

# Phases and facets of 2-colour matter

Jon-Ivar Skullerud  
with Tamer Boz, Seamus Cotter, Leonard Fister  
Pietro Giudice, Simon Hands

Maynooth University

New Directions in Subatomic Physics, CSSM, 10 March 2016

# Outline

## Background

QC<sub>2</sub>D vs QCD

Lattice formulation

## Phase transitions

## Gluon propagator

## Quark propagator

## Summary

# Tony and lattice QCD

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Anthony G. Williams (Adelaide U.), FT. Havers (Florida State U. & Florida State U., SCRI), Jul 1995, 4 pp.  
Published in *Nucl.Phys.Proc.Suppl.* **47 (1996)** 691-694  
ADP-95-48-T-195, C95-07-11  
DOI: [10.1016/0592-5532\(96\)00152-1](#)  
Presented at Conference: [C95-07-11 \(Lattice 1995:0691-694\)](#) [Proceedings](#)  
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- Optimization of Monte Carlo calculations of the effective potential**  
A. Ardehali, Anthony G. Williams (Adelaide U.), May 1997, 26 pp.  
Published in *Phys.Rev.E* **57 (1998)** 6140-6151  
ADP-97-13-T-250  
DOI: [10.1103/PhysRevE.57.6140](#)  
e-Print: [hep-lat/9705021](#) | [PDF](#)  
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- The infrared behavior of the gluon propagator from lattice QCD**  
UKQCD Collaboration (D.B. Leinweber et al.), Feb 1998, 7 pp.  
Published in *In "Adelaide 1998, Nonperturbative methods in quantum field theory"* 97-103  
ADP-98-31-T304  
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- Gluon propagator in the infrared region**  
UKQCD Collaboration (Derek B. Leinweber et al.), Mar 1998, 13 pp.  
Published in *Phys.Rev.D* **58 (1998)** 031501  
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- Lattice gauge theory studies of the gluon propagator**  
UKQCD Collaboration (D.B. Leinweber et al.), Jun 1998, 8 pp.  
ADP-98-52-T320  
Talk given at Conference: [C98-10-14.1](#), Talk given at Conference: [C98-06-01.4 Proceedings](#)  
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PHYSICAL REVIEW D, VOLUME 58, 034504

## Lattice calculation of the strangeness magnetic moment of the nucleon

S. J. Dong and K. F. Liu

Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506

A. G. Williams

Special Research Centre for the Subatomic Structure of Matter and Department of Physics and Mathematical Physics,

University of Adelaide, 5001 Australia

(Received 22 December 1997; published 2 September 1998)

We report on a lattice QCD calculation of the strangeness magnetic moment of the nucleon. Our result is  $G_S^N(0) = -0.38 \pm 0.20$ . The sea contributions from the  $u$  and  $d$  quarks are about 40% larger. However, they cancel to a large extent due to their electric charges, resulting in a smaller net sea contribution of  $-0.007 \pm 0.072$  to the nucleon magnetic moment. As far as the ratios to proton magnetic moment ratio is concerned, this sea contribution tends to cancel out the chiral-quark effect from the  $Z$  graphs and results in a ratio of  $-0.040 \pm 0.046$  which is close to the  $SU(6)$  relation and the experiment. The strangeness Sachs electric magnetic radius  $\langle r_{SE}^2 \rangle$  is found to be small and negative. [S0556-2813(98)01147-7]

PACS numbers: 13.60.Ea, 12.38.Gd, 34.20.Dg

PHYSICAL REVIEW D, VOLUME 58, 031501

## Gluon propagator in the infrared region

Derek B. Leinweber,\* Ken Iwar Skarland,† and Anthony G. Williams†

Special Research Centre for the Subatomic Structure of Matter and The Department of Physics and Mathematical Physics

University of Adelaide, Adelaide 5005, Australia

Claudio Parrinello†

Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom

(UKQCD Collaboration)

(Received 22 March 1998; published 1 July 1998)

The gluon propagator is calculated in quenched QCD for two different lattice sizes ( $16^3 \times 48$  and  $32^3 \times 64$ ) at  $\beta = 6.0$ . The volume dependence of the propagator in the Landau gauge is studied. The smaller lattice is unphysical in violating finite volume and unphysical lattice artifacts. Methods for minimizing these artifacts are developed and applied to the larger lattice data. New structure seen in the infrared region survives these corrections only in the lattice data. This structure seems to rule out a number of models that have appeared in the literature. A fit to a simple analytical form capturing the momentum dependence of the nonperturbative gluon propagator is also reported. [S0556-2813(98)03021-9]

## Background



- ▶ A plethora of phases at high  $\mu$ , low  $T$
- ▶ Based on models and perturbation theory

### Indirect approach

Study QCD-like theories without a sign problem

- ▶ Generic features of strongly interacting systems at  $\mu \neq 0$
- ▶ Check on model calculations, functional methods

QC<sub>2</sub>D vs QCD

- ▶ Baryons are **bosons** (diquarks); superfluid 'nuclear matter'
- ▶ Scalar diquark is pseudo-Goldstone (degenerate with pion)
- ▶ Onset transition at  $\mu_q = m_\pi/2$ , not  $m_N/3$

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### Phase diagram

- ▶ Superfluid phase for  $\mu > m_\pi/2$ : BEC  $\longrightarrow$  BCS?
- ▶ Exotic phases: quarkyonic, spatially varying?
- ▶ Deconfinement at high density, shape of deconfinement line?

## Two-colour quarks and gluons

**Gluodynamics** — SU(2) and SU(3) very similar?

- ▶ Effects of deconfinement on gluon propagation?
- ▶ Gap equation with effective or one-gluon interaction used to determine superconducting gap → more realistic input?

**Quark propagator**

- ▶ Details of phase diagram depend critically on the effective quark mass in the medium.
- ▶ Dynamical quark masses → effective **strange** quark mass?
- ▶ Location of Fermi surface?
- ▶ Direct determination of diquark gap, size of Cooper pairs?

## Lattice formulation

We use **Wilson fermions**:

- ▶ Correct symmetry breaking pattern, Goldstone spectrum
- ▶  $N_f < 4$  needed to guarantee continuum limit
- ▶ No problems with locality, fourth root trick
- ▶ Chiral symmetry buried at bottom of Fermi sea



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$$S = \bar{\psi}_1 M(\mu) \psi_1 + \bar{\psi}_2 M(\mu) \psi_2 - \textcolor{red}{J} \bar{\psi}_1 (C \gamma_5) \tau_2 \bar{\psi}_2^T + \textcolor{red}{J} \psi_2^T (C \gamma_5) \tau_2 \psi_1$$

$$\gamma_5 M(\mu) \gamma_5 = M^\dagger(-\mu), \quad C \gamma_5 \tau_2 M(\mu) C \gamma_5 \tau_2 = -M^*(\mu)$$

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$$\gamma_5 M(\mu) \gamma_5 = M^\dagger(-\mu), \quad C \gamma_5 \tau_2 M(\mu) C \gamma_5 \tau_2 = -M^*(\mu)$$

**Diquark source  $J \equiv \kappa j$**  introduced to

- ▶ lift low-lying eigenmodes in the superfluid phase
- ▶ study diquark condensation without uncontrolled approximations

## Simulation parameters

Name	$\beta$	$\kappa$	$a$	$am_\pi$	$m_\pi/m_\rho$
Coarse	1.9	0.1680	0.18fm	0.65	0.80
Light	1.7	0.1810	0.19fm	0.42	0.61
Fine	2.1	0.1577	0.12fm	0.45	0.81

### $\mu$ -scans, fixed $T$

Ensemble	$N_s$	$N_\tau$	$T$ (MeV)	$\mu a$	$ja$
Coarse	12	24	47	0.25–1.10	0.02, 0.04 (0.03)
	16	24	47	0.30–0.90	0.04
	12	16	70	0.30–0.90	0.04
	16	12	94	0.20–0.90	0.02, 0.04
	16	8	141	0.10–0.90	0.02, 0.04
Fine	16	32	54	0.15–0.80	0.02, 0.03 (0.01)
	16	20	86	0.20–0.60	0.02, 0.03
	16	16	107	0.20–0.60	0.02, 0.03
	16	12	143	0.20–0.60	0.03
Light	12	24	43	0.15–0.90	0.02, 0.04 ( 0.03)

## Simulation parameters

### $T$ -scans, fixed $\mu$ (coarse lattice)

All simulations done on  $16^3 \times N_\tau$  lattices

$\mu a$	$ja$	$N_\tau$
0.0	0.0	4–10
0.35	0.02	4–13, 16
	0.04	4–12, 14, 16
0.40	0.02	5–13, 16
	0.04	4–13
0.50	0.02	6–12, 16
	0.04	4–16, 18, 20
0.60	0.02	6–12, 14, 16
	0.04	6–16, 20

In addition, 300 trajectories were generated at  $ja = 0.03, 0.05$  for  $N_\tau = 9, 10, 11$  at all  $\mu > 0$ .

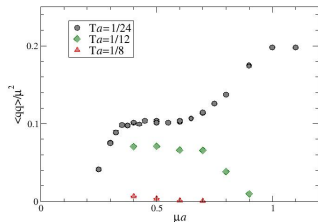
# Diquark condensate — $\mu$ -scan

Coarse lattice

Results shown are for **linear** extrapolation

**Power law**  $\langle qq \rangle = A j^\alpha$  works for  $\mu a \lesssim 0.4$ , with  $\alpha = 0.85 - 0.5$ .

[Effective field theory predicts  $\alpha = \frac{1}{3}$  near onset]



- ▶ **BCS** scaling  $\langle qq \rangle \sim \mu^2$  for  $0.35 \lesssim \mu a \lesssim 0.7$
- ▶ **Melted** at  $T = 141\text{MeV}$  ( $N_\tau = 8$ )

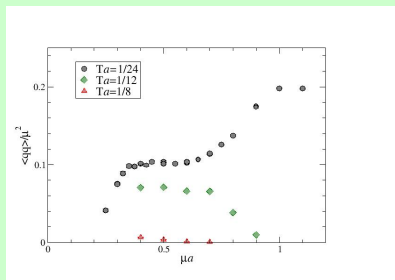
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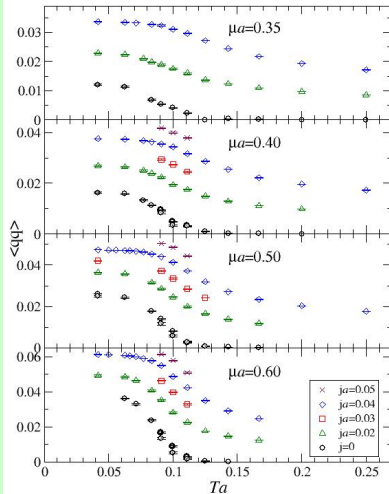
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- ▶ **BCS** scaling  $\langle qq \rangle \sim \mu^2$  for  $0.35 \lesssim \mu a \lesssim 0.7$
- ▶ **Melted** at  $T = 141\text{MeV}$  ( $N_\tau = 8$ )
- ▶ New transition for  $\mu a \gtrsim 0.7$ ?
- ▶ Melting for  $N_\tau = 12, \mu a \gtrsim 0.7$ ?

$N_\tau = 16$  results are very close to  $N_\tau = 24$  results.

# Superfluid to normal transition



$j \rightarrow 0$  extrapolation not fully under control  
Linear form used here

Transition temperatures from inflection points (and  $j \rightarrow 0$ )

$a\mu$	$T_s$ (MeV)
0.35	82(27)
0.40	94(9)
0.50	93(6)
0.60	93(7)

Remarkably constant!

## Deconfinement transition

Polyakov loop  $L$  requires renormalisation,

$$L_R = e^{-F_q/T} = e^{-(F_0+\Delta F)/T} = Z_L^{N_\tau} L_0$$

We use two schemes to determine  $Z_L = \exp(-a\Delta F)$ ,

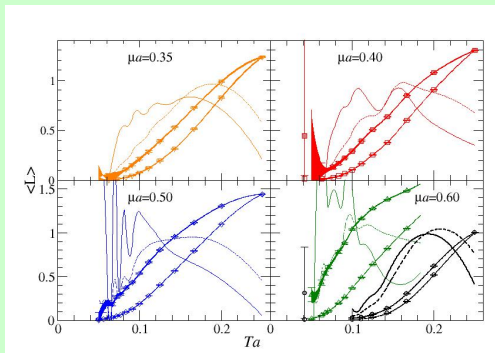
**Scheme A** 
$$L_R(T = \frac{1}{4a}, \mu = 0) = 1,$$

**Scheme B** 
$$L_R(T = \frac{1}{4a}, \mu = 0) = 0.5.$$

We determine the deconfinement temperature (crossover region) from the inflection point (linear region) of  $L_R$



# Deconfinement transition

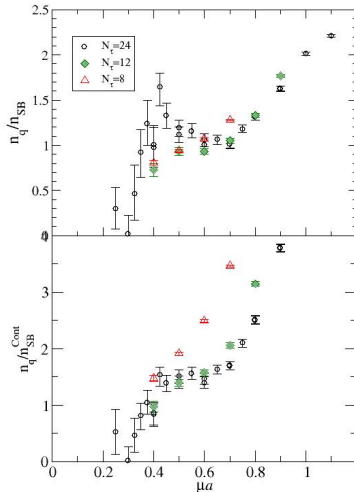


Estimates from [Scheme B](#),  
encompassing Scheme A

$\mu a$	$T_d a$	$T_d$ (MeV)
0.0	0.193(20)	217(23)
0.35	0.140–0.220	157–247
0.40	0.108–0.200	121–225
0.50	0.080–0.200	90–225
0.60	0.060–0.135	67–152

Scheme dependence  $\longleftrightarrow$   
broad crossover?

## Number density

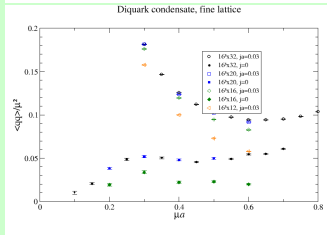
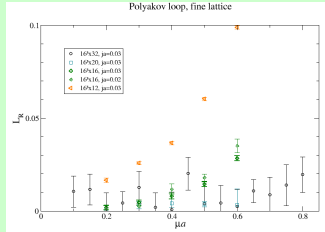
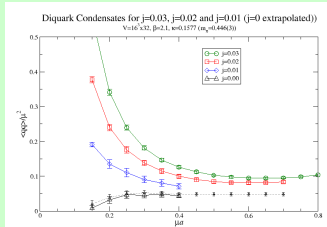


- Normalised to lattice SB or

$$n_{SB}^{Cont} = \frac{4\mu T^2}{3} + \frac{4\mu^3}{3\pi^2}$$

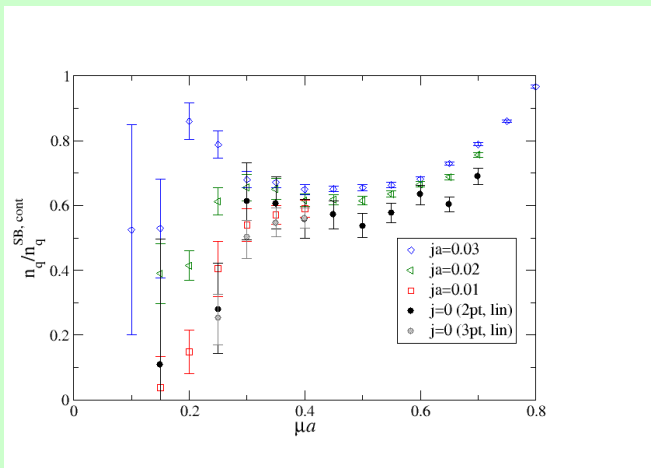
- Close to SB scaling for  $0.4 \lesssim \mu a \lesssim 0.7$ ,  $N_\tau = 24, 12$ .
- Little  $T$ -dependence for  $T \lesssim 100\text{MeV}$ .
- Strong thermal effects for  $N_\tau = 8$
- $N_\tau = 16, 24$  almost identical

## Results from fine ensemble



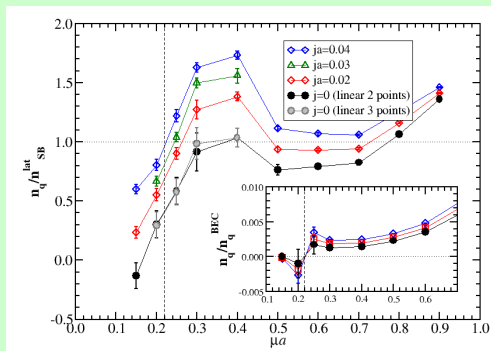
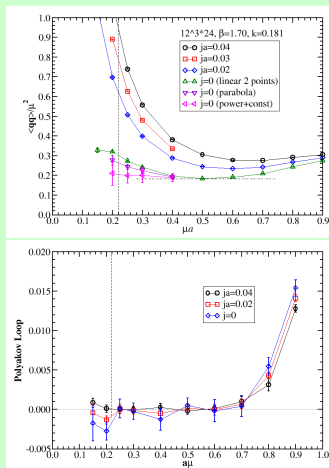
- BCS scaling of diquark condensate **confirmed**
- Similar melting temperature
- Second transition at high  $\mu$  a lattice artefact?
- No deconfinement at low  $T$ ?

## Number density on fine ensemble



The same qualitative features as on coarse ensemble!

## Results from light ensemble



Deviation from BCS scaling:  
BEC window?

## Gluon propagator

- ▶ Essential ingredient in gap equation

$$S^{-1}(p) = S_0^{-1}(p) + Z_2 \int d^4q \Gamma_\mu(p, q) D_{\mu\nu}(q) S(p - q) \gamma_\nu$$

used to determine dynamical fermion mass and  
superfluid/superconducting gap

- ▶ Link to functional methods (DSE, FRG)
- ▶ Electric gluon may signal deconfinement transition

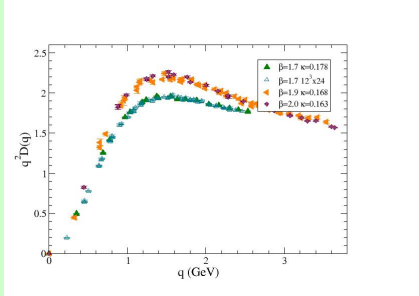
### Tensor structure in medium

$$D_{\mu\nu}(\vec{q}, q_0) = P_{\mu\nu}^T D_M(\vec{q}^2, q_0^2) + P_{\mu\nu}^E D_E(\vec{q}^2, q_0^2) + \xi \frac{q_\mu q_\nu}{(q^2)^2}$$

$$P_{\mu\nu}^M(\vec{q}, q_0) = (1 - \delta_{0\mu})(1 - \delta_{0\nu})(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{\vec{q}^2}),$$

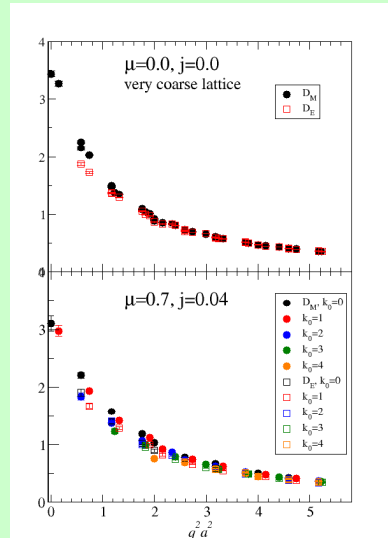
$$P_{\mu\nu}^E(q_0, \vec{q}) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) - P_{\mu\nu}^M(q_0, \vec{q}).$$

## Gluon propagator results

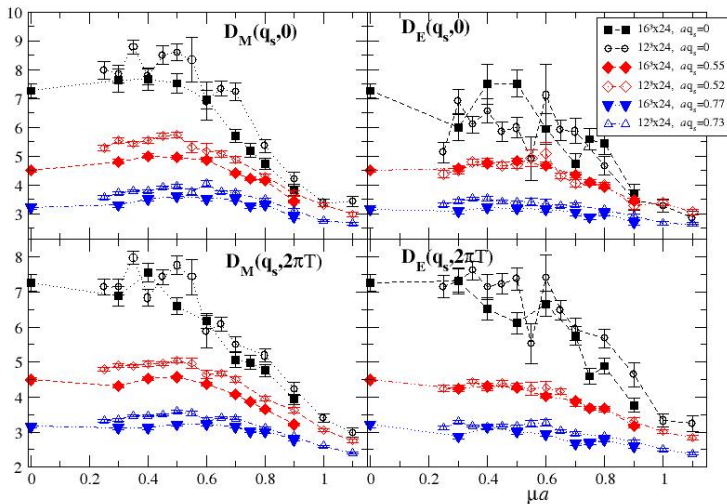


Some finite volume and lattice spacing effects at  $\mu = 0$

In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at high density

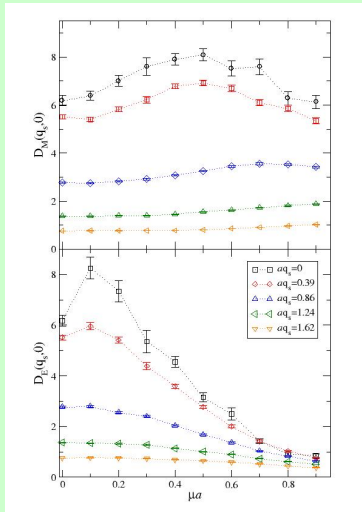


Gluon propagator:  $\mu$ -scans,  $N_\tau = 24$



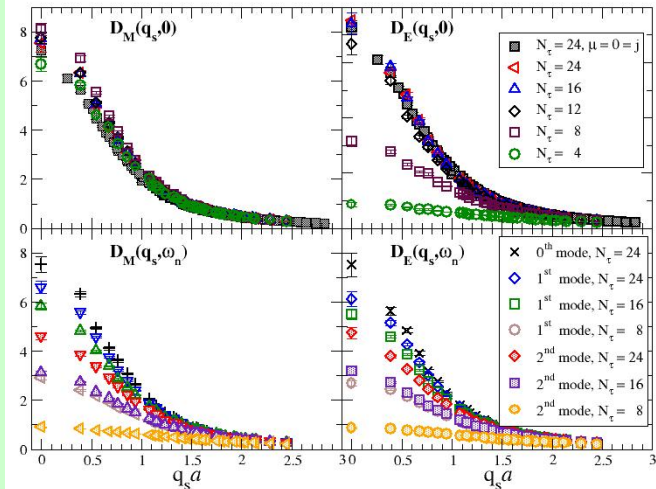


## Gluon propagator: $\mu$ -scans, $N_\tau = 8$



- ▶ Very little volume dependence
- ▶ No  $j$ -dependence observed
- ▶ All modes screened at high  $\mu$ , low  $T$ :  
Weak-coupling theory says static  $D_M$  is unscreened
- ▶ Static magnetic gluon **not** screened at high  $T$

## Gluon propagator: $T$ -scan, $\mu a = 0.5$



## Gluon propagator fits

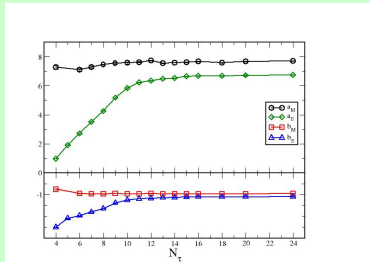
2-parameter fit:

$$D_k^{\text{fit}}(q^2) = \frac{\Lambda^2(q^2 + \Lambda^2 a_k)^{-b_k}}{(q^2 + \Lambda^2)^2}$$

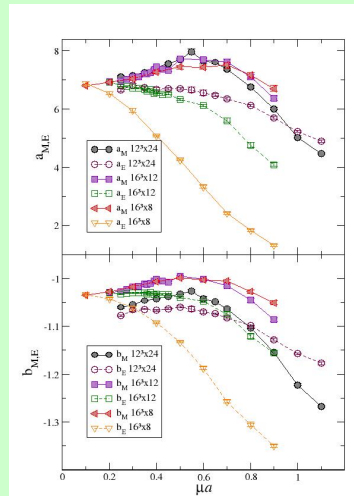
with  $k = M, E$  and

$\Lambda a = 0.999(3)$  from  $\mu = j = 0$  fit

T-scan



$\mu$ -scans



## Quark propagator — tensor structure

$$\begin{aligned}
 S^{-1}(\vec{p}, \tilde{\omega}) &= i\vec{p} A(\vec{p}^2, \tilde{\omega}^2) + i\gamma_4 \tilde{\omega} C(\vec{p}^2, \tilde{\omega}^2) + B(\vec{p}^2, \tilde{\omega}^2) \\
 &\quad + i\gamma_4 \vec{p} D(\vec{p}^2, \tilde{\omega}^2) \\
 S(\vec{p}, \tilde{\omega}) &= i\vec{p} S_a + i\gamma_4 \tilde{\omega} S_c + S_b + i\gamma_4 \vec{p} S_d
 \end{aligned}$$

where  $\tilde{\omega} \equiv p_4 - i\mu$ .

In general the form factors are **complex**!

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 \end{aligned}$$

where  $\tilde{\omega} \equiv p_4 - i\mu$ .

In general the form factors are **complex**!

The  $D$  form factors can be shown to vanish  
 [Rusnak&Furnstahl 1995]

## Gor'kov formalism

Quarks and antiquarks are in the same representation.

Construct Gor'kov spinor

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^T \end{pmatrix} \implies \langle \Psi(x) \bar{\Psi}(y) \rangle \equiv \mathcal{G}(x, y) = \begin{pmatrix} S_N & -S_A \\ \bar{S}_A & \bar{S}_N \end{pmatrix}$$

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Self-energies are diquark gaps  $\Delta$  (superfluid/superconducting)

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Self-energies are diquark gaps  $\Delta$  (superfluid/superconducting)

General tensor structure is the same as for normal components,  
scalar diquark gap is  $\Delta_b$ .

From isospin and charge conjugation symmetry it follows that

$$\bar{S}_N(x, y) = -S_N(y, x)^T, \quad S_A(x, y) = S_A(y, x)^T$$



## Fermi surface and Cooper pairs

### Fermi surface

In a Fermi liquid the Fermi surface is given by

$$\det S^{-1}(\vec{p}_F, p_4 = 0) = 0 \quad \Longleftrightarrow \quad \vec{p}^2 A^2 + \tilde{\omega}^2 C^2 + B^2 = 0$$

### Pole in propagator

## Fermi surface and Cooper pairs

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In gapped phase: zero crossing!

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### Size of Cooper pair

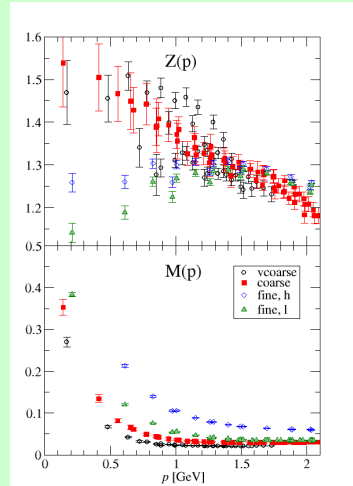
If we know the anomalous propagator  $S_A(x)$  we can compute the size of the Cooper pairs:

$$\xi^2 = \frac{\int d^3x \, \vec{x}^2 \left| \frac{1}{2} \text{Tr}(S_A(x) \Lambda^+) \right|^2}{\int d^3x \, \left| \frac{1}{2} \text{Tr}(S_A(x) \Lambda^+) \right|^2}$$

## Quark propagator results

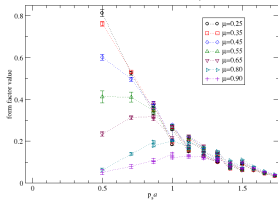
### Quark propagator in vacuum

- ▶ Large lattice spacing dependence
- ▶ Substantial quark mass dependence for  $Z(p)$
- ▶ Unusual  $p$ -dependence in  $Z(p)$
- ▶ infrared suppression recovered in **low-mass** and **continuum** limit?
- ▶  $M(p)$  not yet properly corrected!

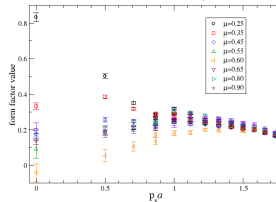


# Normal form factors – PRELIMINARY

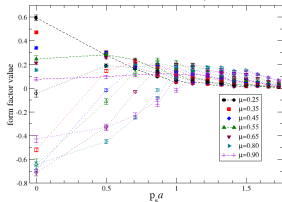
Comparison of the Form Factor  $S_{a, re}$  for Different  $\mu$  Values  
 $V=12^3 \times 24$ ,  $\beta=1.9$ ,  $\kappa=0.1680$ ,  $j=0.04$ ,  $p_1=1/2$



Comparison of the Form Factor  $S_{b, re}$  for Different  $\mu$  Values  
 $V=12^3 \times 24$ ,  $\beta=1.9$ ,  $\kappa=0.1680$ ,  $j=0.04$ ,  $p_1=1/2$

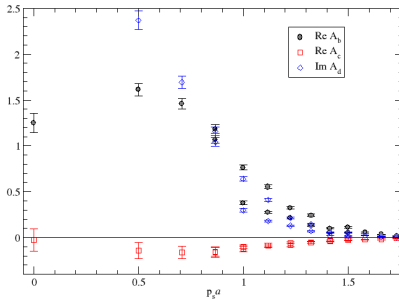


Comparison of the Form Factor  $S_c$  for Different  $\mu$  Values  
 $V=12^3 \times 24$ ,  $\beta=1.9$ ,  $\kappa=0.1680$ ,  $j=0.04$ ,  $p_1=1/2$



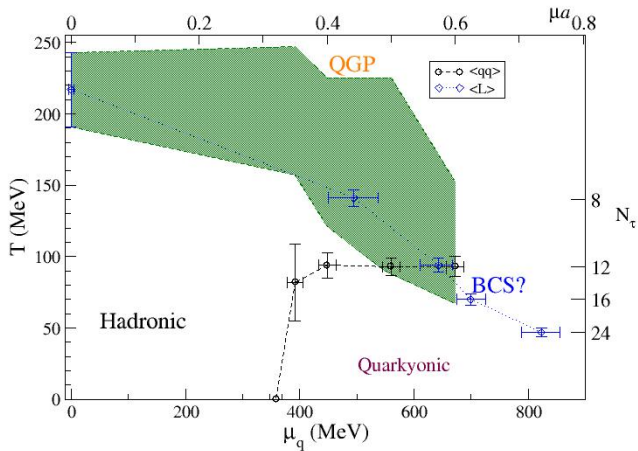
- Dramatic changes in superfluid phase
- Vanishing  $M(0)$  at  $\mu a = 0.65$ ?
- Zero crossing in  $S_c$

# Anomalous form factors [ $\mu a = 0.7, ja = 0.02, n_t = \frac{1}{2}$ ]



- Diquark gap is evident
- Nonzero tensor form factor  $A_d$
- Evidence of vector form factor  $A_c$

## Summary



# Summary

## Evidence for three phases/regions

- ▶ Vacuum/hadronic phase below  $\mu_o = m_\pi/2$ , low  $T$
- ▶ BCS/quarkyonic for intermediate  $\mu$ , low  $T$
- ▶ Deconfined/QGP matter at high  $T$



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- ▶ Possible deconfined superfluid region?
- ▶ Both electric and magnetic gluon screened at high  $\mu$ , low  $T$
- ▶ Static magnetic gluon not screened at high  $T$
- ▶ Dramatic modification of quarks in superfluid region

## Outlook

- ▶ Attempt  $O(2)$  scaling fit for superfluid to normal transition?
  - Requires several larger lattice volumes
- ▶ Full analysis of quark propagator **in progress**
- ▶ Analysis of static quark potential **in progress**
- ▶ Phase scan on light ensemble **in progress**
- ▶ Temperature scans on fine ensemble
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Happy 60th birthday Tony!