x)^\alpha}$$

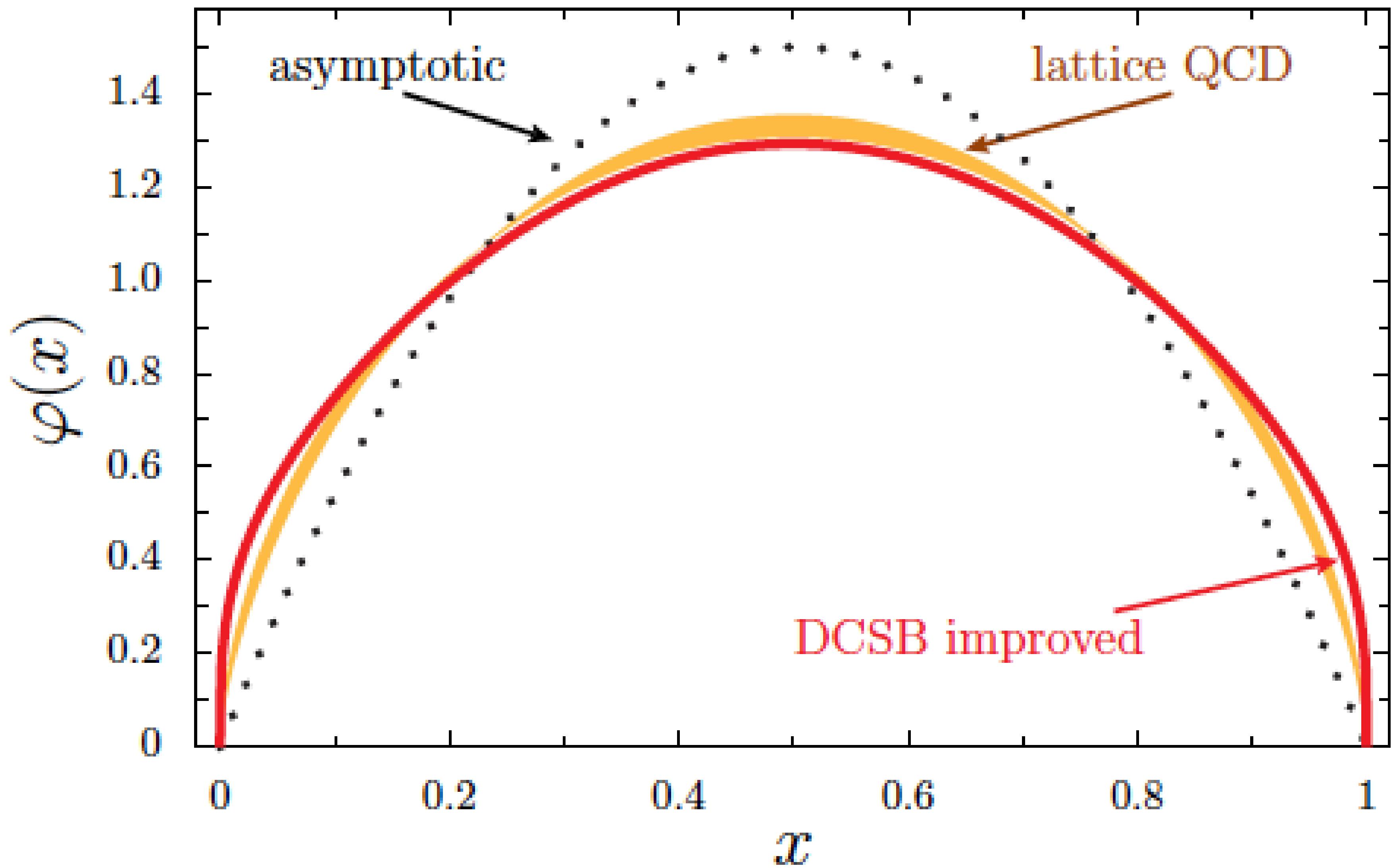
$$\alpha = \mathbf{0.35 + 0.32 - 0.24}$$

$$\langle(\mathbf{2x-1})^2\rangle^{\mathbf{LQCD}}_{\mu=\mathbf{2}} = \mathbf{0.27 \pm 0.04}$$

V. Braun et al., PRD74, 074501 (2006)



Pion Distribution Amplitude



$$\langle\, (2\mathrm{x}-1)^2 \,\rangle^{\mathrm{LQCD}}_{\mu=2\ \mathrm{GeV}} = 0.2361\,(41)\,(39)$$

V. Braun et al., arXiv:1503.03656 [hep=lat]

DSE prediction: 0.251

Excited Pion (1300) Distribution Amplitude

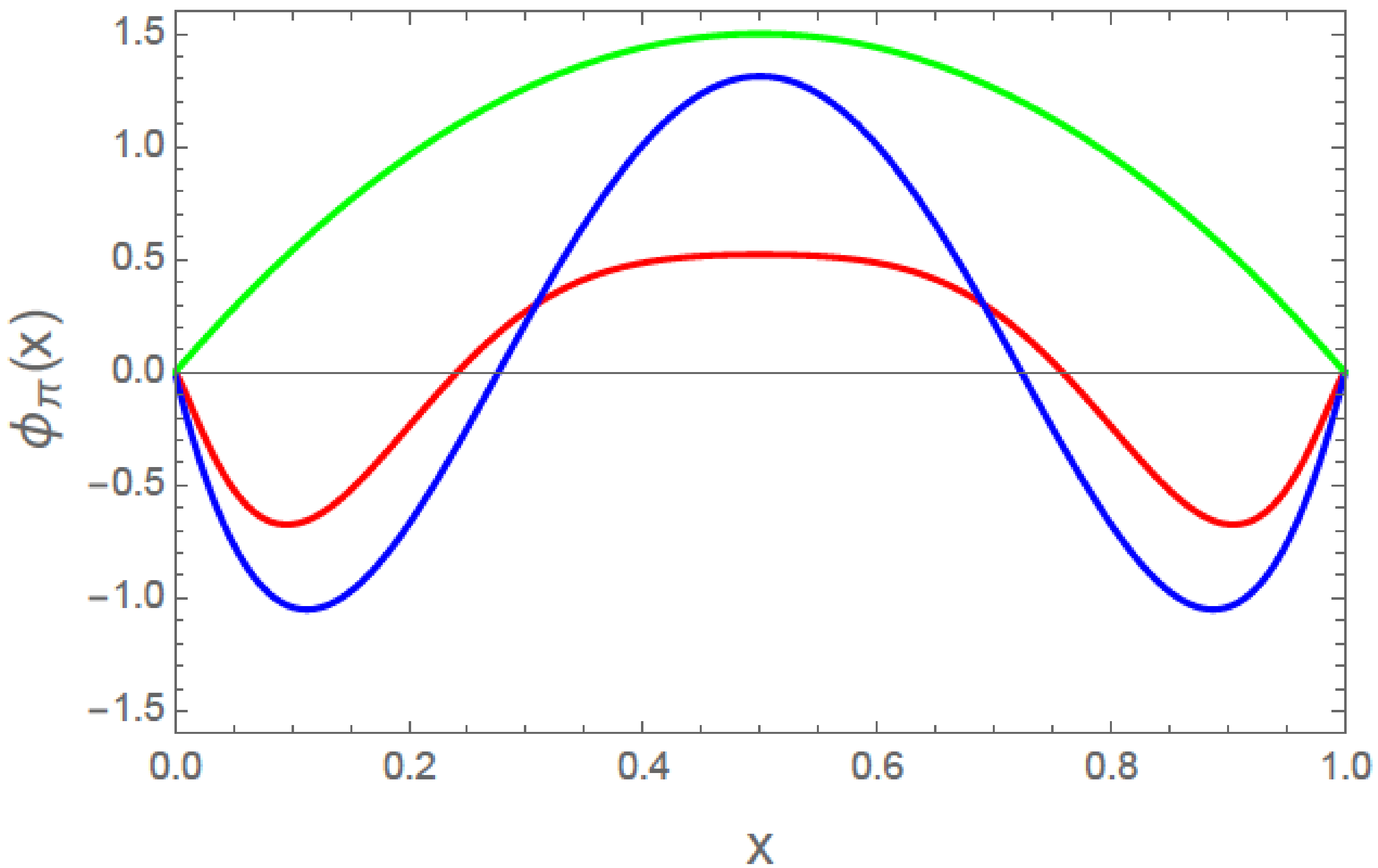


Figure 2: The red line is the calculated radial excited pion distribution amplitude, while the blue line is its conformal limit. The green line is the conformal limit of the ground pion distribution amplitude.

Bo-Lin, L. Chang, C.D.Roberts, H-S., Zong, in progress 2016, Nanjing U.

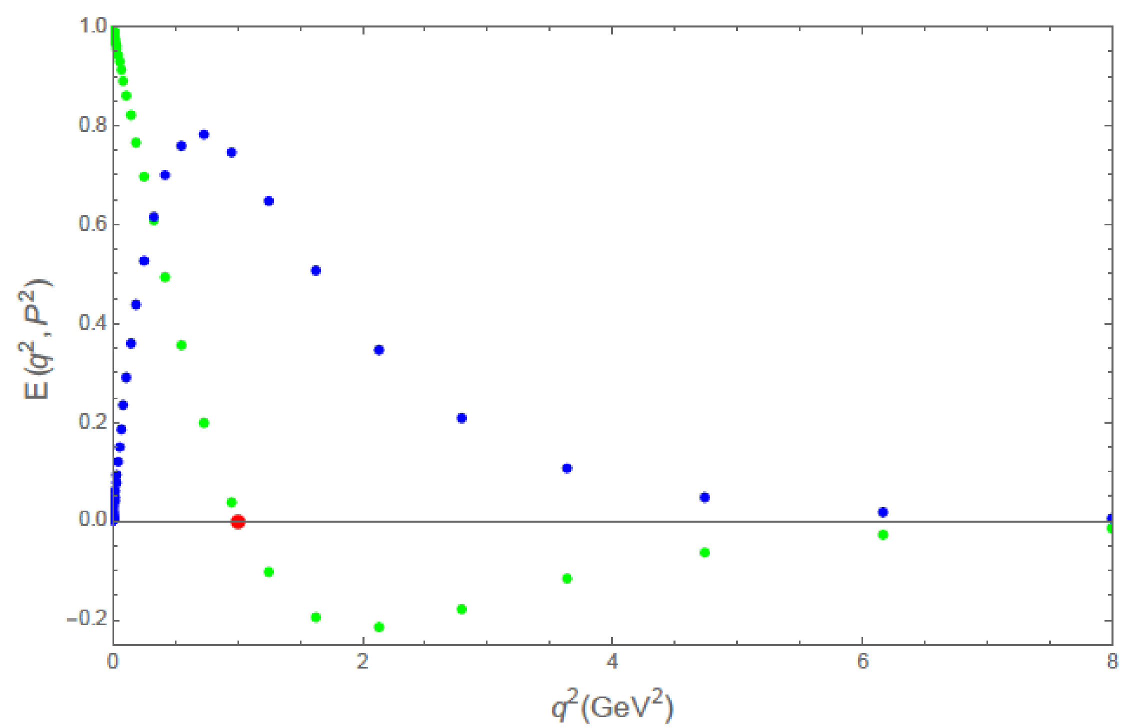


Figure 3: The green dotted line is the zeroth Chebyshev moment of E(q ; P), while the blue dotted one is the second Chebyshev moment.

Kaon Distribution Amplitude

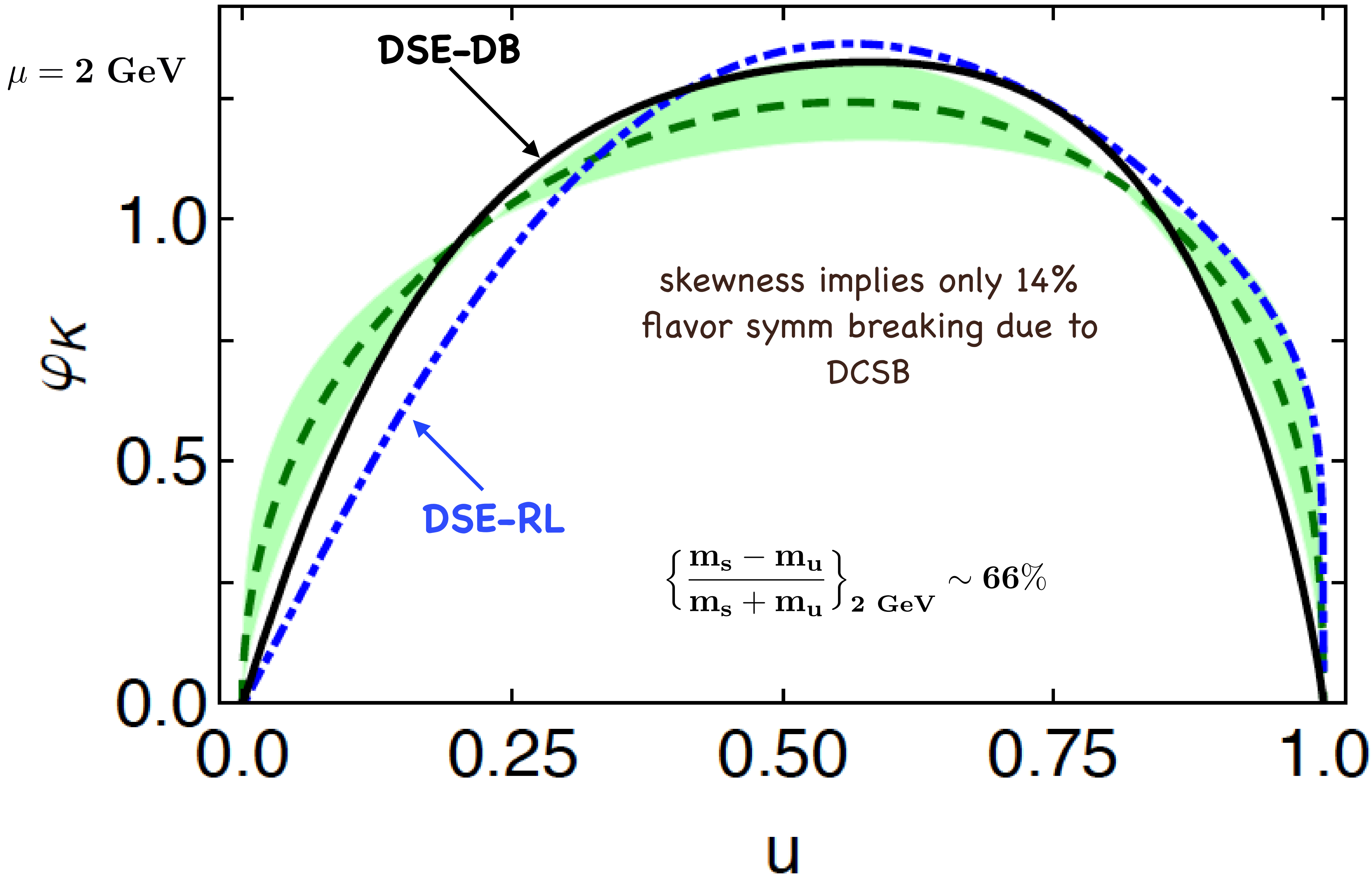
Size of SU(2)xSU(3) spin-flavor symmetry-breaking?

that, as strong interaction bound states whose decay is mediated only by the weak interaction, so that they have a relatively long lifetime, kaons have been instrumental in establishing the foundation and properties of the Standard Model; notably, the physics of CP violation. In this connection the nonleptonic decays of B mesons are crucial because, e.g., the transitions $B^{\pm} \to (\pi K)^{\pm}$ and $B^{\pm} \to \pi^{\pm}\pi^{0}$ provide access to the imaginary part of the CKM matrix element V_{ub}: $\gamma = \mathrm{Arg}(V_{ub}^{*})$ [4]. Factorisation theorems have been derived and are applicable to such decays [5]. However, the formulae involve a certain class of so-called "non-factorisable" corrections because the parton distribution amplitudes (PDAs) of strange mesons are not symmetric with respect to quark and antiquark momenta. Therefore, any derived estimate of γ is only as accurate as the evaluation of both the difference between K and π PDAs and also their respective differences from the asymptotic distribution, $\varphi^{\mathrm{asy}}(u) = 6u(1-u)$. Amplitudes of twist-two and -three are involved. With this motivation, we focus on the twist-two amplitudes herein.

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)

Kaon Distribution Amplitude

C. Shi, L. Chang, C.D. Roberts, S.Schmidt, PCT, H-S. Zong, PLB738, 512 (2014)



R. Arthur, P. Boyle, D. Brommel, M. Donnellan, J. Flynn et al, PRD83, 074505 (2011)

Kaon DA Moments

$\mu = \mathbf{2\ GeV}$

Table 1
Moments ($u_\Delta = 2u - 1$) of the K-meson PDA computed using Eqs. (11) and (12), compared with selected results obtained elsewhere: Refs. [40,41], lattice-QCD; Ref. [10], analysis of lattice-QCD results in Ref. [41]; Refs. [42–46], compilation of results from QCD sum rules; and Ref. [47], holographic soft-wall *Ansatz* for the kaon's light-front wave function. We also list values obtained with $\varphi = \varphi^{asy}$, Eq. (14), and $\varphi = \varphi_{ms}$, Eq. (16), because they represent lower and upper bounds, respectively, for concave distribution amplitudes.

	$\langle u_\Delta^m \rangle$	$m=1$	2	3	4	5	6
DSE-QCD:	RL	0.11	0.24	0.064	0.12	0.045	0.076
	DB	0.040	0.23	0.021	0.11	0.013	0.063
Lattice-QCD:	[40]	0.027(2)	0.26(2)				
	[41]	0.036(2)	0.26(2)				
	[10]	0.036(2)	0.26(2)	0.020(2)	0.13(2)	0.014(2)	0.085(15)
QCD Sum Rules:	[42–46]	0.035(8)					
	[47]	0.04(2)	0.24(1)				
	$\varphi = \varphi_{ms}$	0.33	0.33	0.2	0.2	0.14	0.14
	$\varphi = \varphi^{asy}$	0	0.2	0	0.086	0	0.048

Shi Chao, L. Chang, C.D. Roberts, P.C. Tandy, PLB738, 512 (2014)

Form Factors

The Pion Charge Form Factor: Transition from npQCD to pQCD

$$\mathbf{F_\pi(Q^2 = uv) = \int_0^1 dx \int_0^1 dy\, \phi^\star_\pi(x;Q)\, [T_H(x,y;Q^2)]\, \phi_\pi(y;Q)}$$ + NLO/higher twist.....

---LFQCD, Brodsky, LePage PRD (1980)

$$\mathbf{Q^2 >> \Lambda^2_{QCD} :\ Q^2 F_\pi(Q^2) \rightarrow 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, \omega_\phi^2(Q^2) \ +\ \mathcal{O}(1/Q^2)}$$

$$\mathbf{\omega_\phi(Q^2) = \frac{1}{3}\int_0^1 dx\, \frac{\phi_\pi(x;Q)}{x}}$$

$$\mathbf{\rightarrow 1\ ,\quad Q^2 \rightarrow \infty}$$

$\mathbf{at\ Q^2 \sim 3-4\ GeV^2, \Rightarrow 0.1}$

$\mathbf{JLab\ expt,\quad Theory \Rightarrow 0.45}$

$\mathbf{But, recent\ DSE\ theory \Rightarrow \phi_\pi(x;\mu = 2\ GeV) \Rightarrow \omega_\phi^2 = 3.3}$

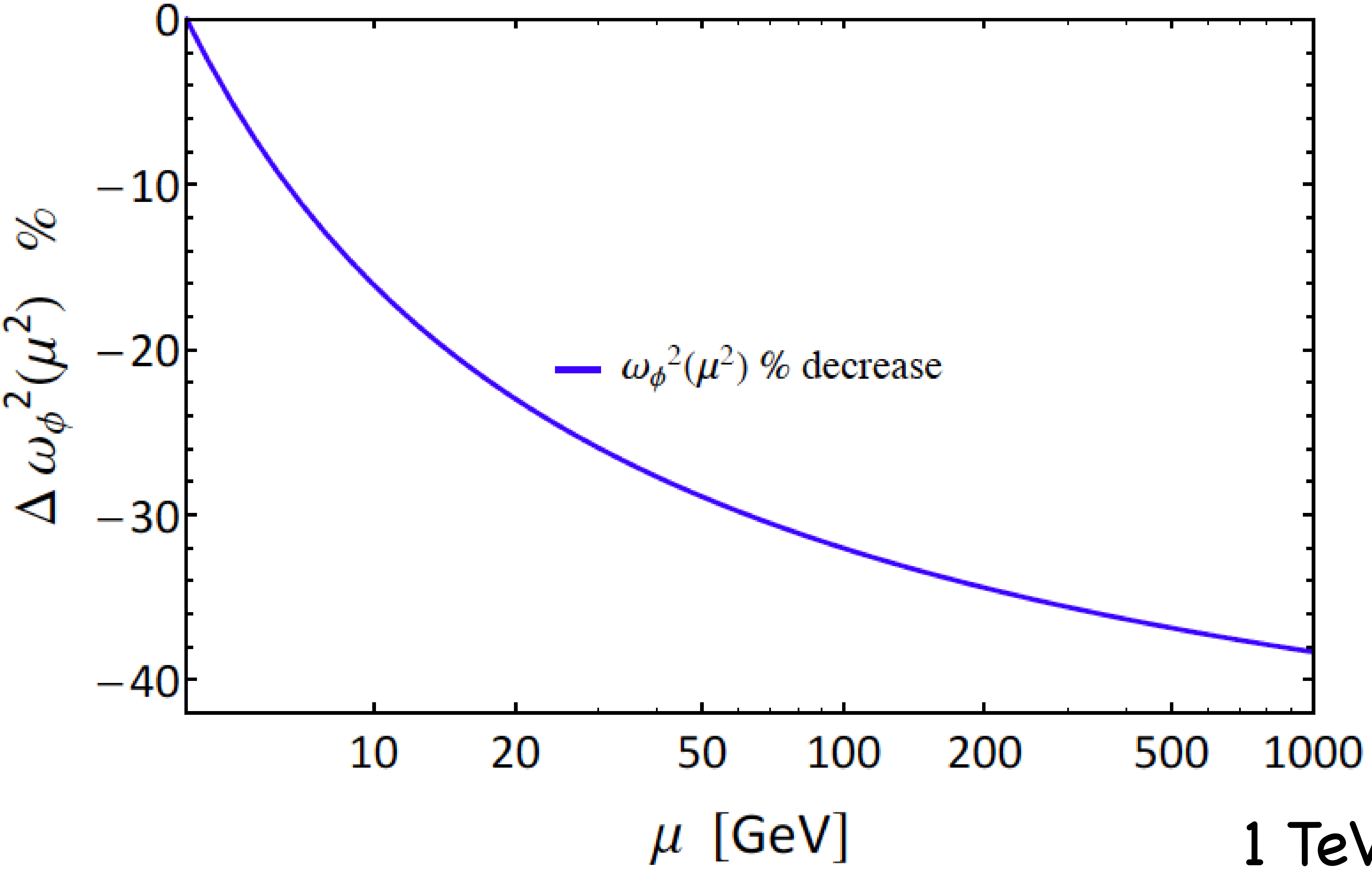
PRL 111, 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I. C. Cloët,[2] C. D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

UV-QCD is not Asymptotic QCD

$$\mathbf{Q^2} >> \mathbf{\Lambda^2_{\rm QCD}} : \ \mathbf{Q^2 F_\pi(Q^2)} \to \mathbf{16}\,\pi\,\mathbf{f_\pi^2}\,\alpha_{\mathbf{s}}(\mathbf{Q^2})\ \omega_\phi^{\mathbf{2}}(\mathbf{Q^2})\ +\ \mathcal{O}(\mathbf{1/Q^2})$$

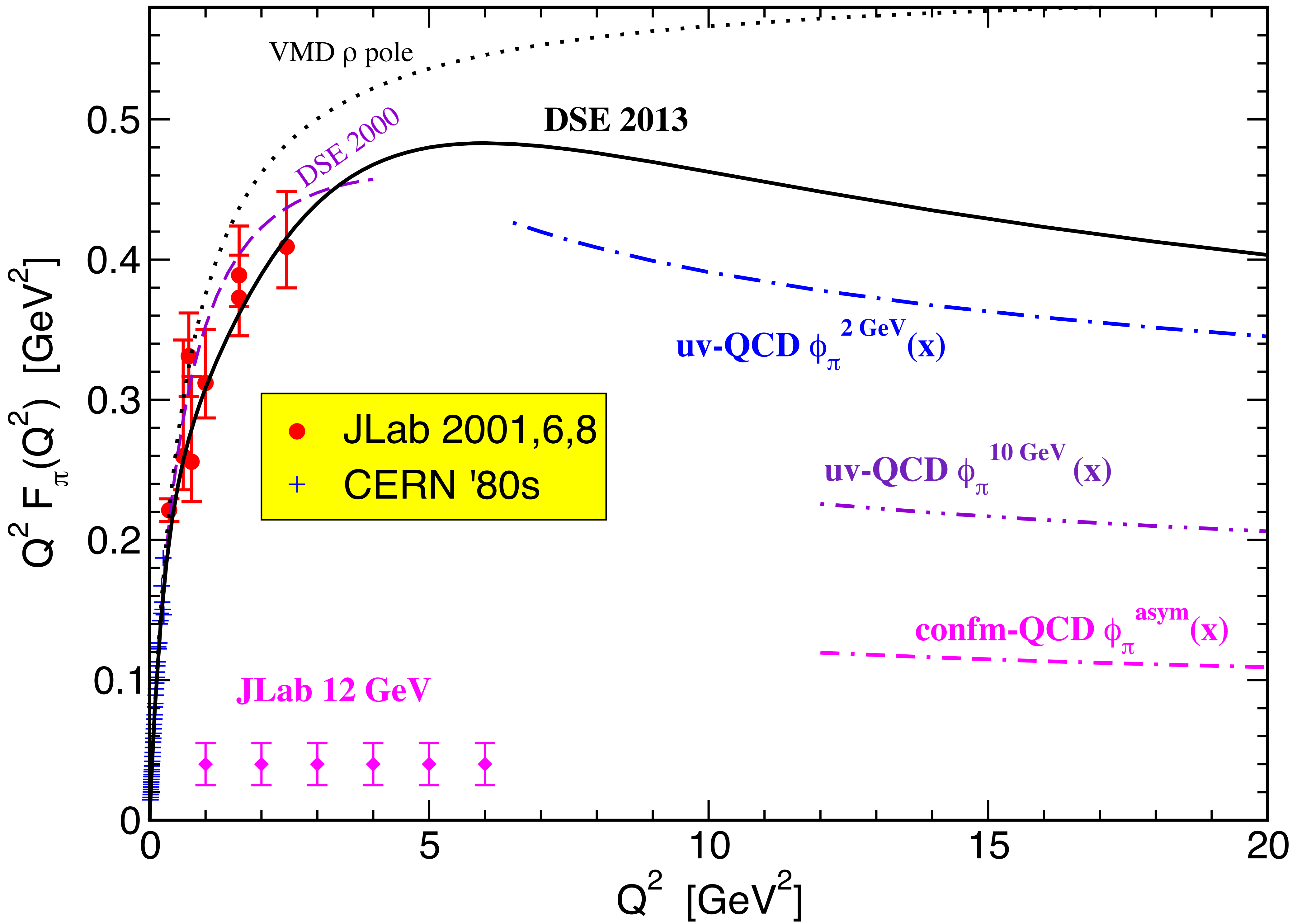


1 TeV

PRL 111, 141802 (2013) PHYSICAL REVIEW LETTERS week ending 4 OCTOBER 2013

Pion Electromagnetic Form Factor at Spacelike Momenta

L. Chang,[1] I. C. Cloët,[2] C. D. Roberts,[2] S. M. Schmidt,[3] and P. C. Tandy[4]

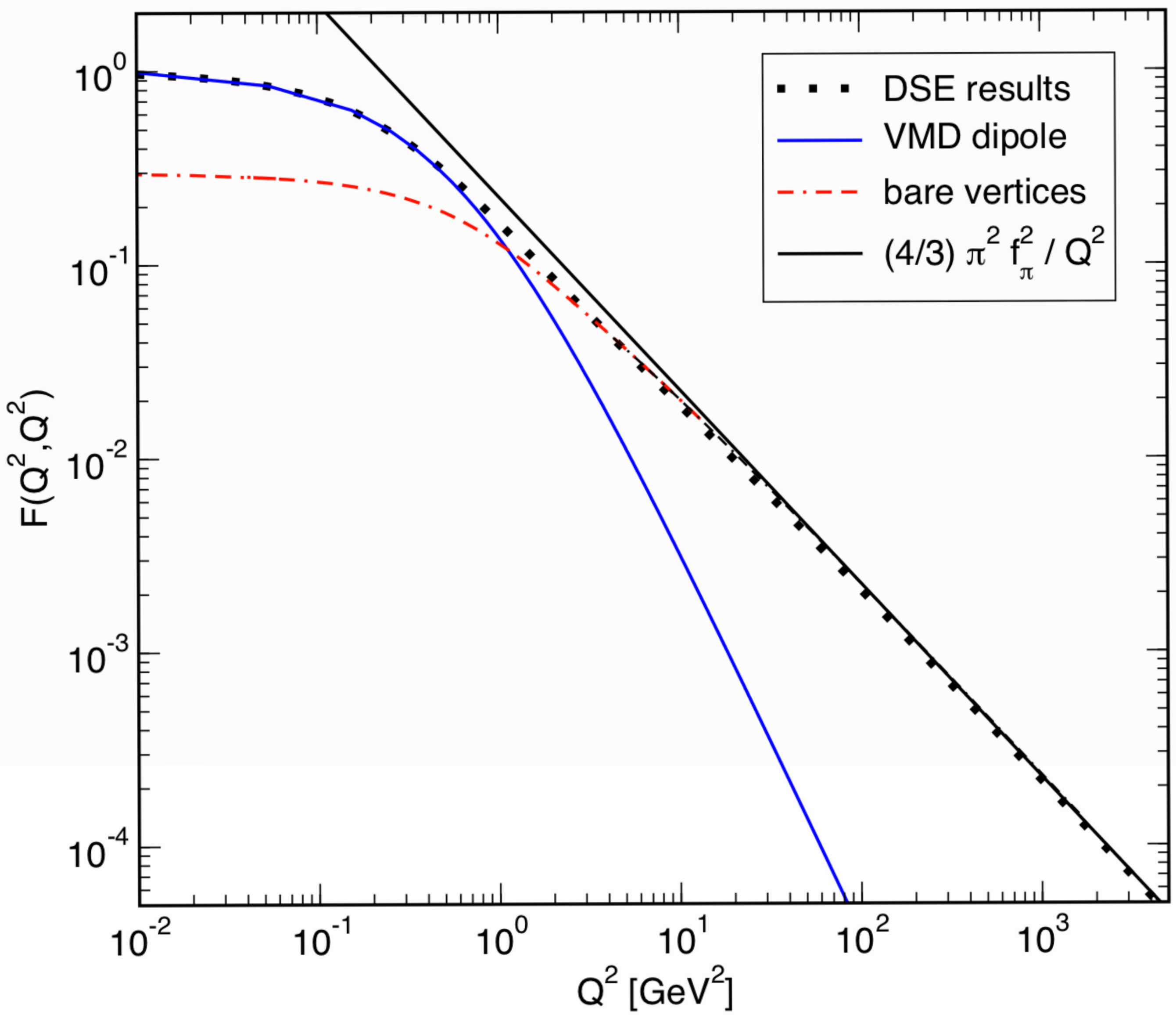


Jab data: G. Huber et al., PRC78, 045203 (2008)

Where Asym FF Could be Calculated, its Power Law was Correct:-

$\gamma^\star\pi\gamma^\star$ Asymptotic Limit

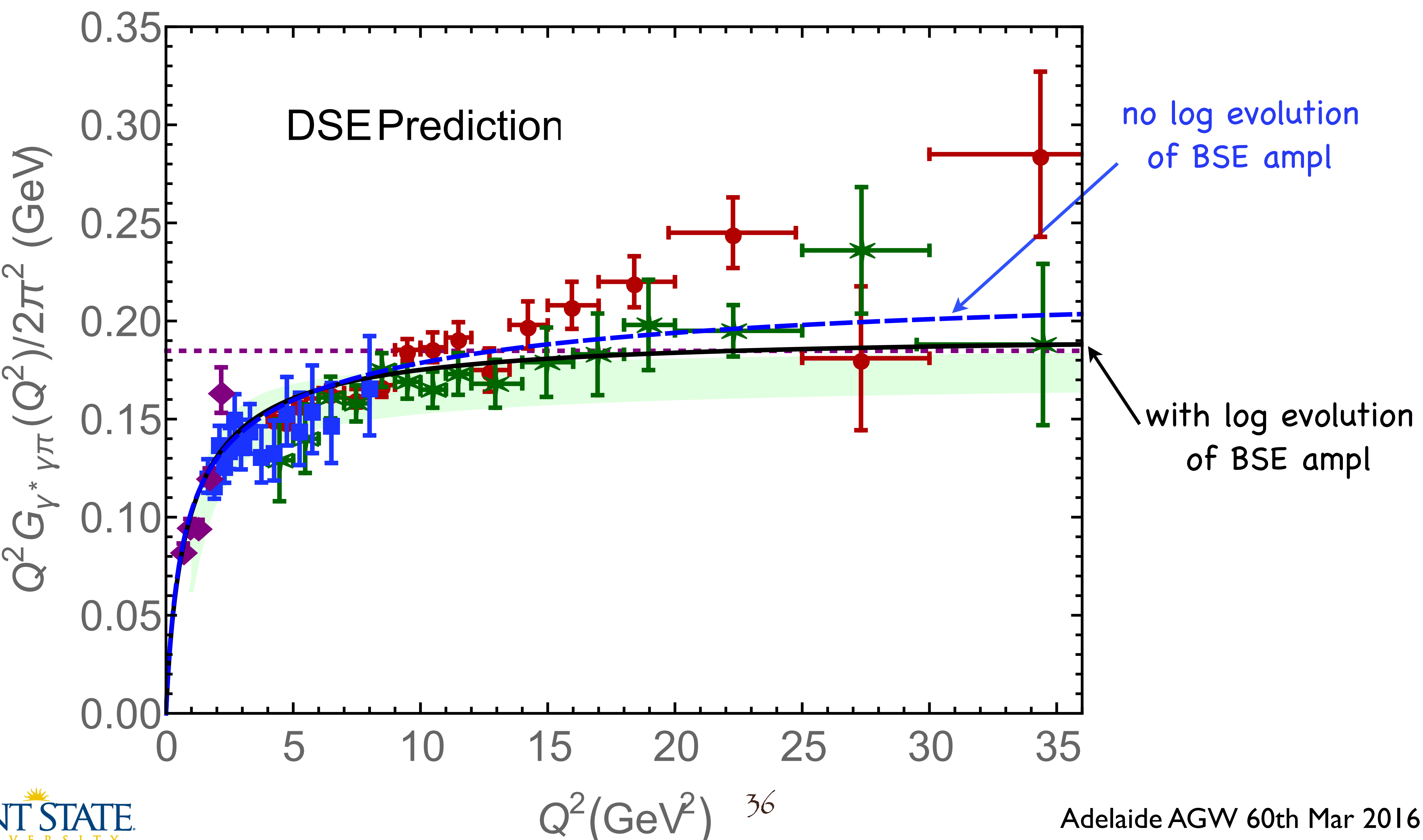
Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE $\Rightarrow$



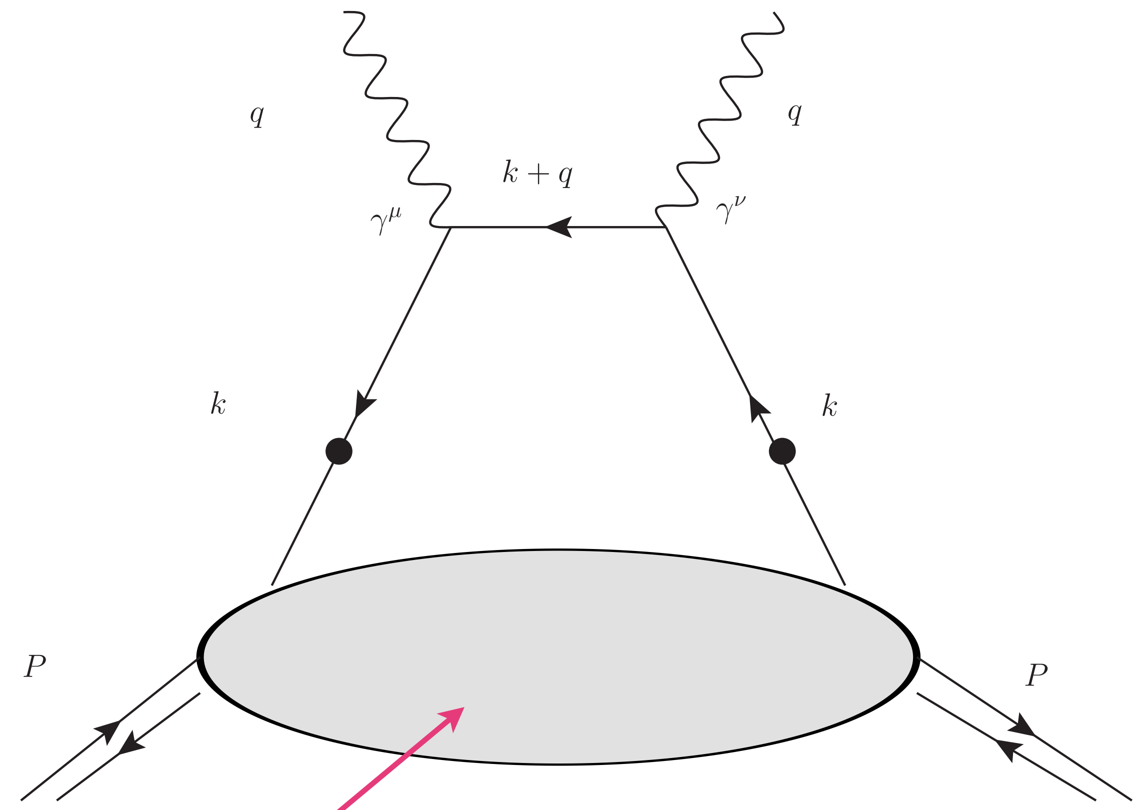
Pion Transition Form Factor

K. Raya, L. Chang, A. Bashir, J.J.Cobos-Martinez, L.X. Gutierez-Guerrero, C.D.Roberts, P.C.Tandy,
arXiv:1510.02799

From unified treatment of DA, elastic FF, and transition FF



Parton Distribution Functions

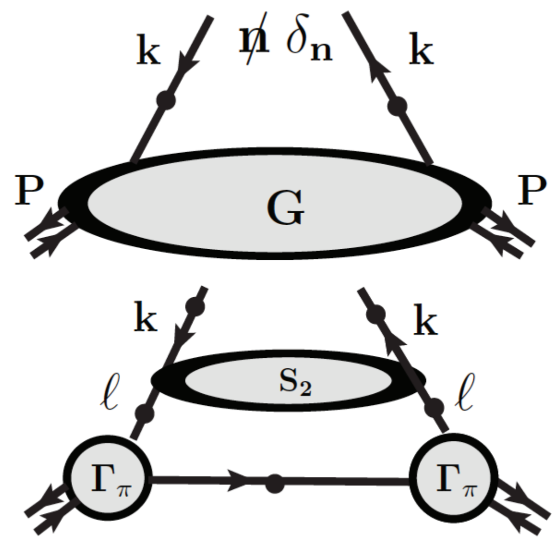


Covariant formulation
and calculation

$$\int \mathbf{d^4q\ \ F(q^2, q\cdot P, q\cdot k, k^2)}$$

The Leading Order PDF

$$\mathbf{q_f(x) = \frac{1}{4\pi}\int d\lambda\ e^{-ixP\cdot n\lambda}\ \langle\pi(P)|\ \bar{\psi}_f(\lambda n)\ \not{n}\ \psi_f(0)\ |\pi(P)\rangle_c}$$



RL DSE:

q(x) From Directly Obtained Moments

$$\langle \mathbf{x^m}\rangle^{\mathrm{RL}}_{\mathbf{v}} = \frac{-\mathbf{N_c}}{\mathbf{2P\cdot n}}\mathrm{tr}\int_\ell \mathbf{\Gamma}_\pi(\ell-\frac{\mathbf{P}}{\mathbf{2}})\ [(\frac{\ell\cdot\mathbf{n}}{\mathbf{P\cdot n}})^{\mathbf{m}}\,\mathbf{n}\cdot\partial_\ell\mathbf{S}(\ell)]\ \mathbf{\Gamma}_\pi(\ell-\frac{\mathbf{P}}{\mathbf{2}})\ \mathbf{S}(\ell-\mathbf{P})$$

$$\mathbf{Method\ can\ easily\ exceed\ the\ Lattice-QCD\ practical\ limit: m=3}$$

Pion PDFs—Expt "Data" Parameterizations

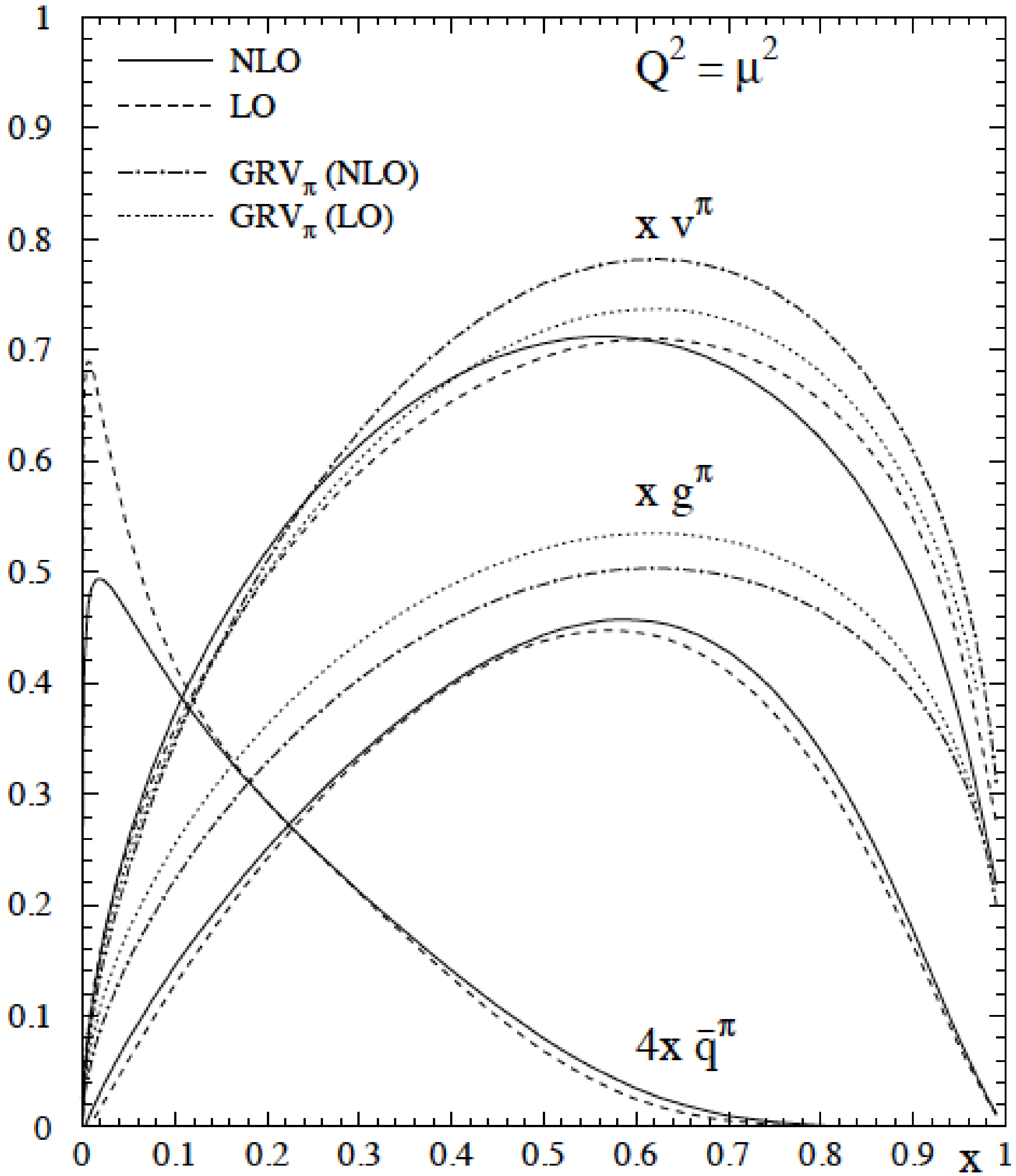


Fig. 1. The valence and valence–like input distributions $xf^\pi(x,Q^2=\mu^2)$ with $f=v,\bar{q},g$ as compared to those of GRV$_\pi$ [5]. Notice that GRV$_\pi$ employs a vanishing SU(3)$_{\text{flavor}}$ symmetric $\bar{q}^\pi$ input at $\mu^2_{\text{LO}} = 0.25$ GeV^2 and $\mu^2_{\text{NLO}} = 0.3$ GeV^2 [5]. Our present SU(3)$_{\text{flavor}}$ broken sea densities refer to a vanishing s^π input in (3), as for GRV$_\pi$ [5]

Eur. Phys. J. C 10, 313–317 (1999)
Digital Object Identifier (DOI) 10.1007/s100529900124

Pionic parton distributions revisited

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Aicher, Schafer, Vogelsang,
arXiv:1009.2481
soft gluon resummation

Estimate 1-Pion Loop Contribution to Pion PDF

$$\pi^+ \;:\; \langle \mathbf{x^1} \rangle_\mu = \int_0^1 \mathbf{dx\,x}\,\{\mathbf{u} + \mathbf{\bar{u}_{sea}} + \mathbf{\bar{d}} + \mathbf{d_{sea}} + \mathbf{g(x)}\} \approx \mathbf{2\langle x\,q_v(x)\rangle + 4\langle x\,q_{sea}(x)\rangle + \langle x\,g(x)\rangle = 1}$$

$$\mathbf{u = u_v + u_{sea}}\;, \quad \mathbf{\bar{d} = \bar{d}_v + \bar{d}_{sea}}$$

Empirical GRS/ASV $\Rightarrow$ universal $\mathbf{q_v(x)},\ \mathbf{q_{sea}(x)}$ at $\mu = 0.630$ GeV

$$\mathbf{\Gamma_\pi = \sqrt{1-\alpha^2}\,\Gamma^{RL}_{q\bar{q}} + \alpha\,\Gamma_{\pi q\bar{q}}}$$

CPT: 18% effect

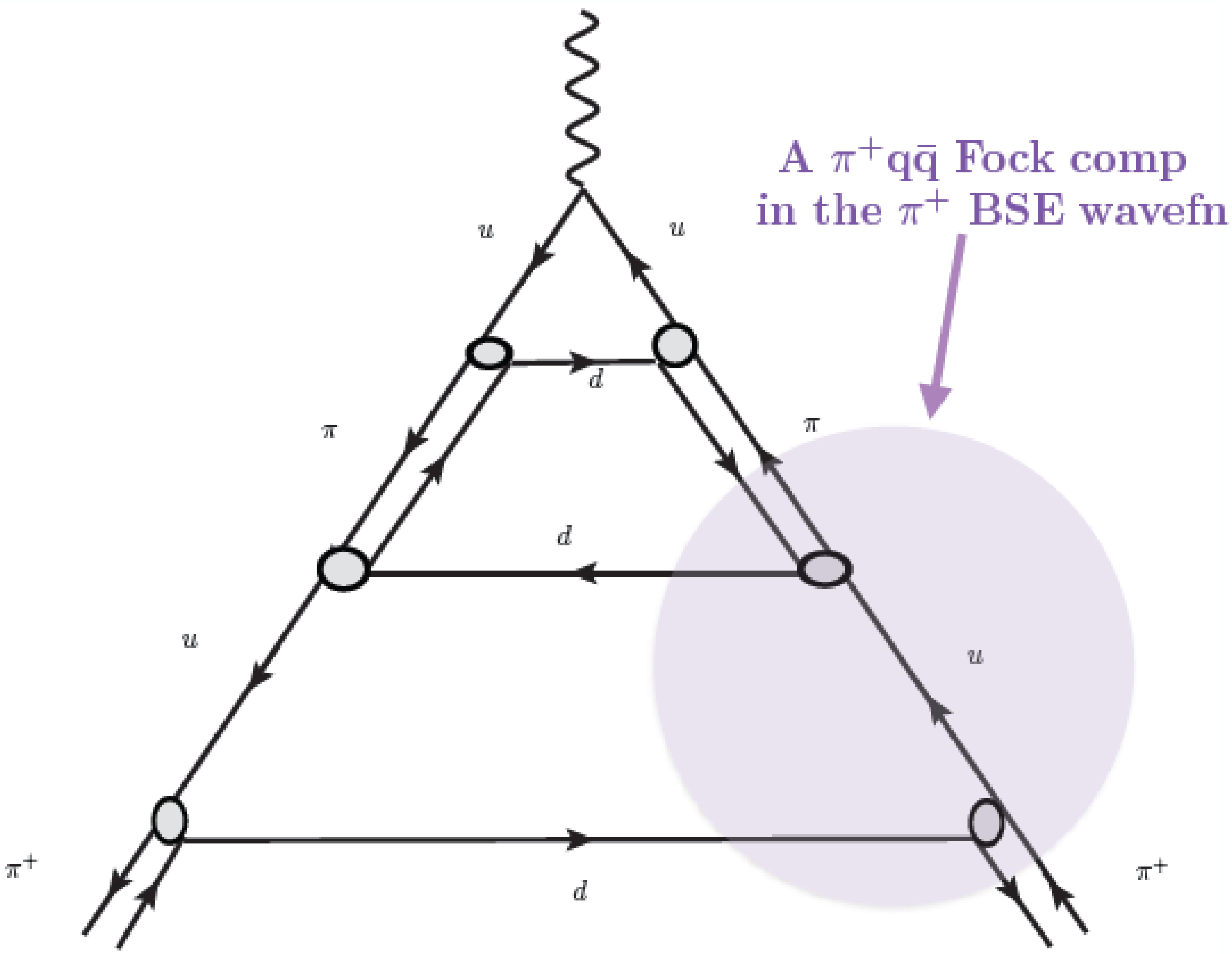
$$\mathbf{r^2_{ch} = (1-\alpha^2)\,r^2_{RL} + \boxed{\alpha^2\,r^2_{\pi-lp}}}$$

DSE-RL: $\mathbf{r^2_{RL} = r^2_{ch} \Rightarrow \alpha^2 = 18\%}$

PDF Consequence:

$$\mathbf{q_v(x) = (1-\alpha^2)\,q^{RL}(x) + q_v^{\pi-lp}(x)}$$

with $\mathbf{\langle q_v^{\pi-lp}(x)\rangle = \alpha^2 = 0.18}$



Adelaide AGW 60th Mar 2016

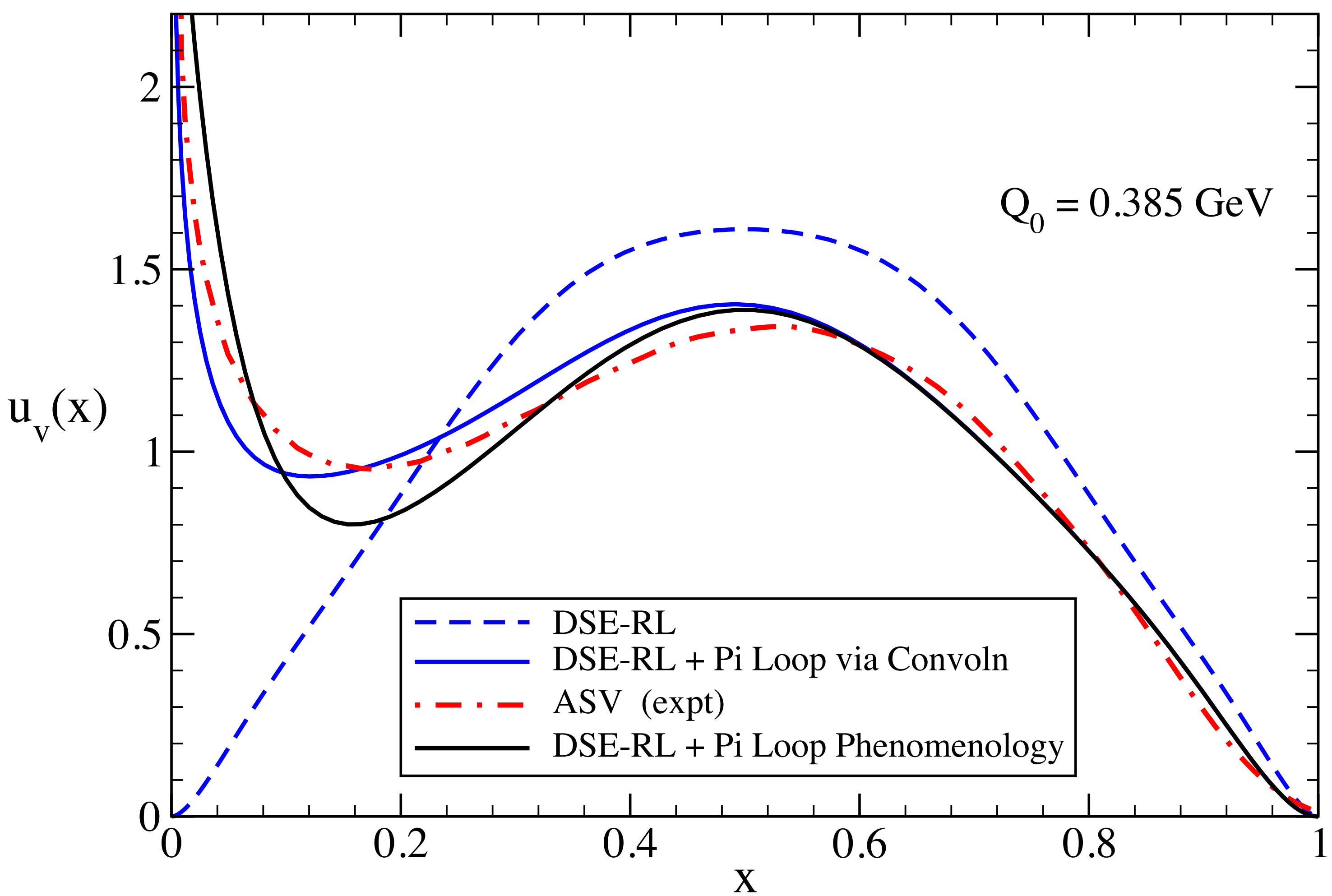
Analysis of Pion Parton Momentum Sum Rule

TABLE II: Momentum fraction sum rule from this work at scale $Q_0 = 0.630$ GeV corresponding to the ASV [13] compilation.

	$2\,q_{\rm val}^{\rm RL}$	$2\,q_{\rm val}^{\rm DSE}$	$4\,q_{\rm sea}^{\rm ASV}$	gluon	Total
$\langle x \rangle_\pi$	0.770	0.649	0.0498	0.300	0.999

K. Khitrin, P. Tandy, in progress (2015)

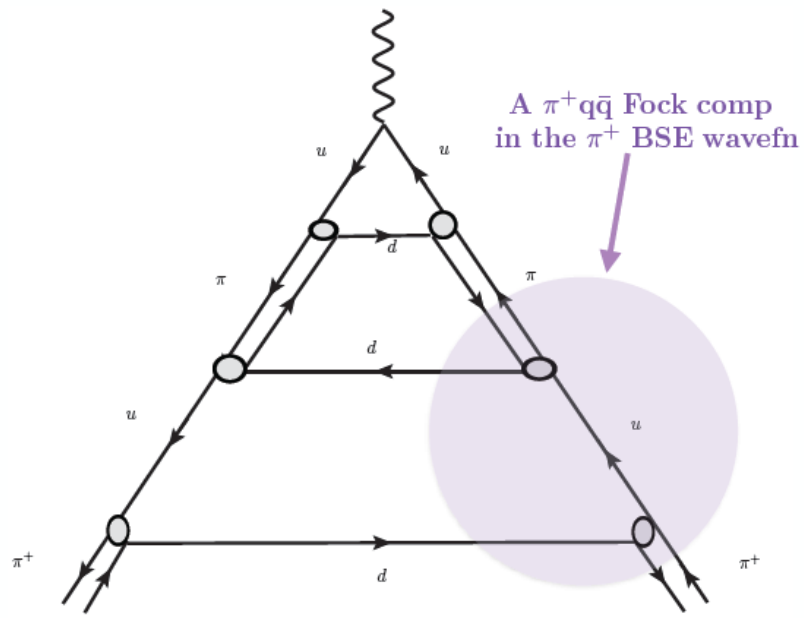
Modern empirical expt parameterization:
Aicher, Shafer, Vogelsang, (ASV) PRL 105, 252003 (2010)



Convolution Model for q(x) from virtual pi loop

$$\mathbf{q_v^{\pi-lp}(x)} \sim \mathcal{P}_{\mathbf{q/T}}(\mathbf{x}) = \int_{\mathbf{x}}^{\mathbf{1}} \mathbf{dy}\ \mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})\ \mathcal{P}_{\mathbf{q}/\pi}(\tfrac{\mathbf{x}}{\mathbf{y}})\ ,$$

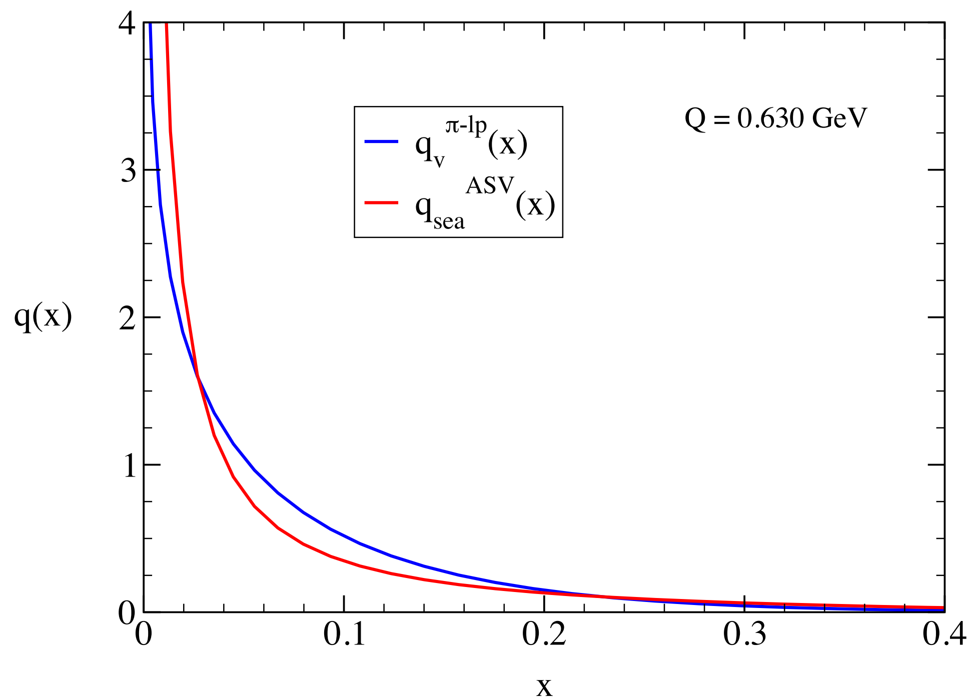
$$\mathbf{T} = \ \mathbf{target} = \pi\ \mathbf{here}$$



$\mathcal{P}_{\pi/\mathbf{T}}(\mathbf{y})$ **should strongly favor** $\mathbf{y} \leq \frac{\mathbf{m}_\pi}{\mathbf{2M_q+m_\pi}} \approx \mathbf{0.2}$,

$\mathcal{P}_{\mathbf{q}/\pi}(\frac{\mathbf{x}}{\mathbf{y}})$ **is self – consistently determined**

Result is strongly constrained



Summary

- X. Ji's space-like correlator approach to PDFs—a model investigation. Spurious anti-quark contributions seem unavoidable if Pz < 2 GeV. For x > 0.8, need Pz > 4 GeV for confidence in the qualitative shape. Further work in progress.

- Parton Distribution Amplitudes (pion, kaon). DSE approach shows good contact with available lattice-QCD moments. Flavor symmetry breaking in kaon DA made quantitative.

- Pion Transition & Elastic Form Factors DSE TFF calculation for all Q^2—- agrees with Belle not BaBar. DSE elFF——Connection with asymptotic QCD reconciled. Identify that the ultraviolet partonic behavior is within reach of proposed JLab pion FF experiments.

- Parton Distribution Functions (pion). Qualitative behavior of empirical data fits reproduced by DSE q-qbar + pion loop analysis.

- Time to declare we understand the pion and kaon in QCD ?

Congratulations Tony !

The End