Tony Williams Fest:
“New Directions in Subatomic Physics”
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POLARIZED QUARK HADRONIZATION

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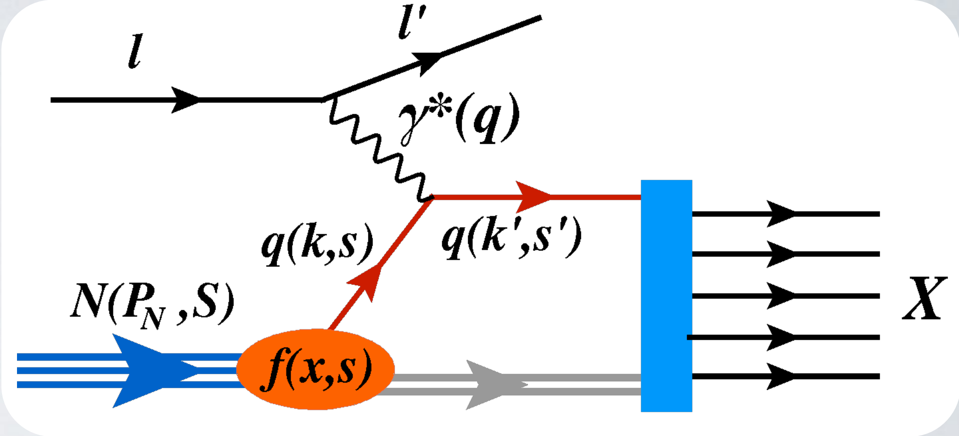
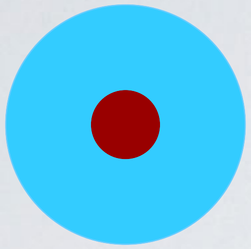
Outlook

- *Introduction and Motivation.*
- *Overview of NJL-jet and the MC implementation.*
- *The recent progress on modelling polarised quark hadronisation.*
- *Conclusions.*

NUCLEON PARTON DISTRIBUTION FUNCTIONS



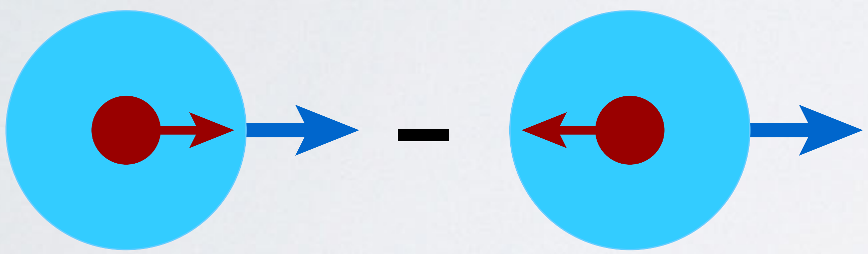
- *Unpolarized* quark in *Unpolarized* nucleon.



$$f_1^q(x, Q^2)$$

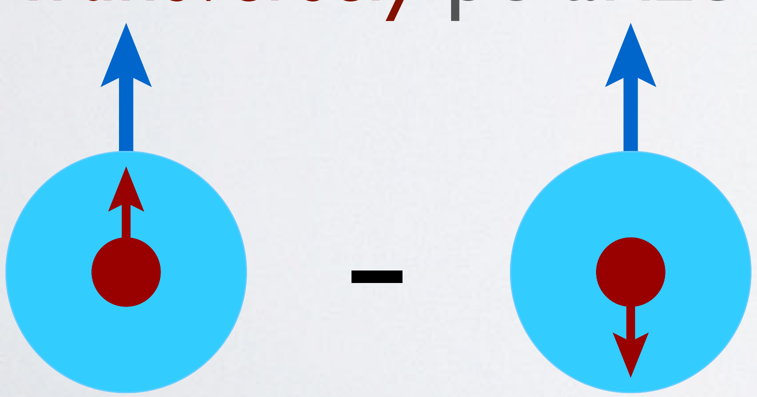
- ***The momentum and the spin of the partons are correlated with the polarization of the nucleon!***

- *Longitudinally* polarized quark in *Longitudinally* polarized nucleon.



$$g_{1L}^q(x, Q^2)$$

- *Transversely* polarized quark in *Transversely* polarized nucleon.



$$h_1^q(x, Q^2)$$

Only for quarks (leading twist)!

Chiral-odd: Suppressed in Inclusive DIS

Tensor Charge

$$\delta q_v(Q^2) = \int_0^1 dx\ h_1^{q_v}(x, Q^2)$$

3D Nucleon Structure with TMD PDFS

- ❖ ***TMDs: Momentum Space***
- ❖ ***GPDs: Impact Parameter***
- ❖ ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon!***
- ❖ *Leading Order TMD PDFs*

✦ Survive after TM integration!

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

Boer-Mulders

Worm Gear

Pretzelocity

Sivers

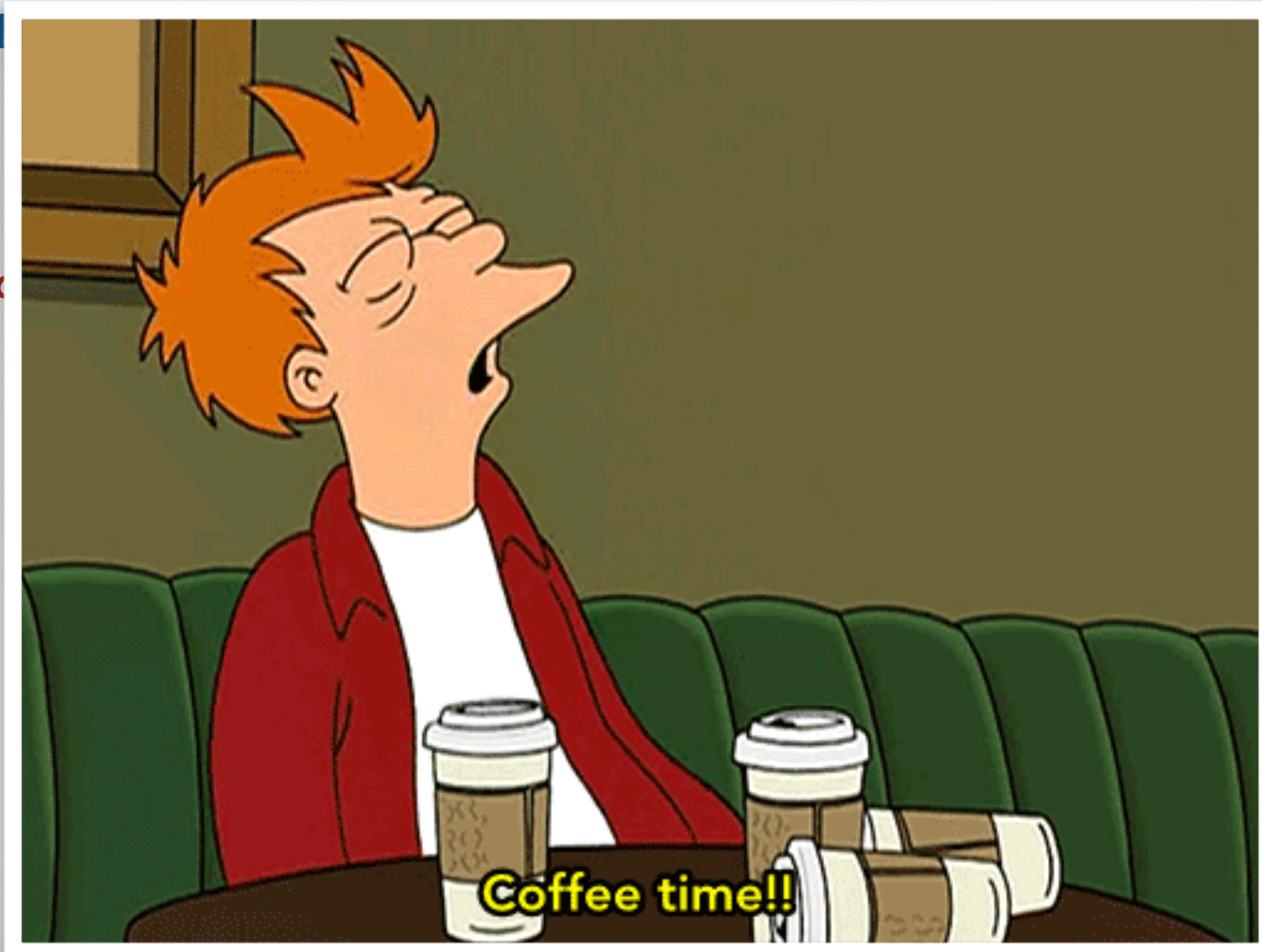
Worm Gear

3D Nucleon Structure with TMD PDFS

❖ ***TMI***

❖ ***The*** ***le***
with

❖ *Lead*



rs

ar

y

PDFs FROM QUARK-QUARK CORRELATOR

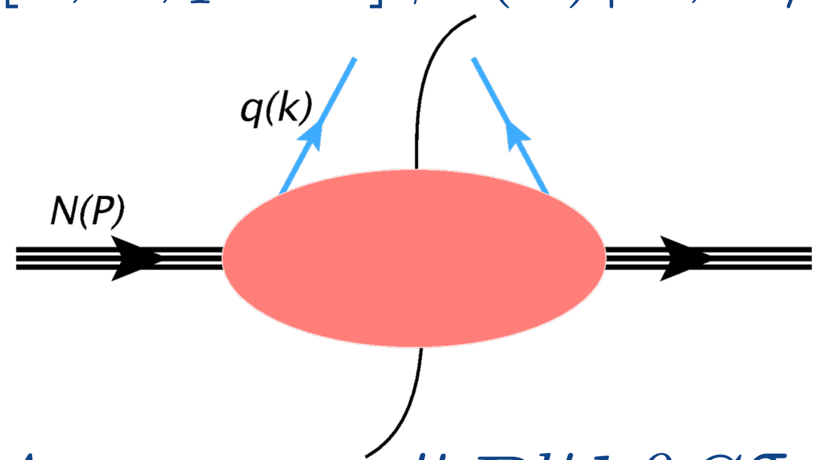
Mulders, Tangerman, NPB 461 197-237 (1996). Bacchetta et al., JHEP 0702, 093 (2007).

- ***Quark-Quark Correlation Function***

$$\Phi_{ij}(P,S;k) = \frac{1}{(2\pi)^4}\int d^4x e^{ik\cdot x}\langle P,S|\bar{\psi}_j(0)\mathcal{L}[0,x;path]\psi_i(x)|P,S\rangle$$

- ***Gauge Link***

$$\mathcal{L}[0,x;path] = \mathcal{P}exp\left(-ig\int_0^x ds^\mu A_\mu(s)\right)$$



- ***Hermiticity of Fields, P and C invariance***

$$\Phi(P,S;k) = A_1 + A_2\not{P} + A_3\not{k} + \ldots + A_{12}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma$$

- ***Traces of the correlator with a Dirac operators***

$$\Phi^{[\Gamma]}(x,\vec{k}_T) \equiv \frac{1}{2}\int dk^-\ Tr[\Phi\Gamma]|_{k^+=xP^+}$$

- ***Leading-order in M/P^+(twist two) expansion***

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma}k_{T\rho}S_{T\sigma}}{M}f_{1T}^\perp \qquad \Phi^{[\gamma^+\gamma^5]} = S_L g_{1L} - \frac{\vec{k}_T\cdot\vec{S}_T}{M}g_{1T}$$

$$\Phi^{[i\sigma^{\alpha+}\gamma^5]} = S_T^\alpha h_1 + S_L\frac{k_T^\alpha}{M}h_{1L}^\perp - \frac{(k_T^\alpha k_T^\rho - \frac{1}{2}k_\perp^2 g_T^{\alpha\rho})S_T^\rho}{M^2}h_{1T}^\perp - \frac{\epsilon_T^{\alpha\rho}k_{T\rho}}{M}h_1^\perp$$

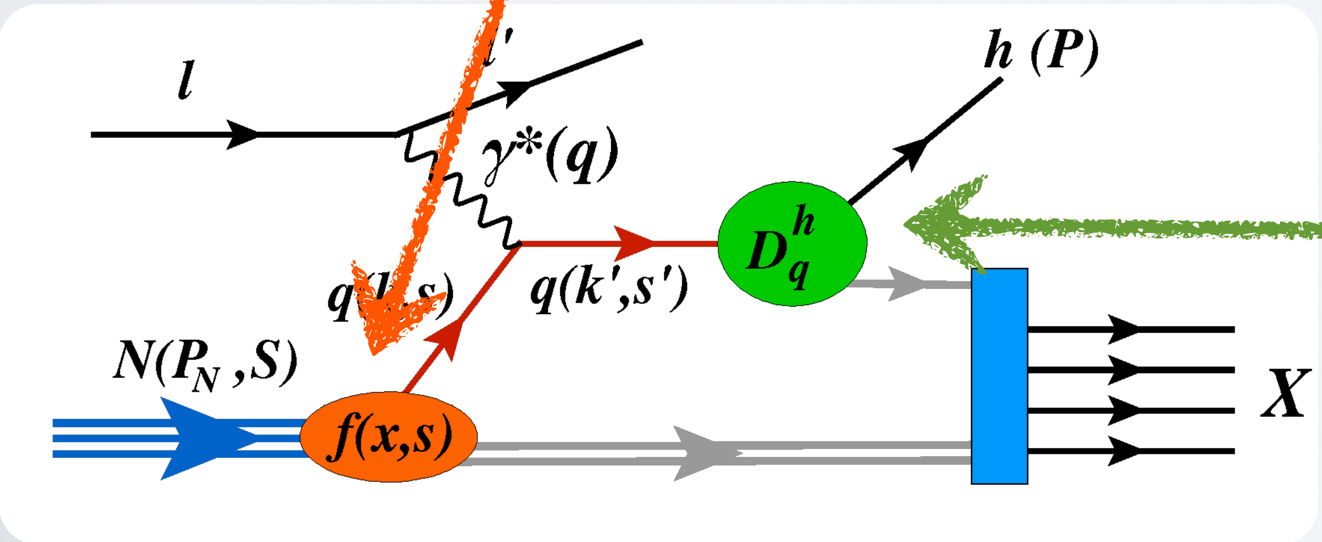
PDFS WITH TRANSVERSE MOMENTUM DEPENDENCE

- ***The transverse momentum (TM) of the parton can couple with both its own spin and the spin of the nucleon:***
- ***TMD PDFs***

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	h_1 $h_{1T}^\perp$

✦ Survive after TM integration!

- ***Accessible in SIDIS Process***
- ***NEED TMD Fragmentation Functions***



q/h	U
U	D_1
L	
T	$H_1^\perp$

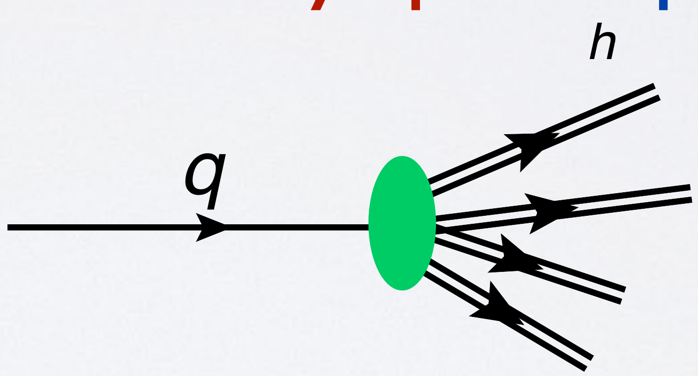
✳unpol/spinless h!

FRAGMENTATION FUNCTIONS

- The cross-sections of DIS processes can be factorized into *"hard scattering"* parts calculable in pQCD and *"soft"*, non-perturbative universal functions encoding *parton distribution in hadrons (PDFs)* and parton hadronization: *Fragmentation Functions (FF)*.

  - Unpolarized FF is the number density of hadron h with LC momentum fraction z, produced by quark q:

$$\boxed{D_q^h(z, Q^2)}$$



- where z is the light-cone momentum fraction of the parton carried by the hadron and Q is the scale of factorization.

$$\boxed{z = \frac{p^-}{k^-} \approx z_h = \frac{2E_h}{Q}}$$

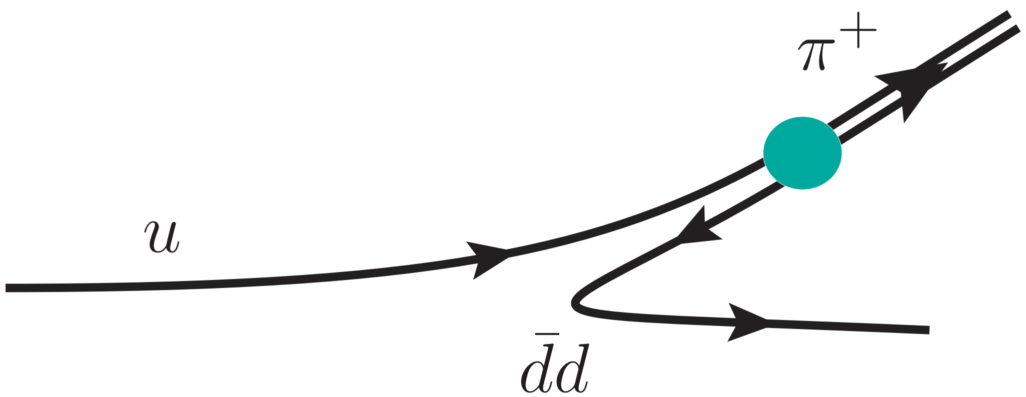
$$\boxed{a^{\pm} = \frac{1}{\sqrt{2}}(a^0 \pm a^3)}$$

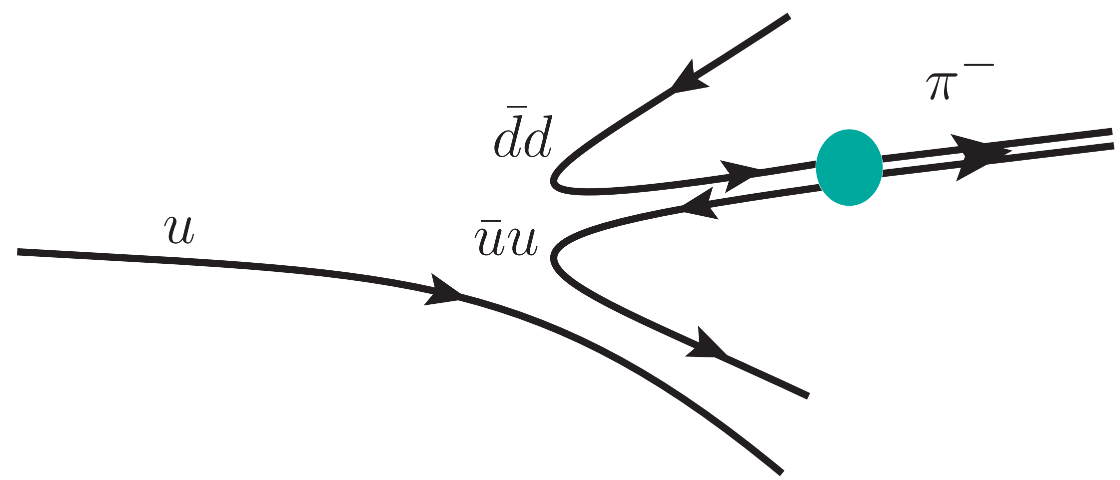
Favoured and Unfavored FFs.

- **Favoured**: the produced hadron has a valence quark of the same flavour.
- **Unfavoured** (disfavoured): NO valence quark of the same flavour.

$$u \to \pi^+ (u\bar{d}) \quad u \to K^+ (u\bar{s}) \qquad u \to \pi^- (\bar{u}d) \quad u \to K^- (\bar{u}s)$$

- **Using a *naive* quark model picture for minimal configuration:**





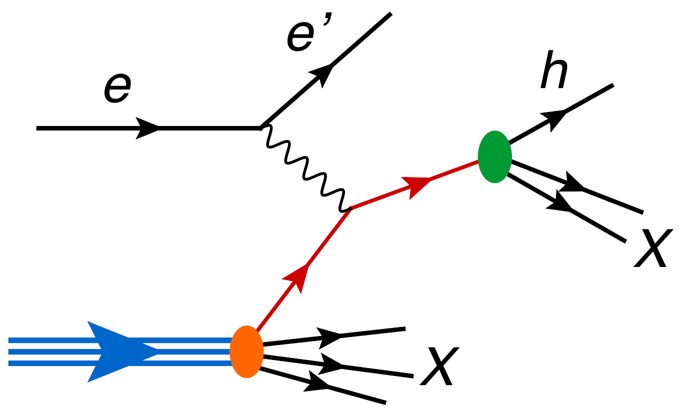
- **Symmetries:**
  - ✦ Charge Conjugation: $q \Leftrightarrow \bar{q}$

$$\boxed{\begin{aligned} D_u^{\pi^+} &= D_{\bar{u}}^{\pi^-} \\ D_s^{K^-} &= D_{\bar{s}}^{K^+} \end{aligned}}$$

  - ✦ Isospin: $u \Leftrightarrow d$

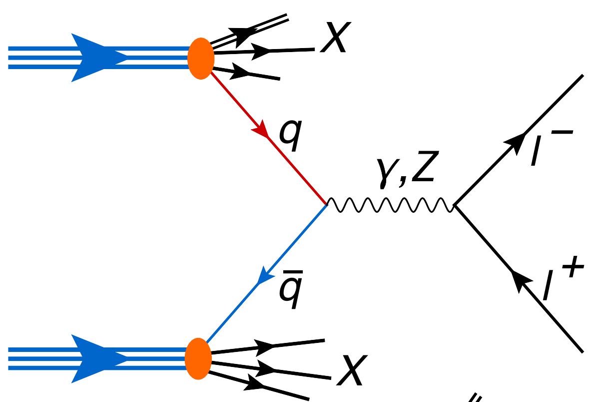
$$\boxed{\begin{aligned} D_u^{\pi^+} &= D_d^{\pi^-} \\ D_u^{K^+} &= D_d^{K^0} \end{aligned}}$$

FACTORIZATION AND *UNIVERSALITY*



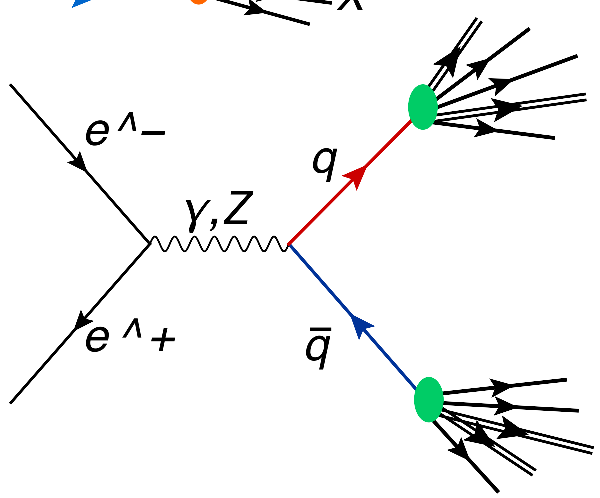
- SEMI INCLUSIVE DIS (SIDIS)

$$\sigma^{eP\to ehX} = \sum_q f_q^P \otimes \sigma^{eq\to eq} \otimes D_q^h$$



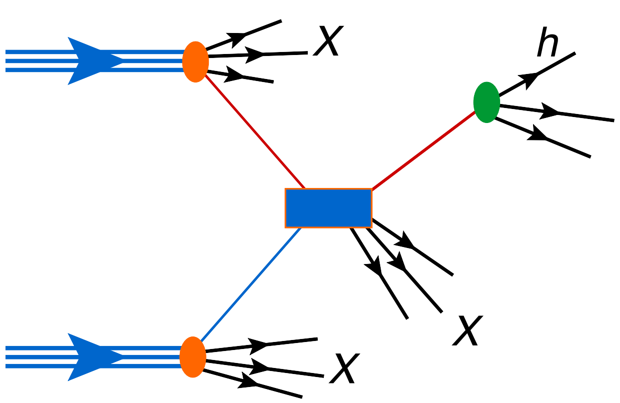
- DRELL-YAN (DY)

$$\sigma^{PP\to l^+l^-X} = \sum_{q,q'} f_q^P \otimes f_{\bar{q}}^P \otimes \sigma^{q\bar{q}\to l^+l^-}$$



- e^+e^-

$$\sigma^{e^+e^-\to hX} = \sum_q \sigma^{e^+e^-\to q\bar{q}} \otimes (D_q^h + D_{\bar{q}}^h)$$



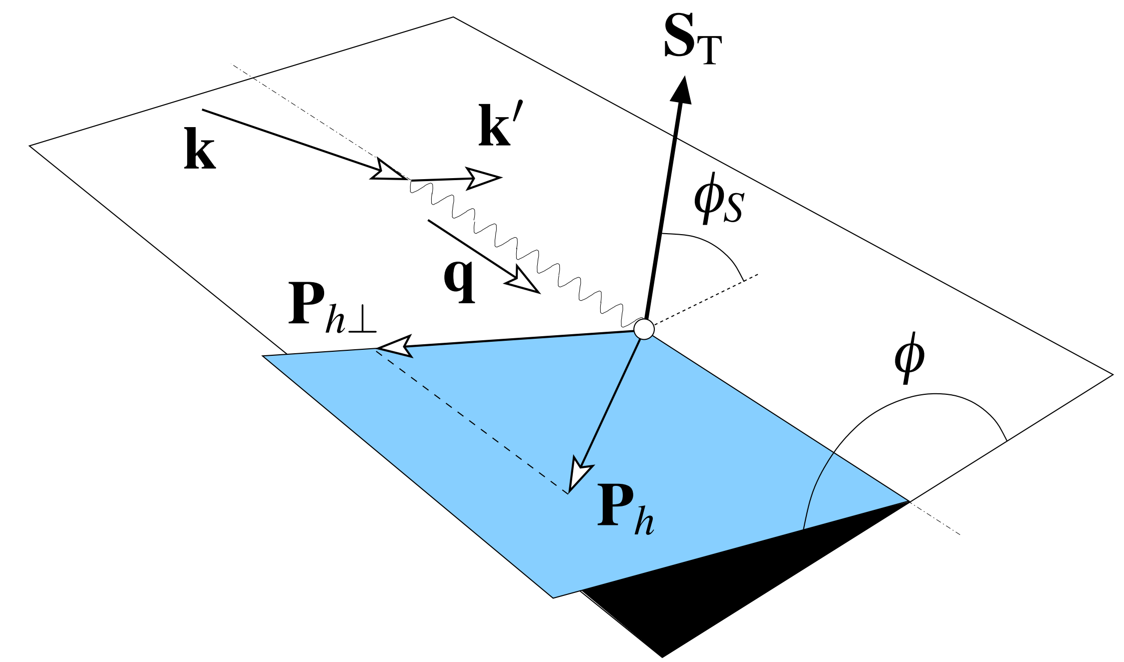
- Hadron Production

$$\sigma^{PP\to hX} = \sum_{q,q'} f_q^P \otimes f_{q'}^P \otimes \sigma^{qq'\to qq'} \otimes D_q^h$$

TMDs from SIDIS $e\,P \to e'\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- *For polarized SIDIS cross-section there are **18 terms** in leading twist expansion:*



$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} \sim F_{UU,T} + \varepsilon F_{UU,L} + \ldots$$

$$+|\boldsymbol{S}_\perp|\left[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + ..\right]$$

Collins term

- Access the structure functions via *specific* modulations.
- LO Matching to *convolutions* of PDFs and FFs: $P_T^2 \ll Q^2$

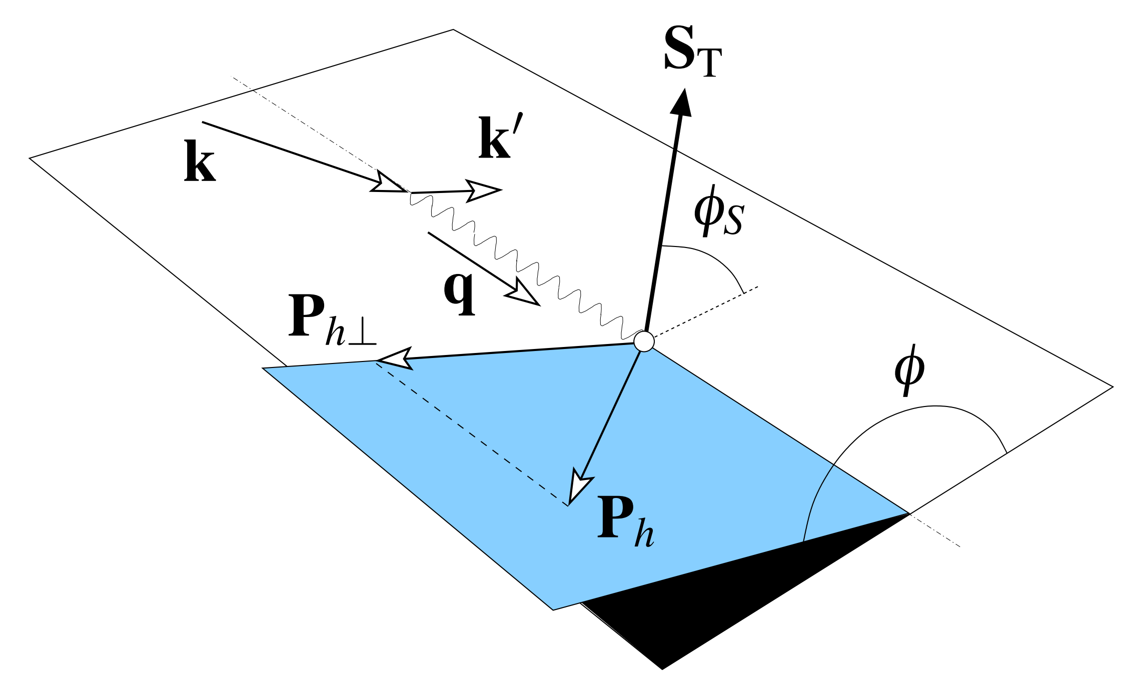
$$F_{UU,T} \sim \mathcal{C}[f_1\;D_1] \qquad F_{UT}^{\sin(\phi_h+\phi_S)} \sim \mathcal{C}[h_1\;H_1^\perp]$$

- ***NEED Collins Fragmentation Function to access Transversity PDF from SIDIS!*** *[BELLE (II) , BaBar]*

TMDs from SIDIS $e\,P \to e'\,h\,X$

A. Bacchetta et al., JHEP08 023 (2008).

- *For polarized SIDIS cross-section there are* ***18 terms*** *in leading twist expansion:*

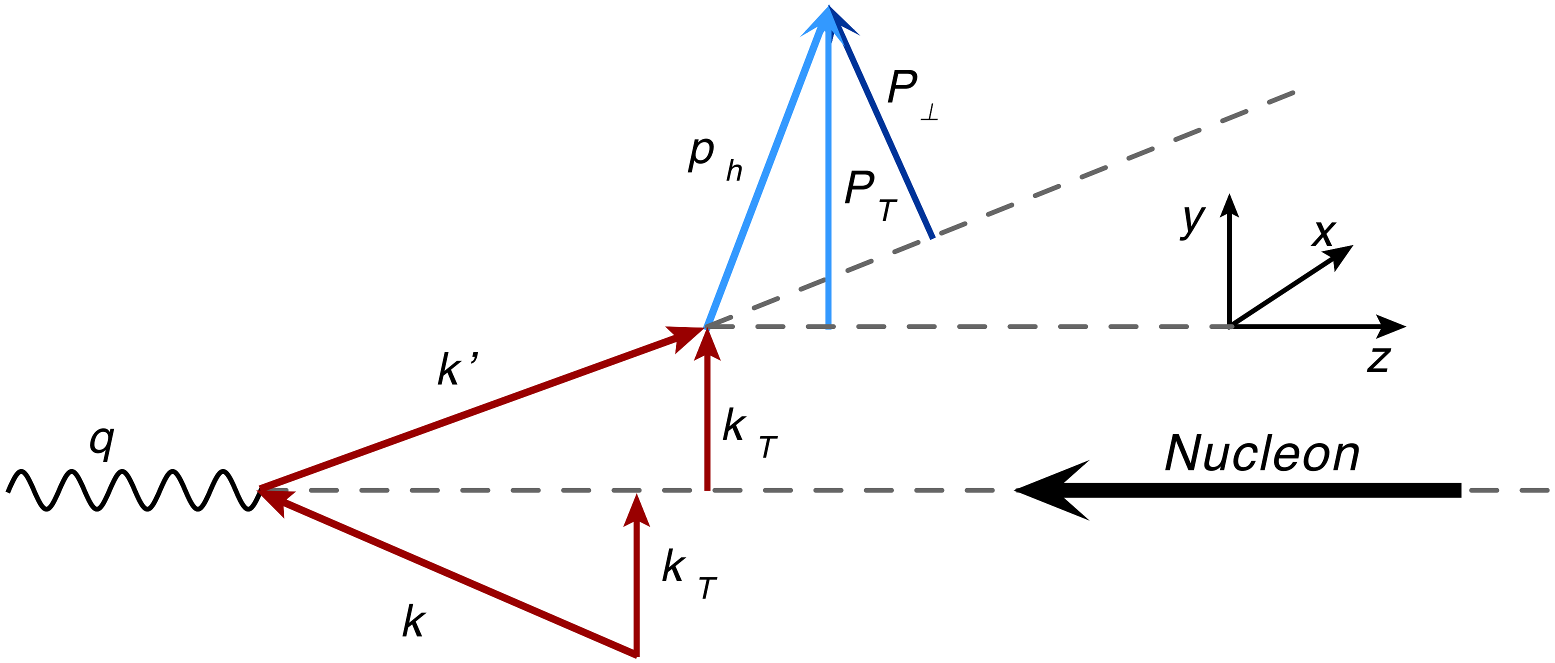


$$\mathcal{C}[f\ g\ \ldots] \equiv \sum_q e_q^2 \int d^2\vec{k}_T\ d^2\vec{P}_\perp\ f\ g\ \ldots\ \delta^2(\vec{P}_T - \vec{P}_\perp - z\vec{k}_T)$$

$$\int d^2\mathbf{k}_\perp f(x, k_\perp^2) = f(x)$$

$$\int d^2\mathbf{P}_\perp D(z, P_\perp^2) = D(z)$$

$$\vec{P}_T = \vec{P}_\perp + z\vec{k}_T$$

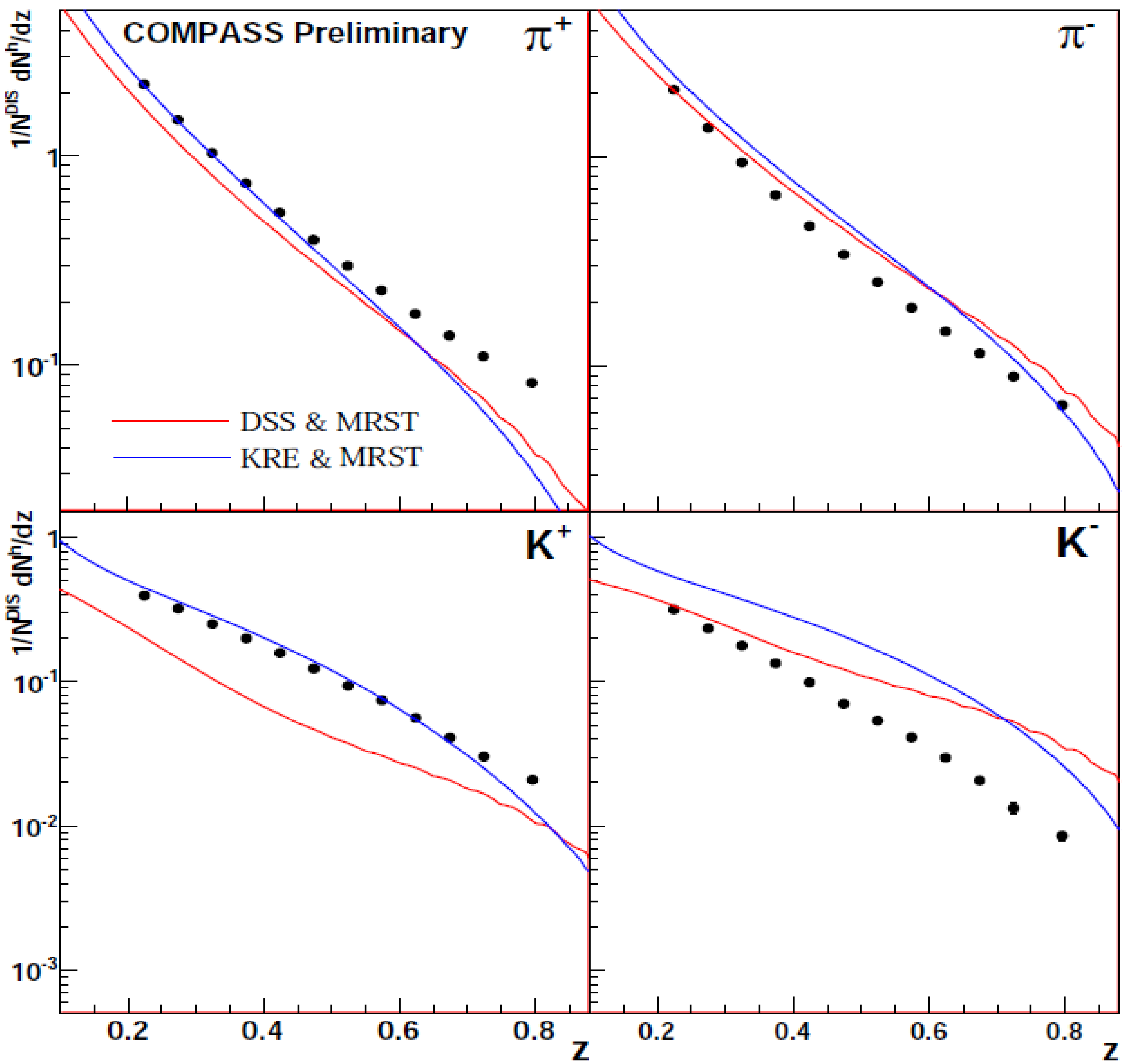


Unfavored FFs NOT well known!

Hadron Multiplicities

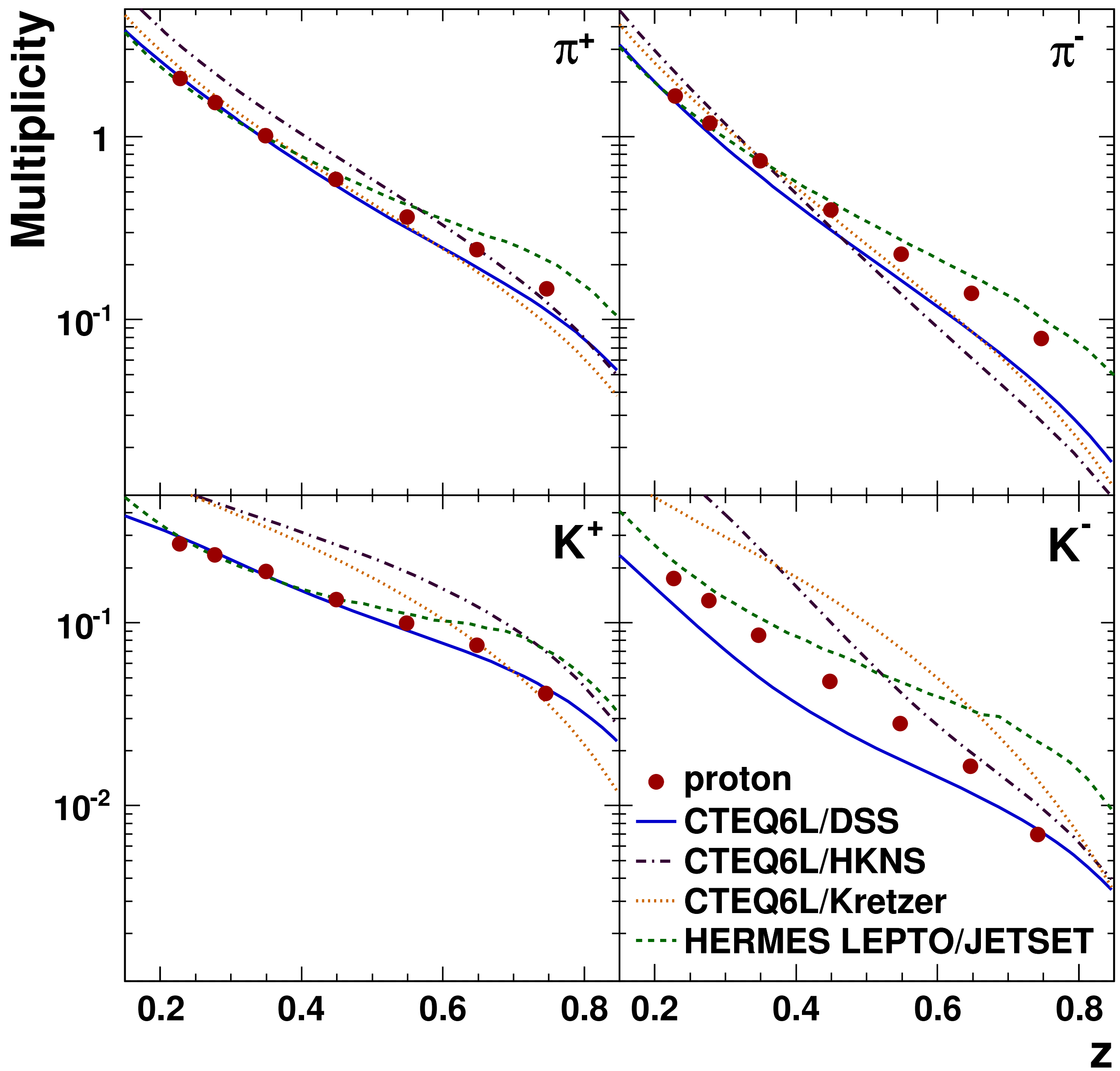
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.



- Also results from HERMES

Phys. Rev. D 87, 074029 (2013)

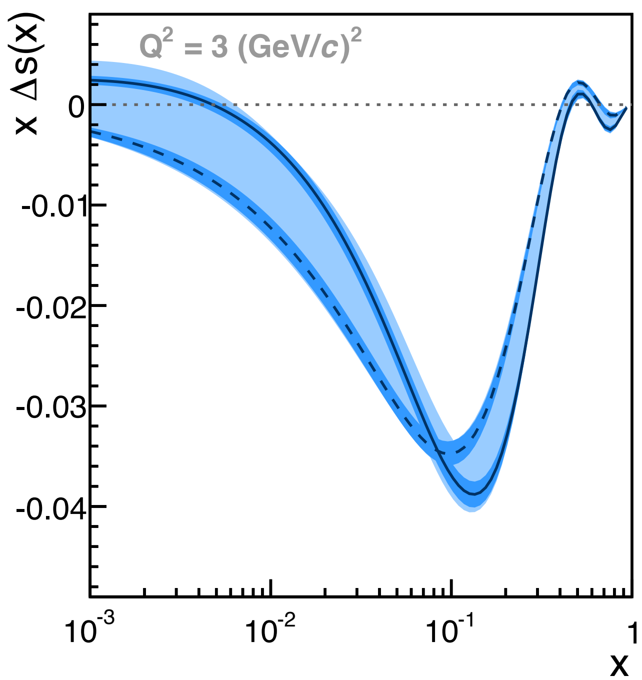


Impact of FF uncertainties on extracted PDFs

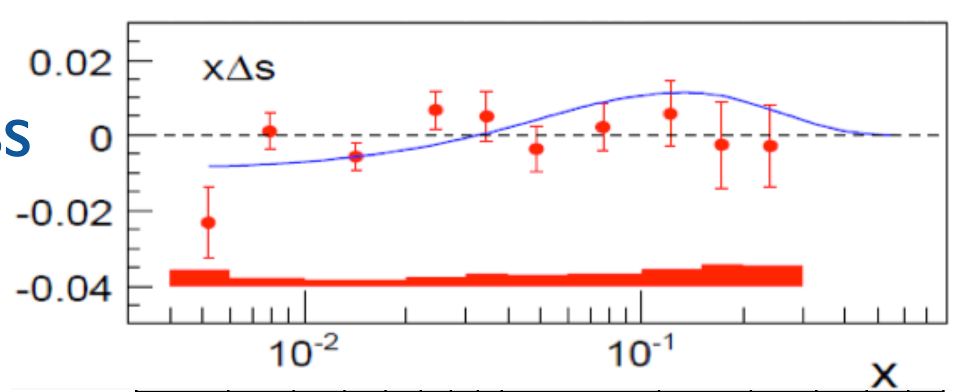
‣ Δs puzzle: DIS vs SIDIS.

Platchkov: Talk in Chile, 2016.

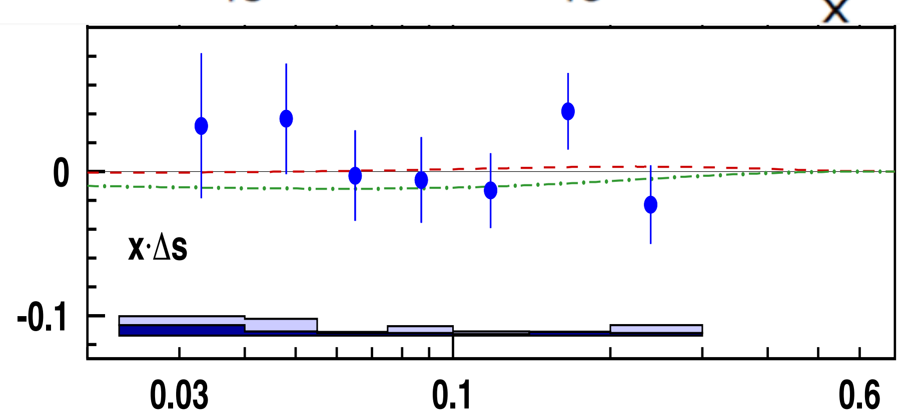
DIS COMPASS



SIDIS COMPASS



SIDIS HERMES

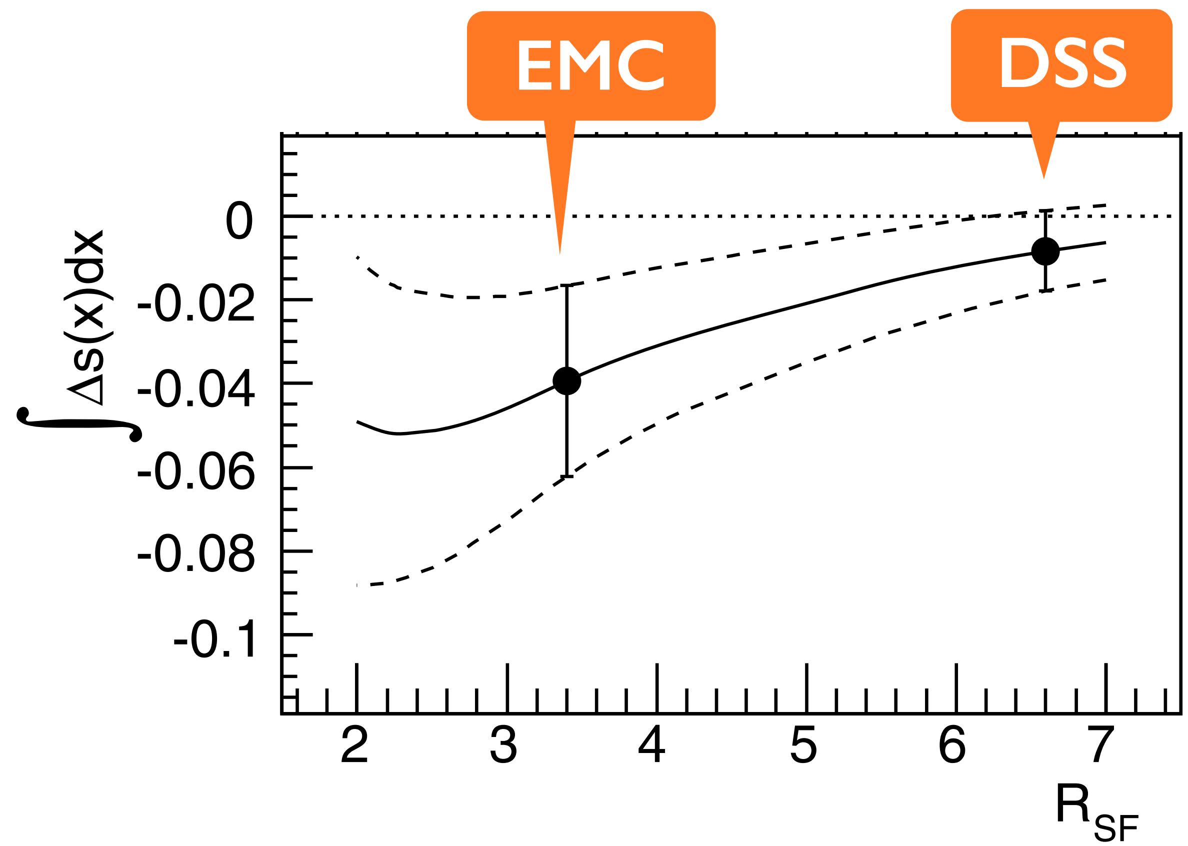


‣ Impact on extracted Δs

COMPASS: PLB 693 (2010) 227–235.

$$A_1^h(x,z) = \frac{\sum_q e_q^2(\Delta q(x) D_q^h(z) + \Delta\bar{q}(x) D_{\bar{q}}^h(z))}{\sum_q e_q^2(q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z))}.$$

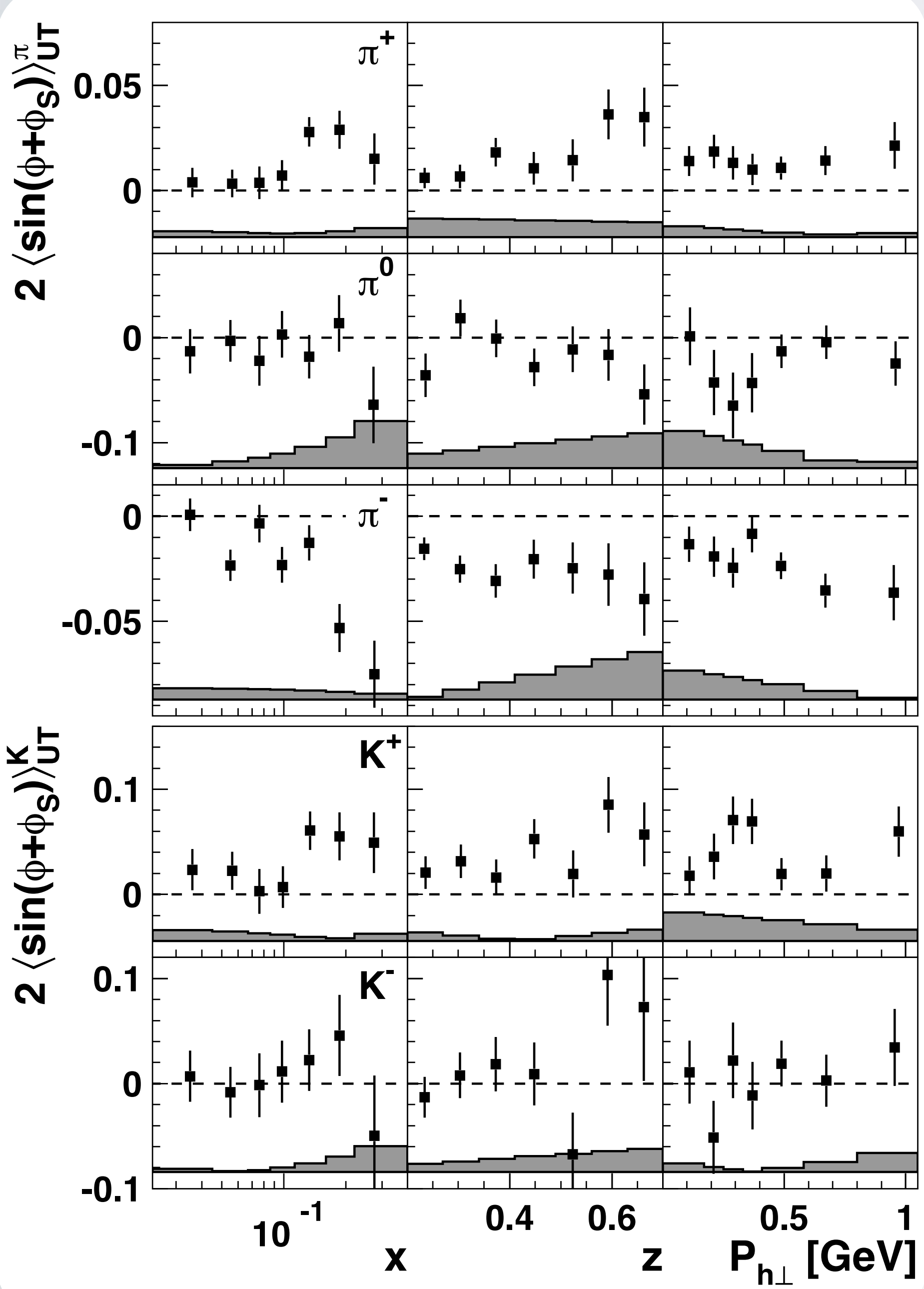
$$R_{UF} = \frac{\int D_d^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}, \qquad R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)\,dz}{\int D_u^{K^+}(z)\,dz}.$$



EMPIRICAL EXTRACTIONS OF TRANSVERSITY

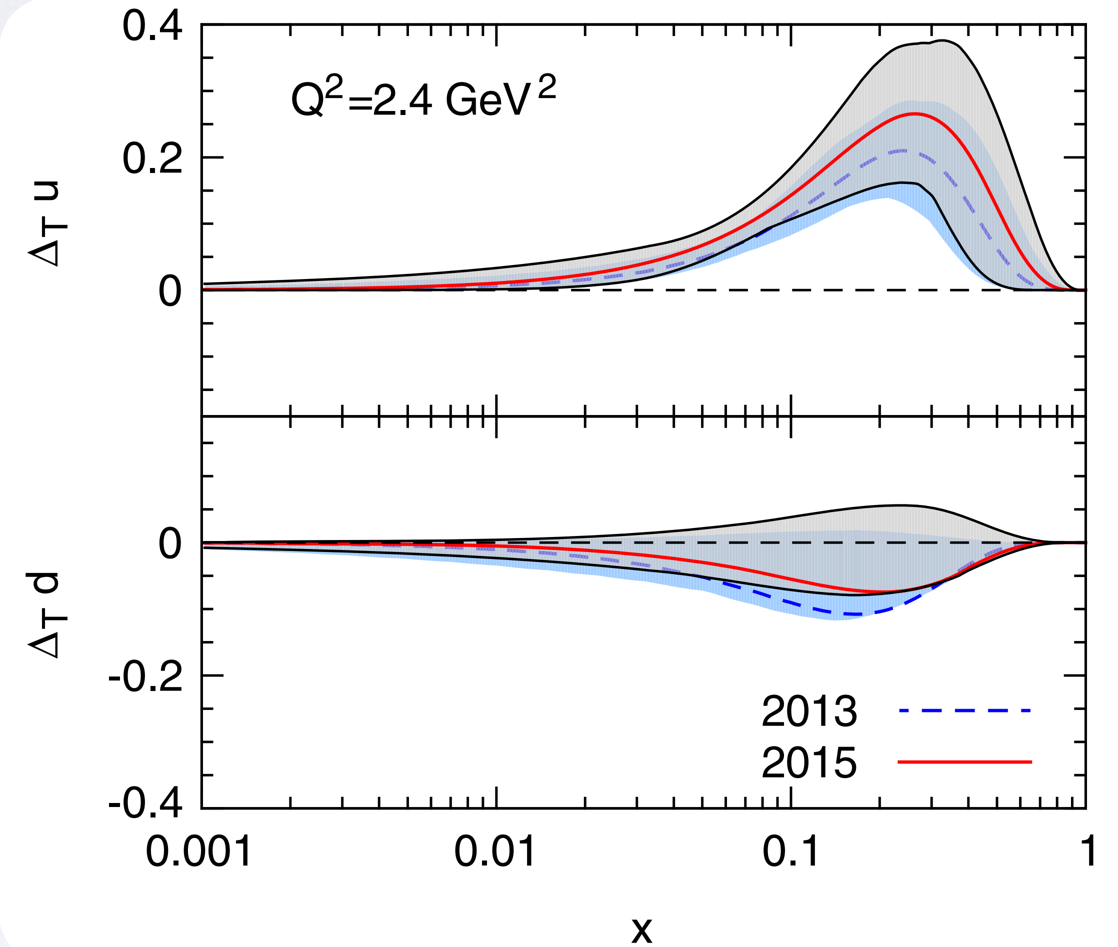
- ***SIDIS at HERMES***
  PLB693 (2010) 11-16.

$$\langle \sin(\phi + \phi_S) \rangle_{UT}^{h} \sim \frac{\mathcal{C}[h_1^q \; H_{1q}^{\perp h/q}]}{\mathcal{C}[f_1^q \; D_1^{h/q}]}$$



- Opposite sign for the charged pions.
- Large positive signal for K^+.
- Consistent with 0 for π^0 and K^-.

❖ ***Fits to HERMES, COMPASS and BELLE/BaBar:*** PRD 92, 114023 (2015).

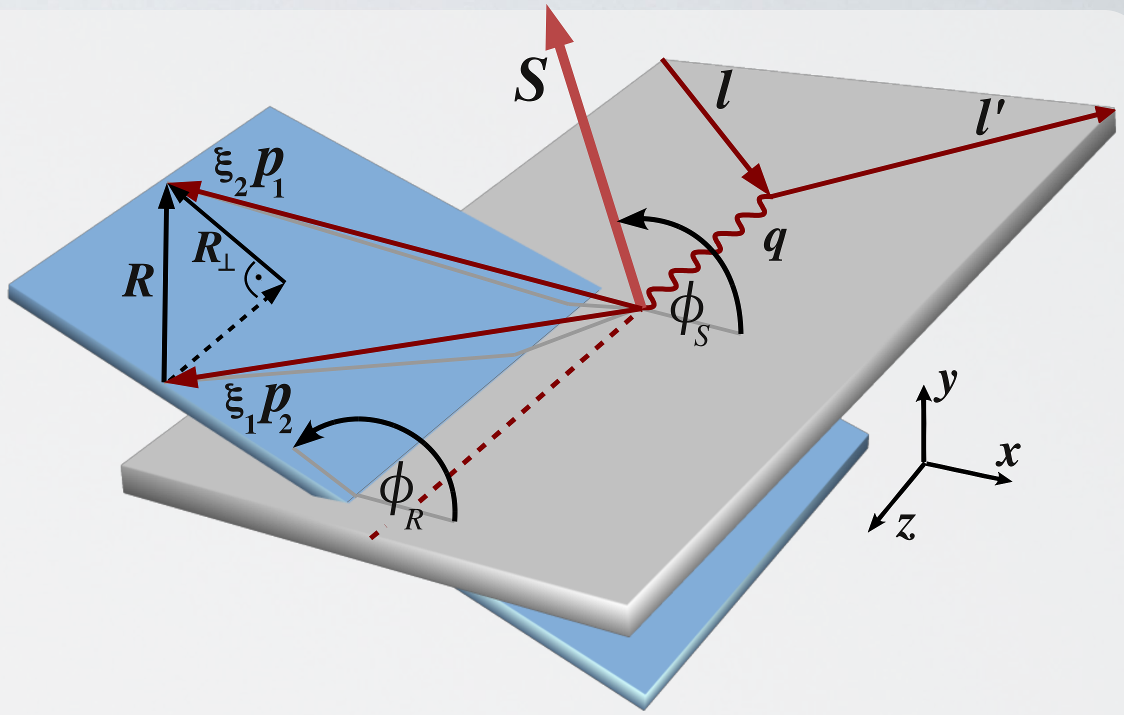


- **Still Large Uncertainties!**
- **Simplistic Approximations !**

ACCESS TO TRANSVERSITY PDF From DiFF in *SIDIS*

M. Radici, et al: PRD 65, 074031 (2002).

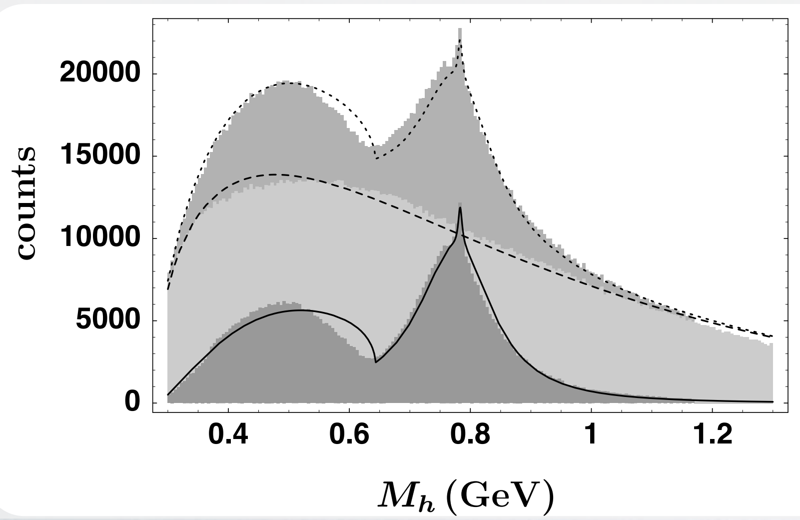
- In two hadron production from polarized target the cross section factorizes ***collinearly*** **- no TMD!**
- Allows clean access to ***transversity***.
- ***Unpolarized*** and ***Interference*** Dihadron FFs are needed!

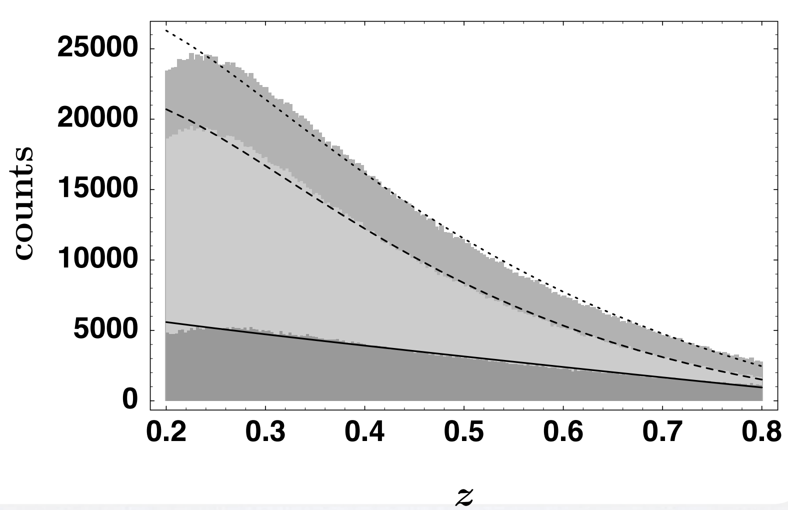


$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \;\; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \;\; D_1^q(z, M_h^2)}$$

- **Empirical Model for D_1^q has been *fitted to PYTHIA simulations*.**

A. Bacchetta and M. Radici, PRD 74, 114007 (2006).

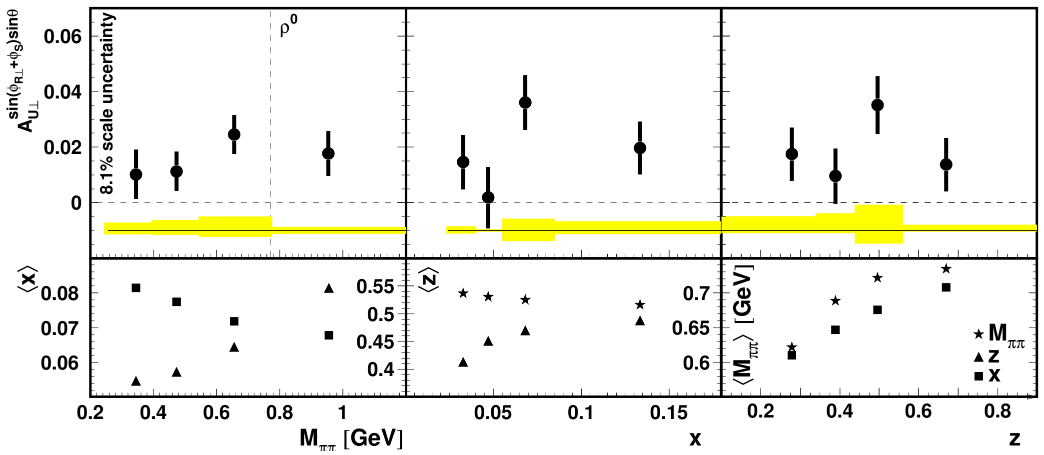




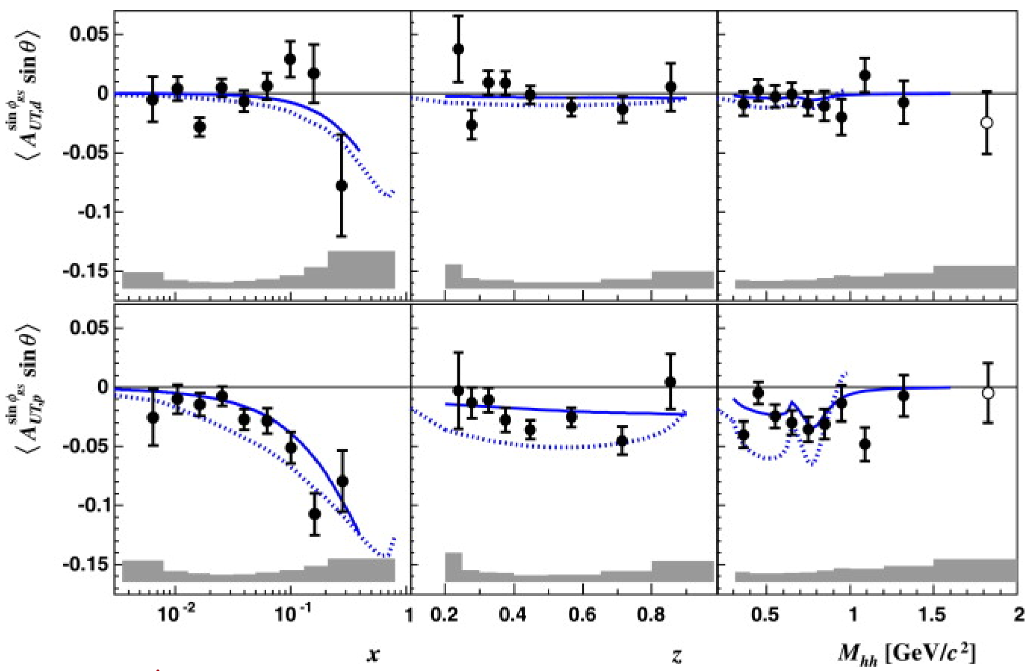
Experiments:
BELLE,
HERMES,
COMPASS.

Empirical Extractions of Transversity from 2h Data

- *SIDIS at HERMES*

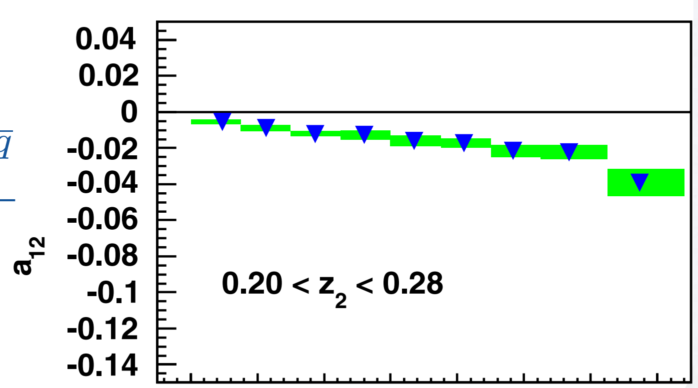


- *SIDIS at COMPASS*

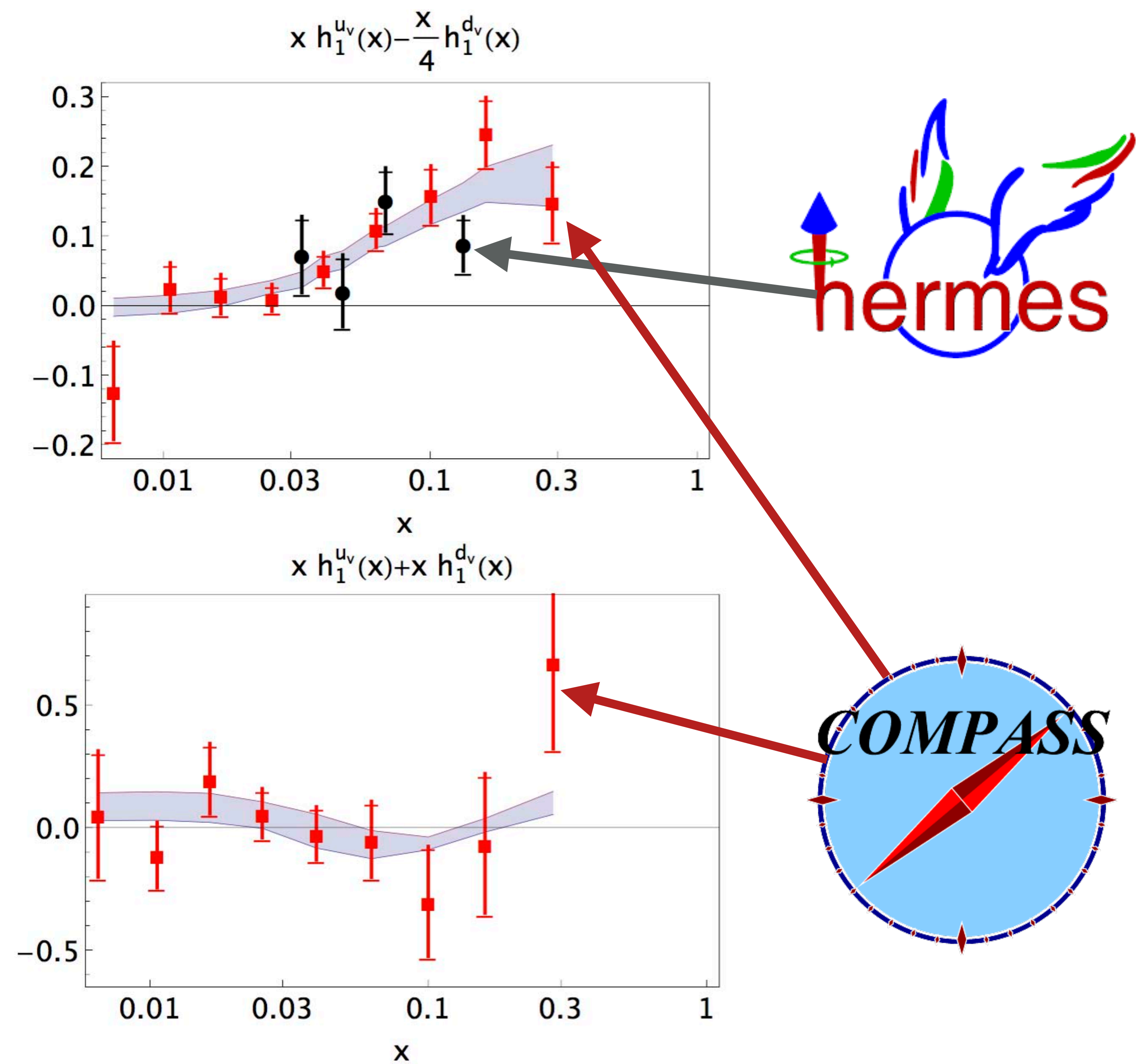


- e^+e^- at BELLE for *two* 2h

$$a_{12} \sim \frac{\sum_q H^{\sphericalangle q}\, H^{\sphericalangle \bar{q}}}{\sum_q D_1^q\, D_1^{\bar{q}}}$$



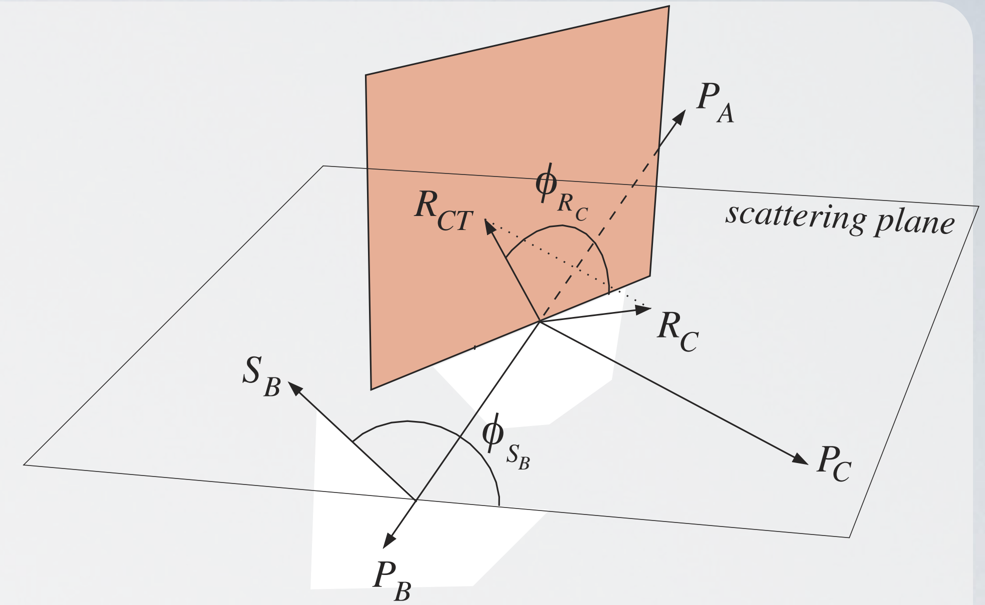
- *Fits to HERMES, COMPASS* ***using*** *BELLE DiFF:* Radici et al: JHEP 1505 (2015) 123.



- **Empirical parametrizations of IFF.**
- **Rely on unpolarised DiFF Model!**

ACCESS TO TRANSVERSITY PDF From DiFF in $P^{\uparrow}P$

Bacchetta, Radici: PRD 70, 094032 (2004).



- In two hadron production from polarized $P^{\uparrow}P$ *assume* the cross section factorizes.
- Access to *transversity* in collinear kinematics in hadron pair from the *same quark jet*.

$$\frac{d\sigma^{PP^{\uparrow}} - d\sigma^{PP^{\downarrow}}}{d\sigma^{PP^{\uparrow}} + d\sigma^{PP^{\downarrow}}} = \frac{d\sigma_{UT}}{d\sigma_{UU}} = \sin(\phi_{RS}) A_{UT}$$

$$A_{UT} \sim \frac{\boxed{\sum_{a,b,c,d}} \int dx_a dx_b \; f_1^{a/P}(x_a) \boxed{h_1^{b^{\uparrow}/P^{\uparrow}}(x_b)} \frac{d\hat{\sigma}^{ab^{\uparrow}\to c^{\uparrow}d}}{d\hat{t}} \boxed{H_{1,c}^{\sphericalangle}(z,M)}}{\boxed{\sum_{a,b,c,d}} \int dx_a dx_b \; f_1^{a/P}(x_a) \boxed{f_1^{b/P}(x_b)} \frac{d\hat{\sigma}^{ab\to cd}}{d\hat{t}} \boxed{D_{1,c}(z,M)}}$$

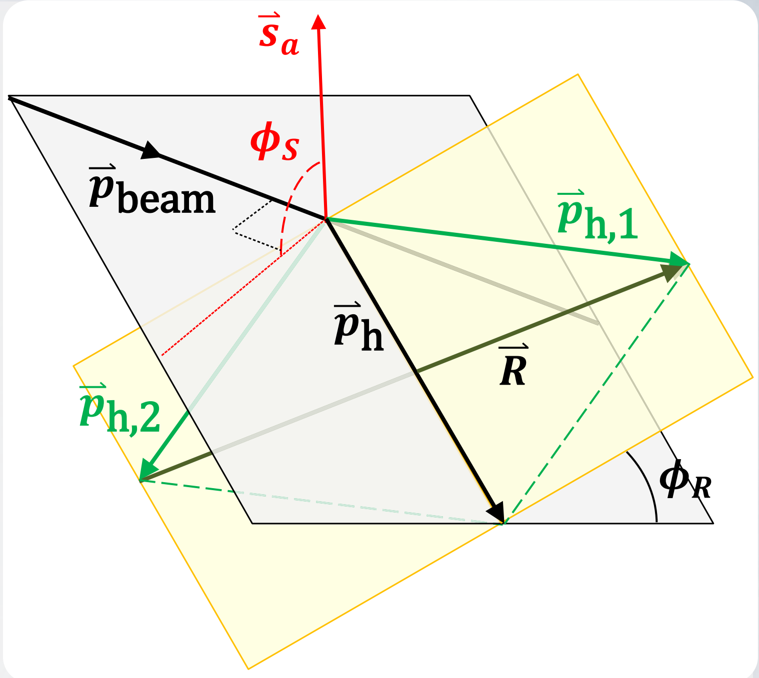
- Again, we *need DiFFs* to extract transversity!
- No *u dominance for P* (no *charge weighting* like in SIDIS).

Measurements in *STAR for P$^{\uparrow}$P at* $\sqrt{s} = 200\text{GeV}$

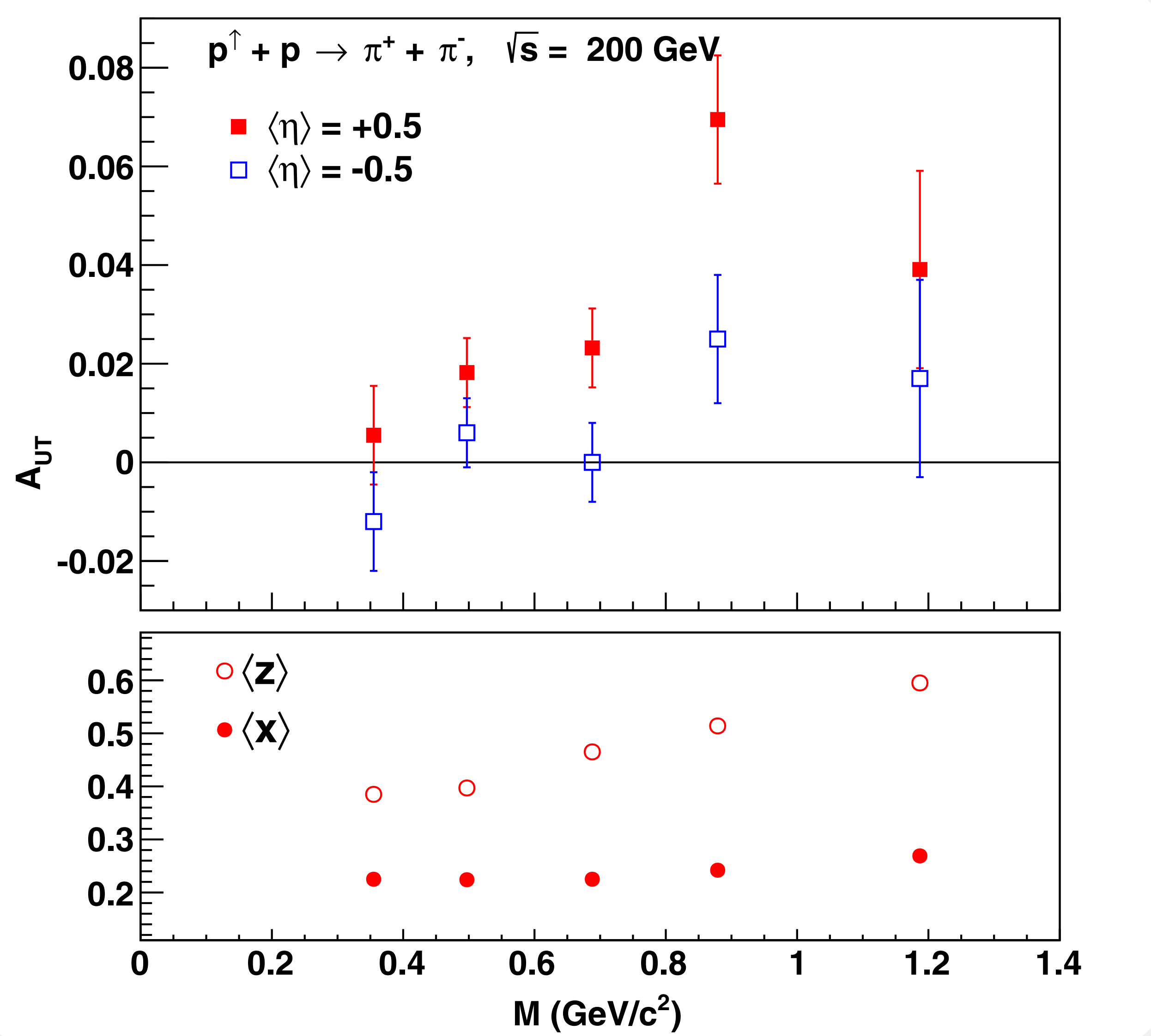
STAR: Phys.Rev.Lett. 115, 242501 (2015).

- **First-ever measurements of transversity in *PP*.**

$$\frac{N^{\uparrow}(\phi_{RS}) - r\cdot N^{\downarrow}(\phi_{RS})}{N^{\uparrow}(\phi_{RS}) + r\cdot N^{\downarrow}(\phi_{RS})} = P_{\text{beam}} A_{UT} \sin(\phi_{RS})$$



- **Signal in excess of 5σ at high pT, $\eta > 0.5$.**



Modelling Hadronization with Spin: The Objectives.

NJL-jet Complete Hadronization Model with Spin. Input from QCD-inspired effective QuarkModel.

1) **Predictions for full set of polarised FFs.**
   - *Quantitative* extract. of fav. and unfav. polarised TMD FF.
   - Include resonance productions and decays.
   - Should explain possible connections between single and dihadron FFs!
   - *The correspondence to FFs in limited z region ($z > z_0$).*

2) **Interpretation in Full Event Generators: PYTHIA…**
   - Number density interpretation.
   - Iterative picture: spin transfer! Should be adaptable to the MC framework.
   - *Should not break any of the unpolarised observables! (PYTHIA "Wisdom" accumulated over decades.)*

How to obtain (TMD) FFs?

✦ Phenomenological Extractions from Experiment.

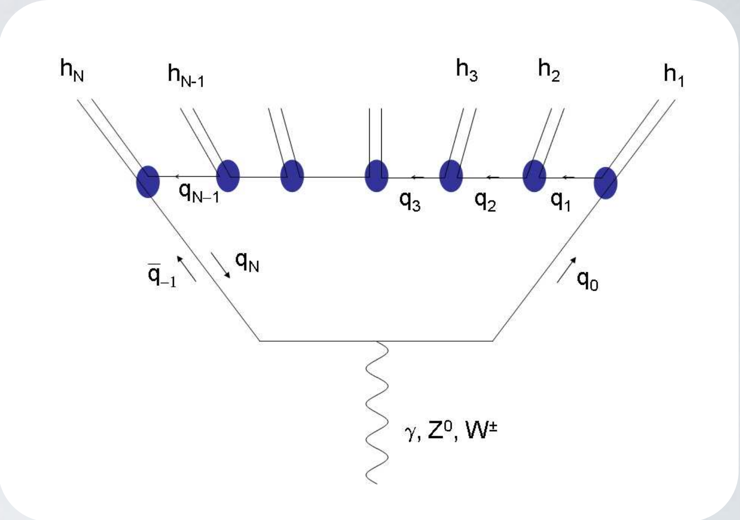
- **Use phenomenological parametrizations of *PDFs/FFs* to fit the *SIDIS/e+e-* x-sections for producing *h*.**
- ***Limited physical insight into the hadronization process.***
- ***Still have to model the contributions of non-DIS processes, etc.***

✦ Models

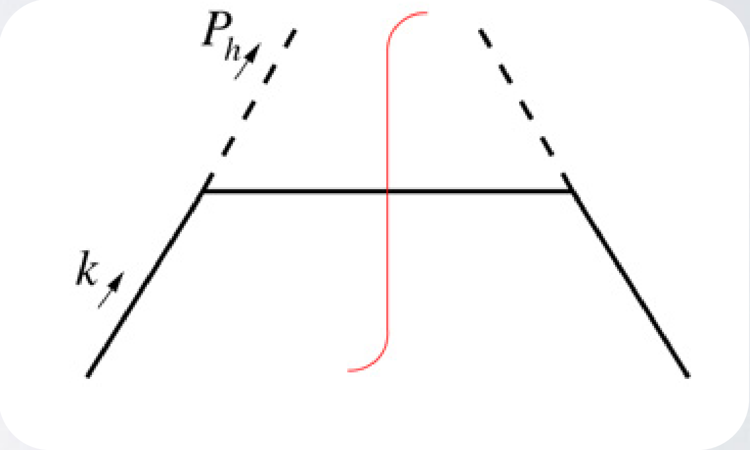
- **Non-quantifiable model approximations.**
- **Only applicable in certain scenarios (**when Jupiter aligns with Mars**).**
- ***Often provide only partial information: only leading hadron/ favoured FFs, etc.***

(SOME of the) MODELS FOR FRAGMENTATION

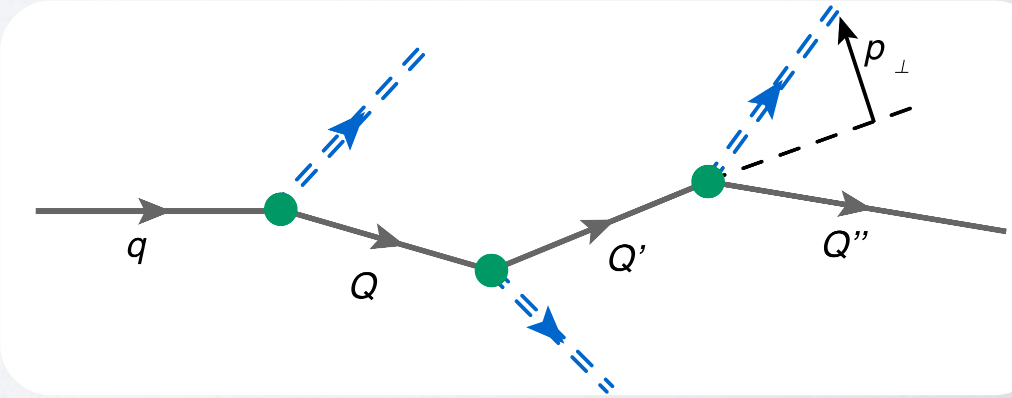
- *Lund String Model*
  - *Very Successful* implementation in *JETSET, PYTHIA*.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.

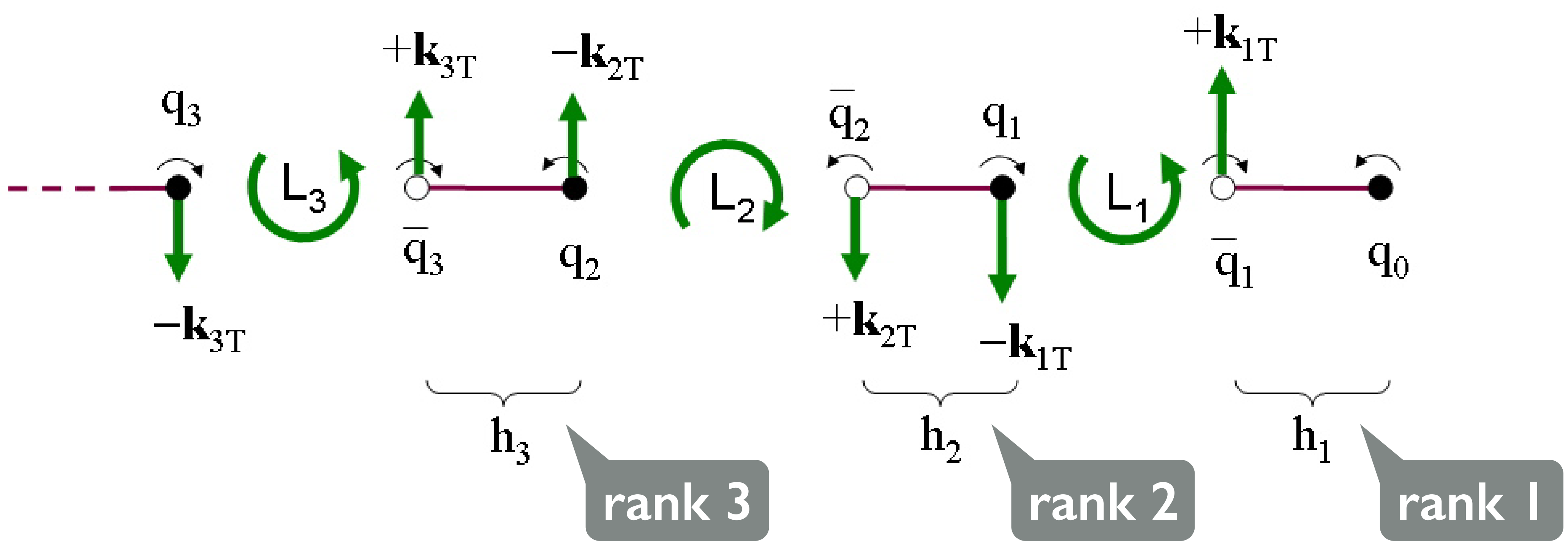


- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.

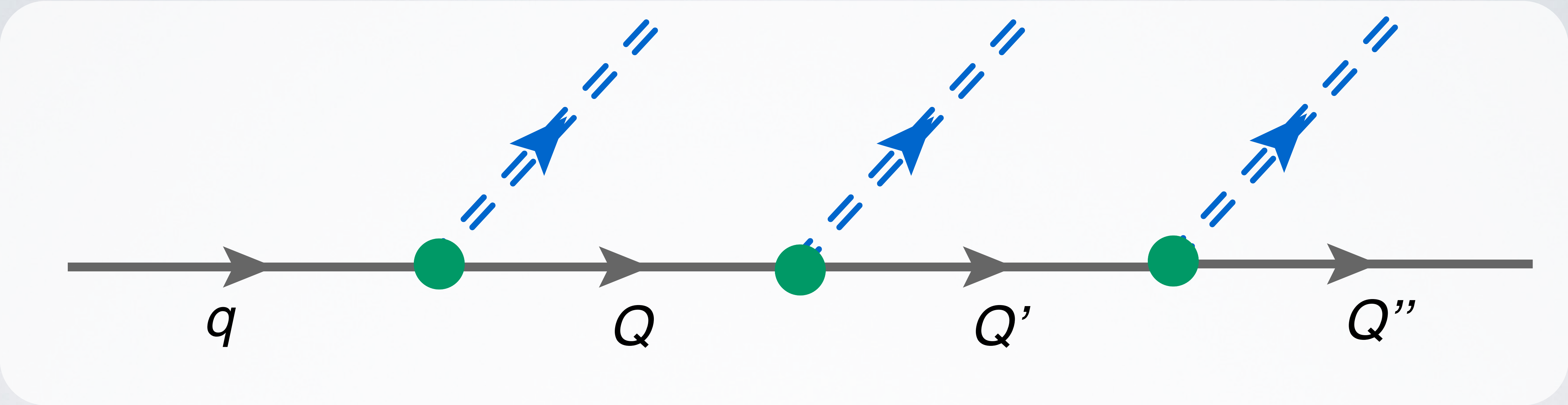


Artru Model

- $q\bar{q}$ **created in** $^3P_0$ **state.**
- **Local compensation of TM.**



- No quantitative results for Collins FFs: implies opposite signs for favoured and unfavored. (Omitting complications from favoured production at rank 2, etc .)
- *Simple* and intuitive *quantum-mechanical* picture.



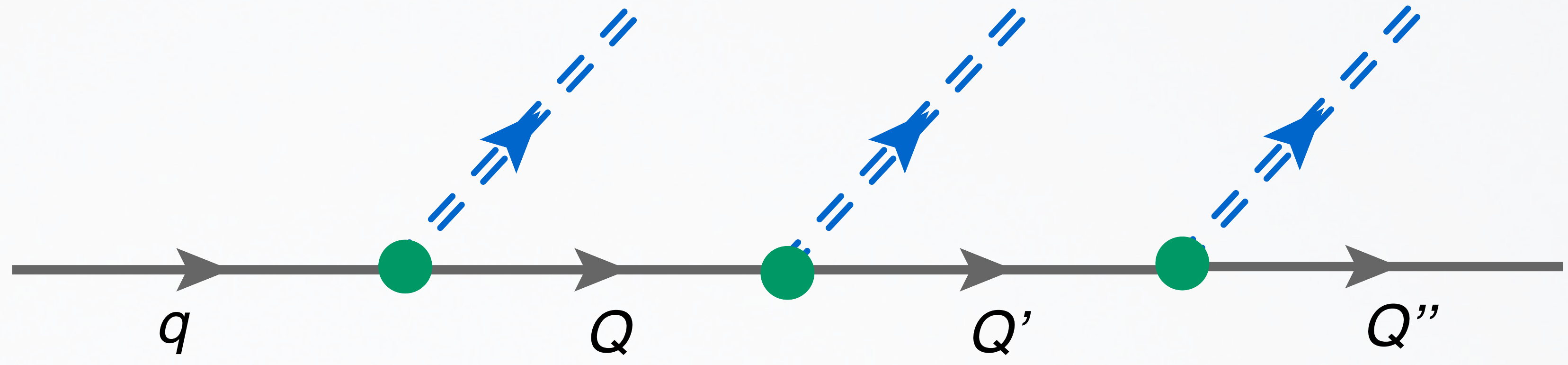
THE NJL-jet MODEL

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



$$D_q^h(z) = \hat{d}_q^h(z) + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h(\frac{z}{y})\frac{1}{y}$$

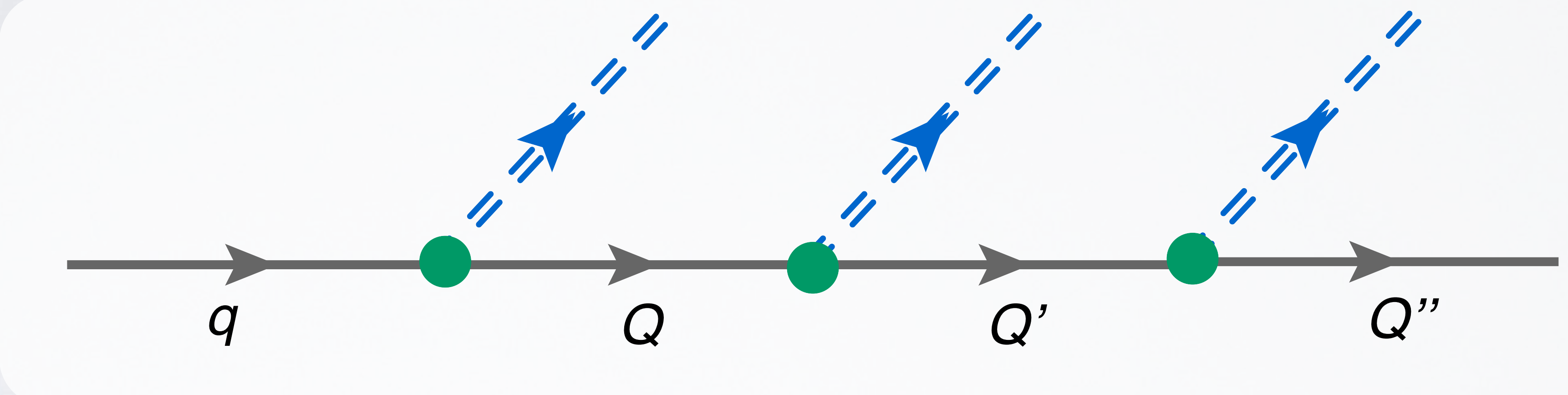
$$\hat{d}_q^h(z) = \hat{d}_q^{Q'}(1-z)|_{h=\bar{Q}'q}$$

THE QUARK JET MODEL

Field, Feynman, Nucl.Phys.B136:1,1978.

Assumptions:

- Number Density interpretation
- No re-absorption
- ∞ hadron emissions



Probability of finding hadron h with mom. frac. [z, z+dz] in a jet of quark q

The probability scales with mom. fraction

$$D_q^h(z)dz = \hat{d}_q^h(z)dz + \int_z^1 \hat{d}_q^Q(y)dy \cdot D_Q^h\left(\frac{z}{y}\right)\frac{dz}{y}$$

Prob. of emitting at step 1

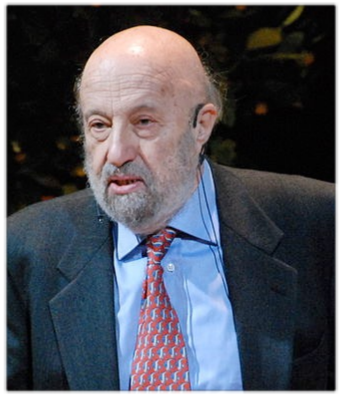
Prob. of mom. [y, y+dy] is transferred to jet at step 1.

NAMBU--JONA-LASINIO MODEL

Yoichiro Nambu and Giovanni Jona-Lasinio:

"Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I"

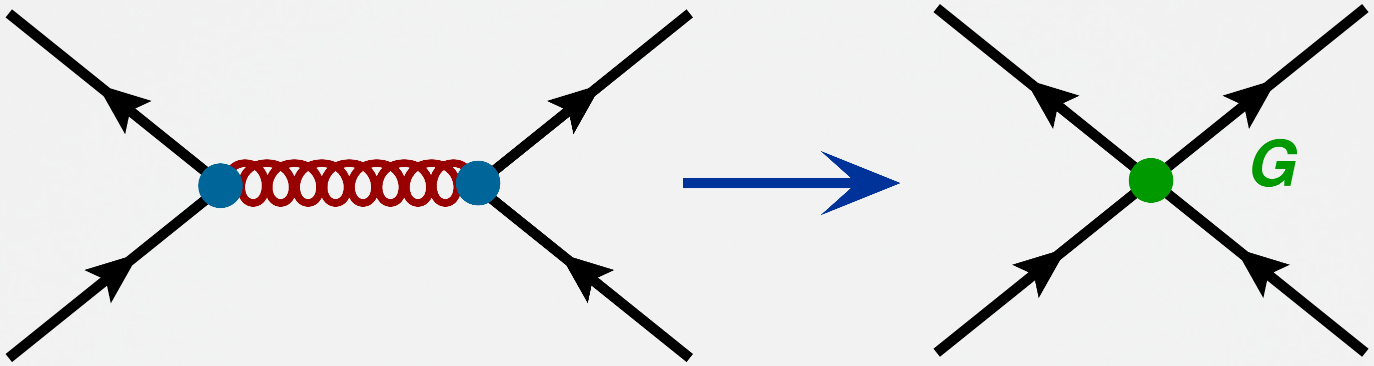
Phys.Rev. 122, 345 (1961)



Effective Quark model of QCD

- Effective Quark Lagrangian

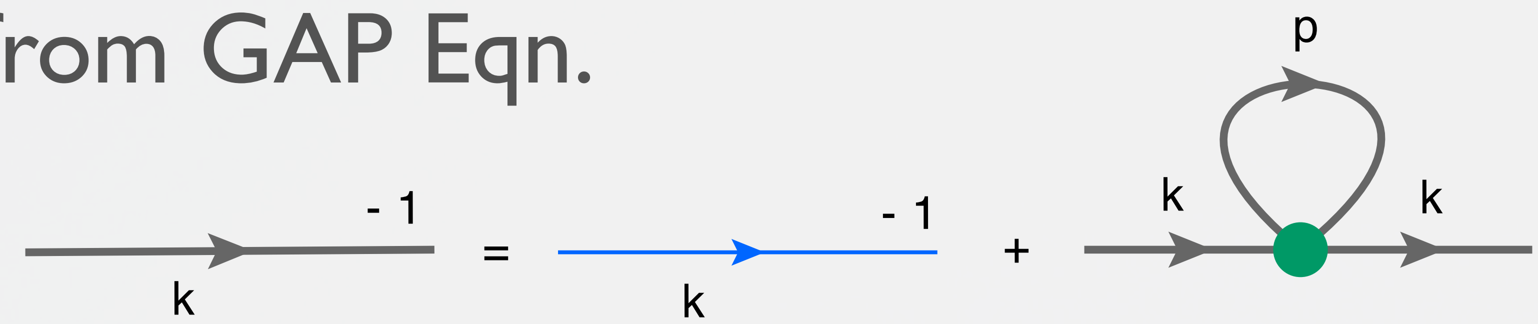
$$\mathcal{L}_{NJL} = \overline{\psi}_q(i\partial\!\!\!/ - m_q)\psi_q + G(\overline{\psi}_q\Gamma\psi_q)^2$$

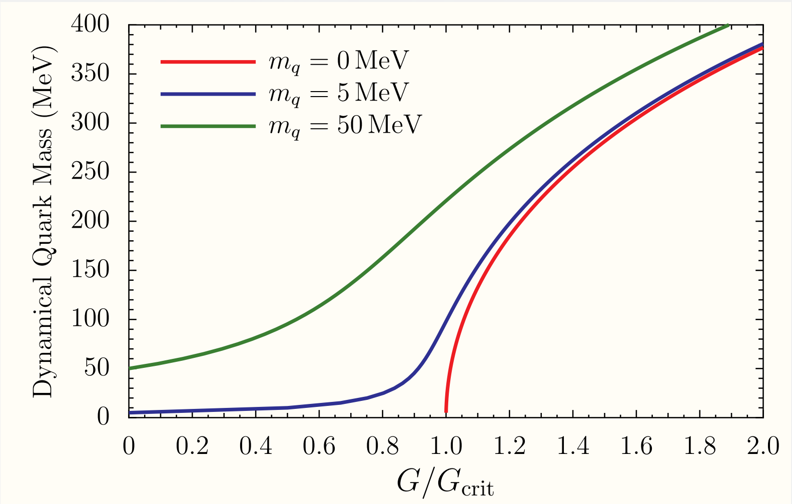


- Low energy chiral effective theory of QCD.
- Covariant, has the same flavor symmetries as QCD.

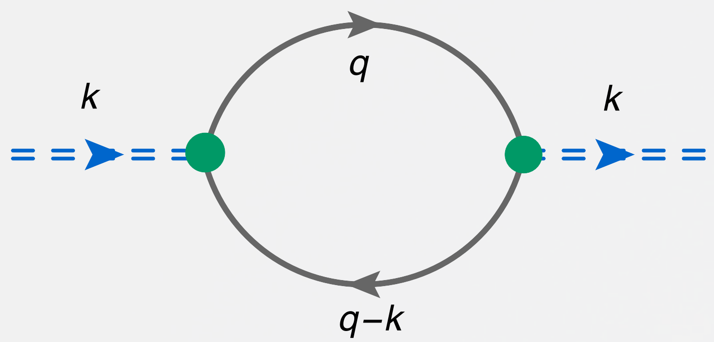
NAMBU--JONA-LASINIO MODEL

•Dynamically Generated Quark Mass from GAP Eqn.

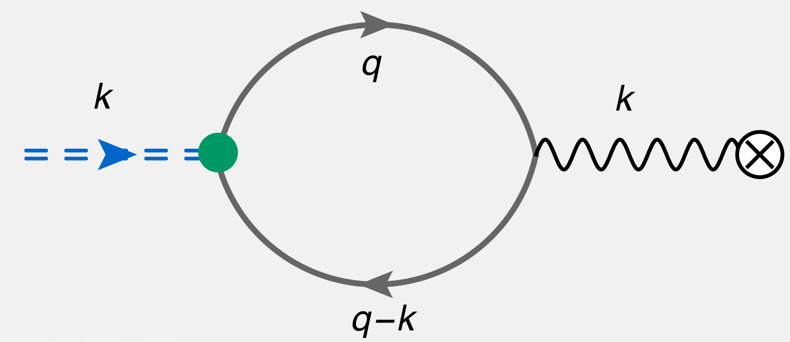




•Pion mass and quark-pion coupling from t-matrix pole.



•Pion decay constant



Fixing Model Parameters

•Use Lepage-Brodsky Invariant Mass cut-off regularisation scheme.

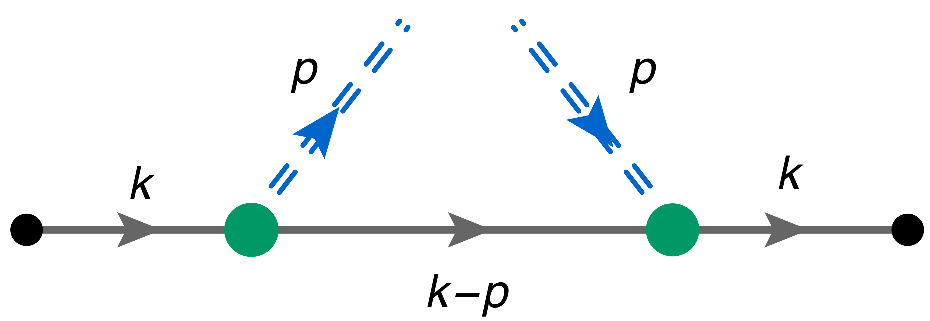
$$M_{12} \leq \Lambda_{12} = \sqrt{\Lambda_3^2 + M_1^2} + \sqrt{\Lambda_3^2 + M_2^2}$$

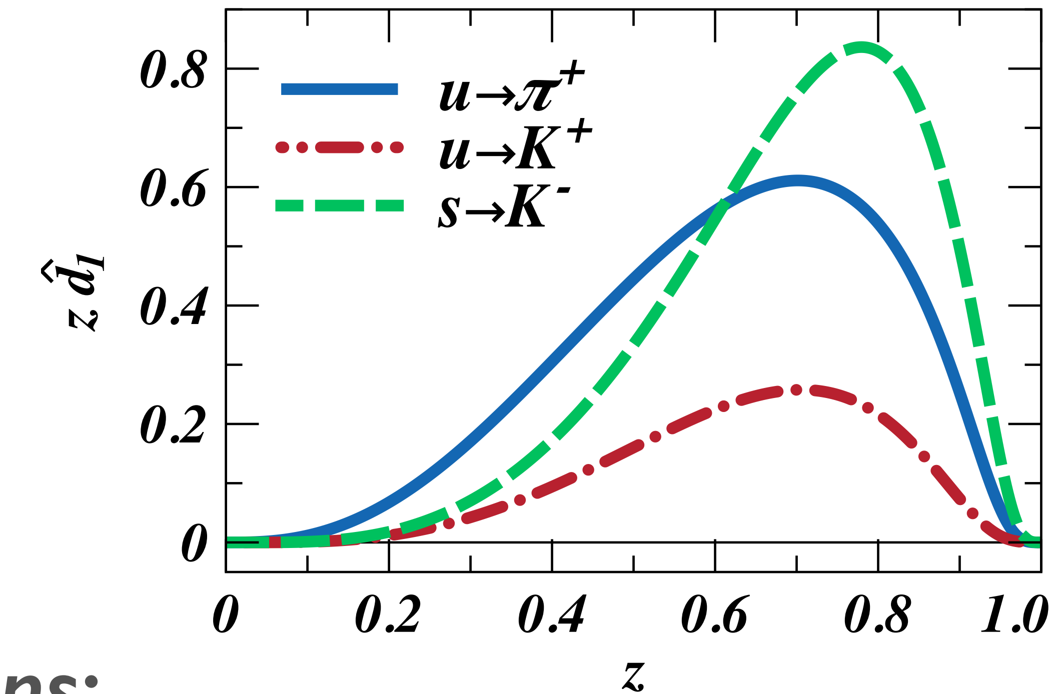
• Choose a $M_{u(d)}$ and use physical f_π, m_π, m_K to fix model parameters $\Lambda_3,\ G, M_s$ and calculate g_{hqQ}.

SOLUTIONS OF THE INTEGRAL EQUATIONS

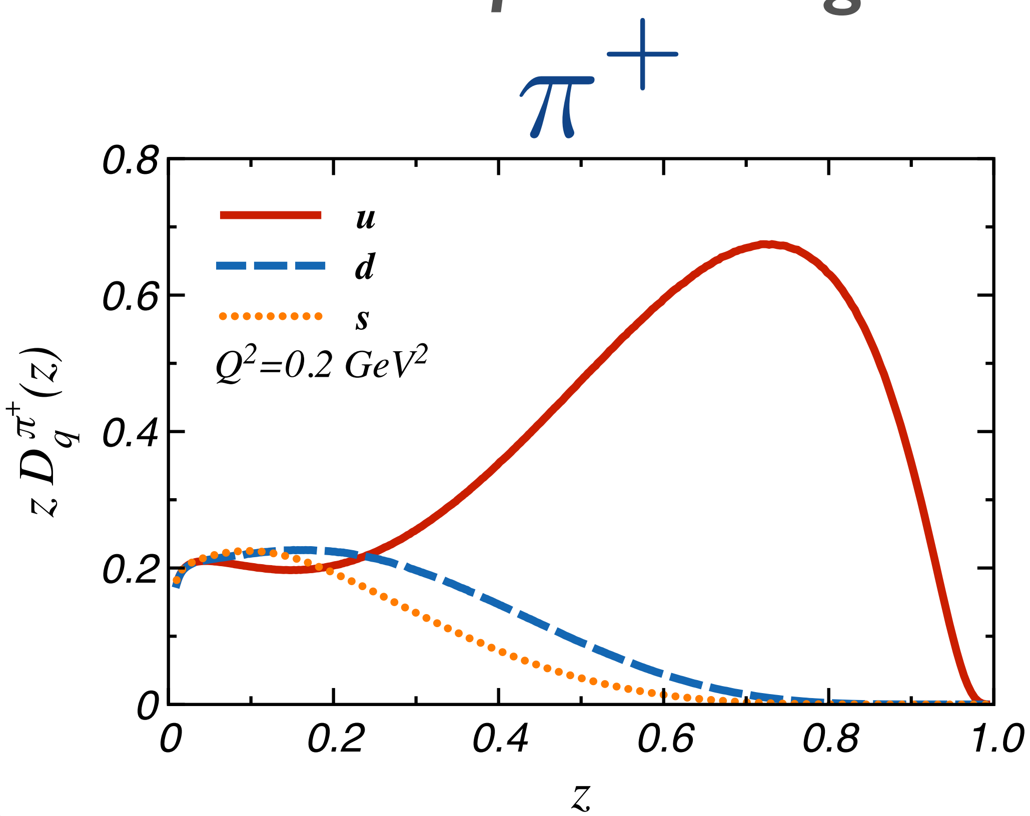
H.M., Thomas, Bentz, PRD. 83:074003, 2011

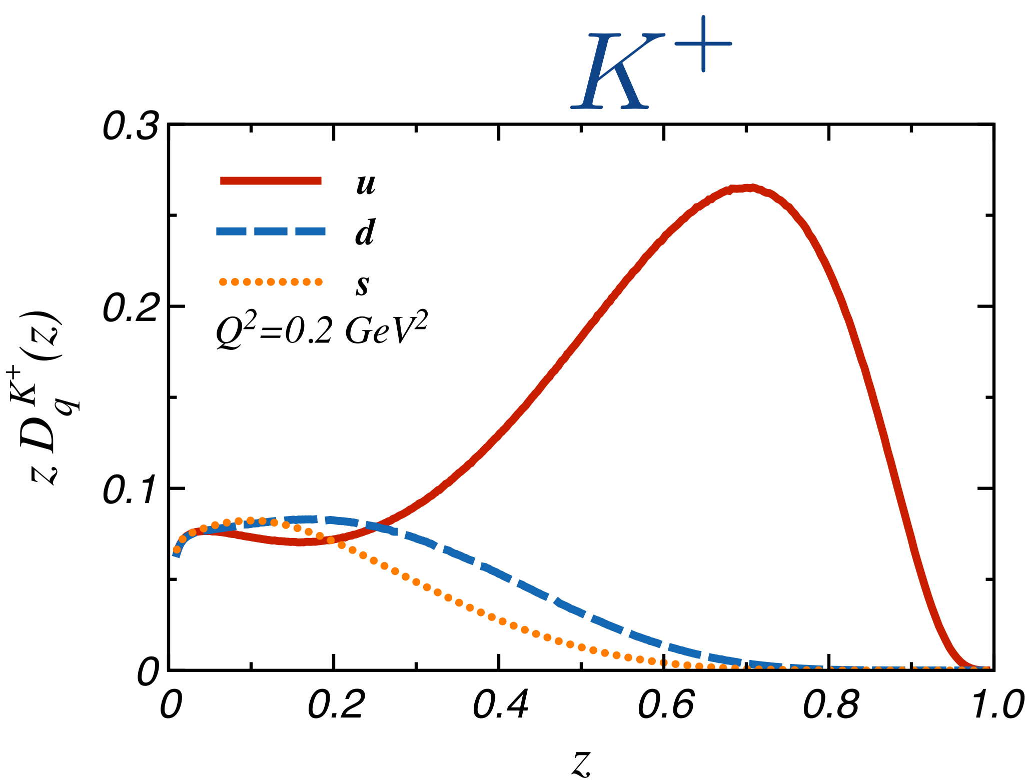
✦ ***Input elementary probabilities from NJL:***





✦ ***Solutions of the integral equations:***





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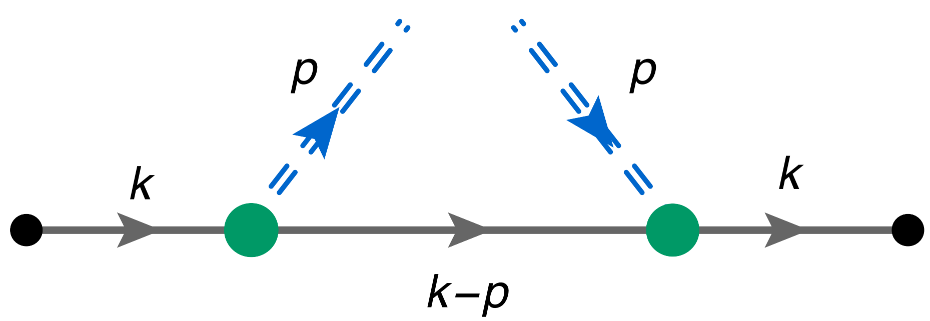
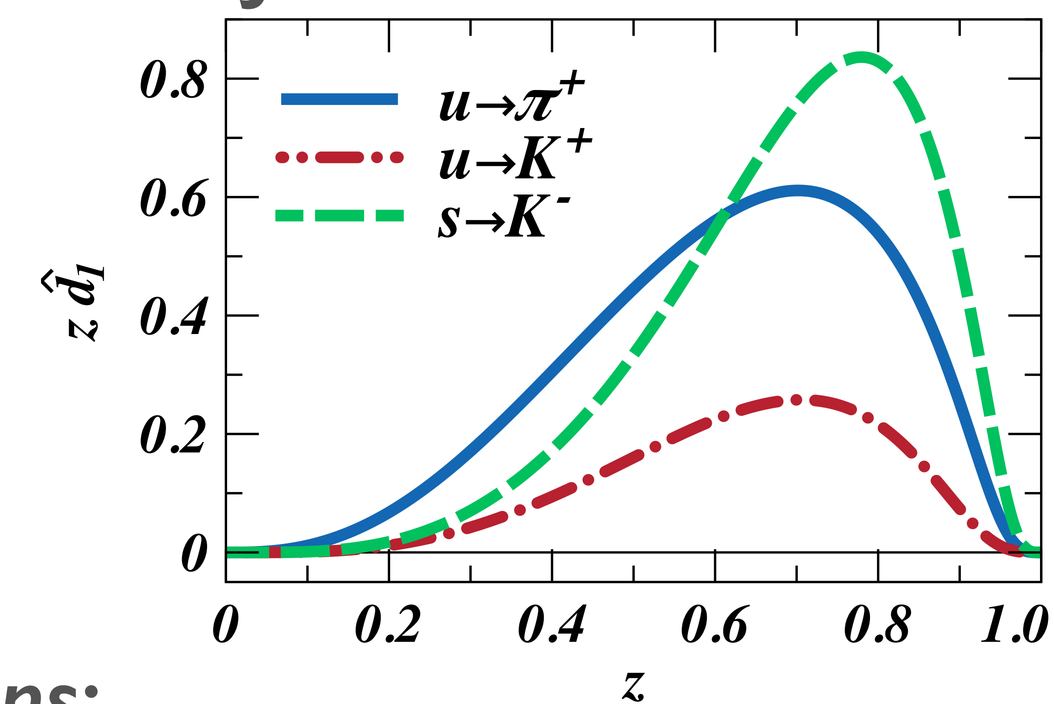
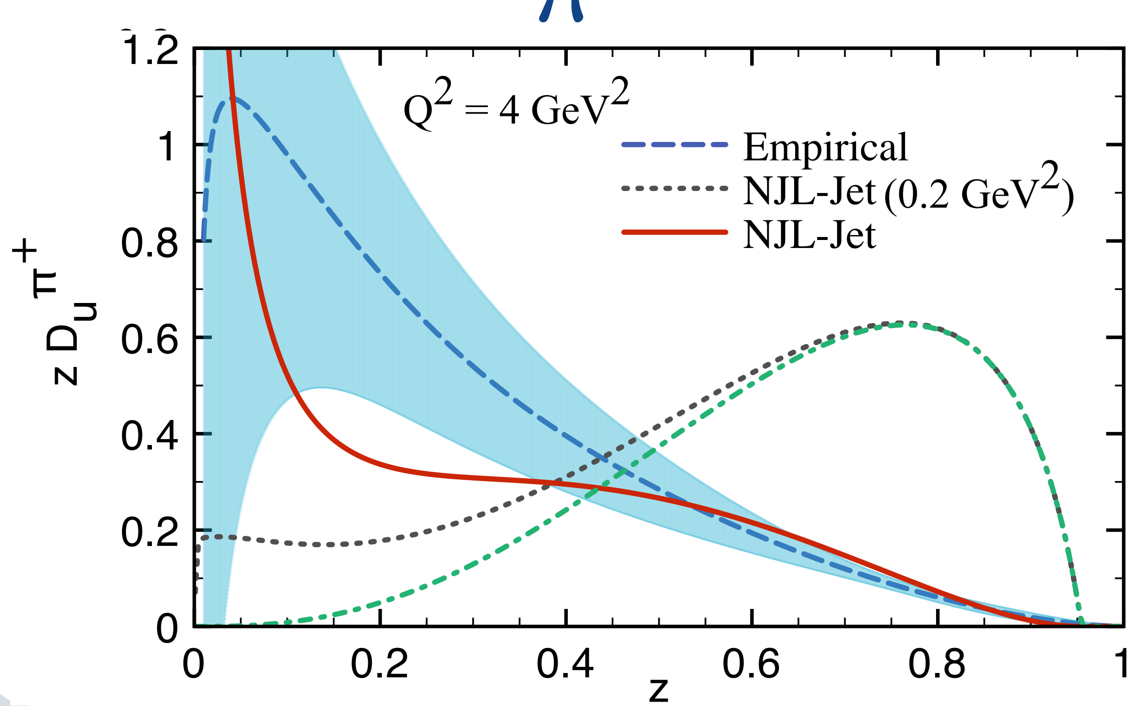


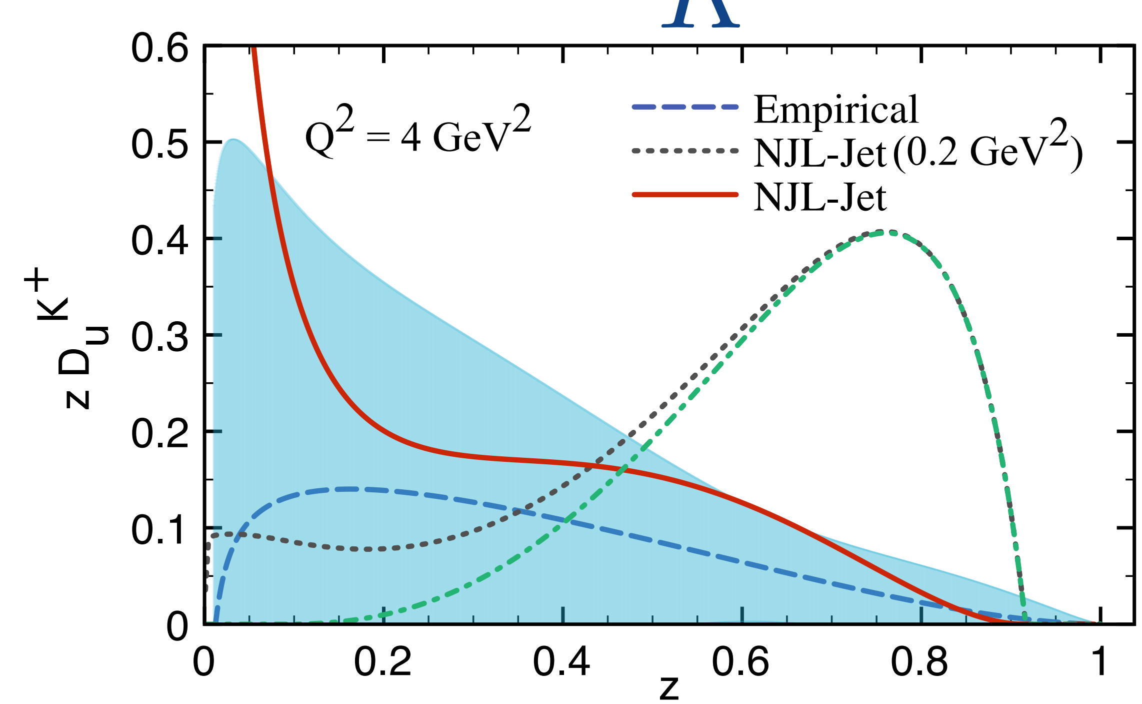


✦ ***Solutions of the integral equations:***

π^+



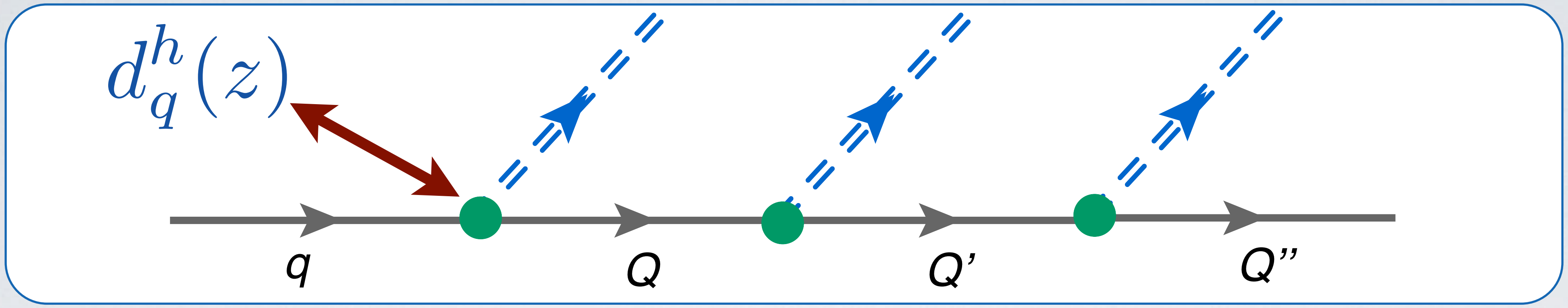
K^+



COLLINEAR FRAGMENTATIONS FROM MC

H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

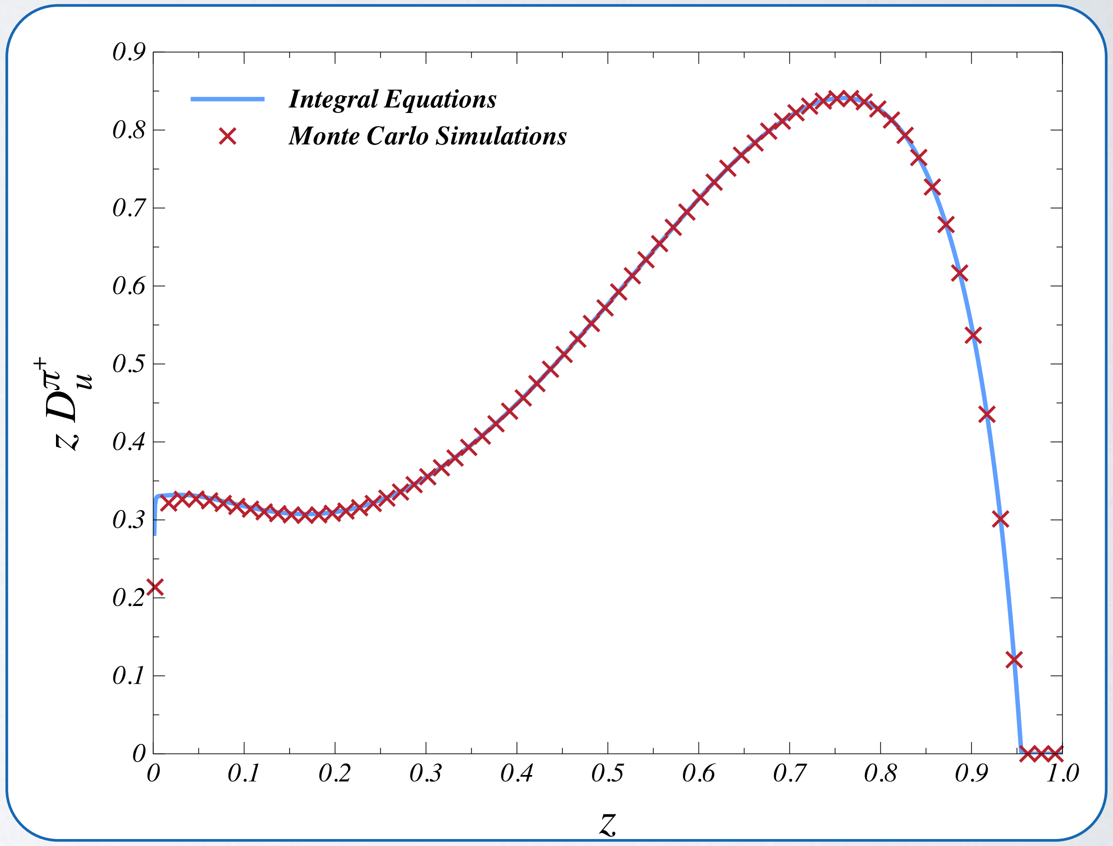
- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.

- ***CONSERVE***: Momentum and Quark Flavor in each step.

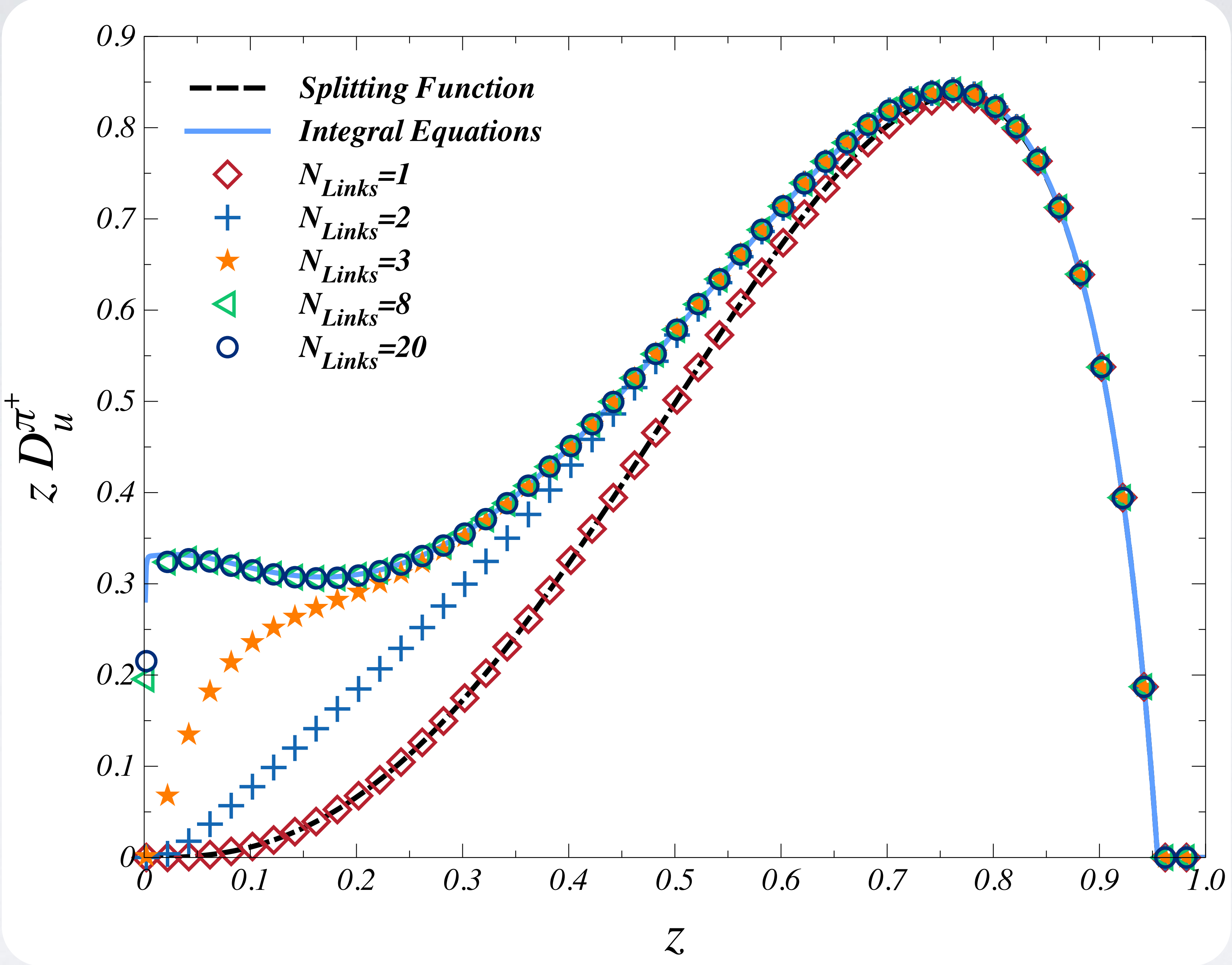
- Repeat for decay chains with the same initial quark.



$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

DEPENDENCE ON NUMBER OF EMITTED HADRONS

- Restrict the number of emitted hadrons, N_{Links} in MC.



- We reproduce the splitting function and the full solution perfectly.
- The low z region is saturated with just a few emissions.

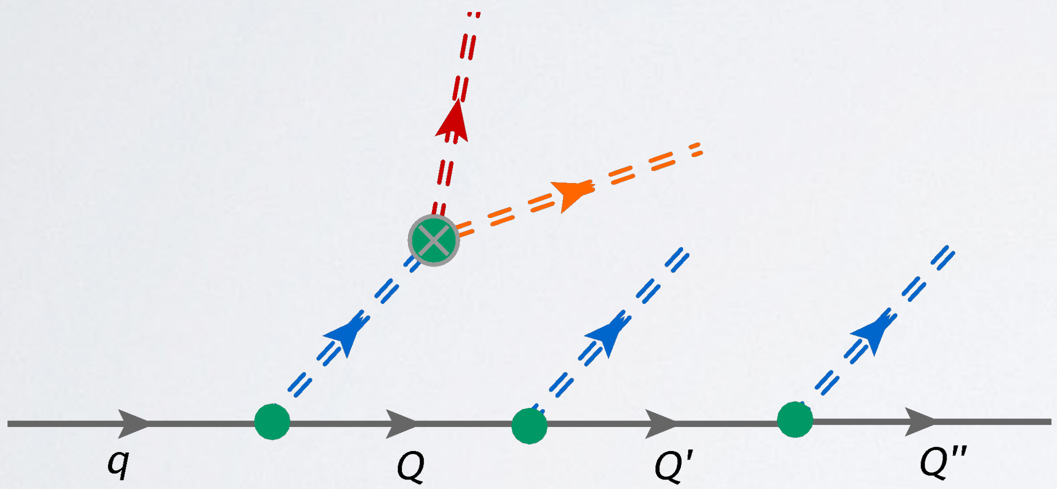
MORE CHANNELS

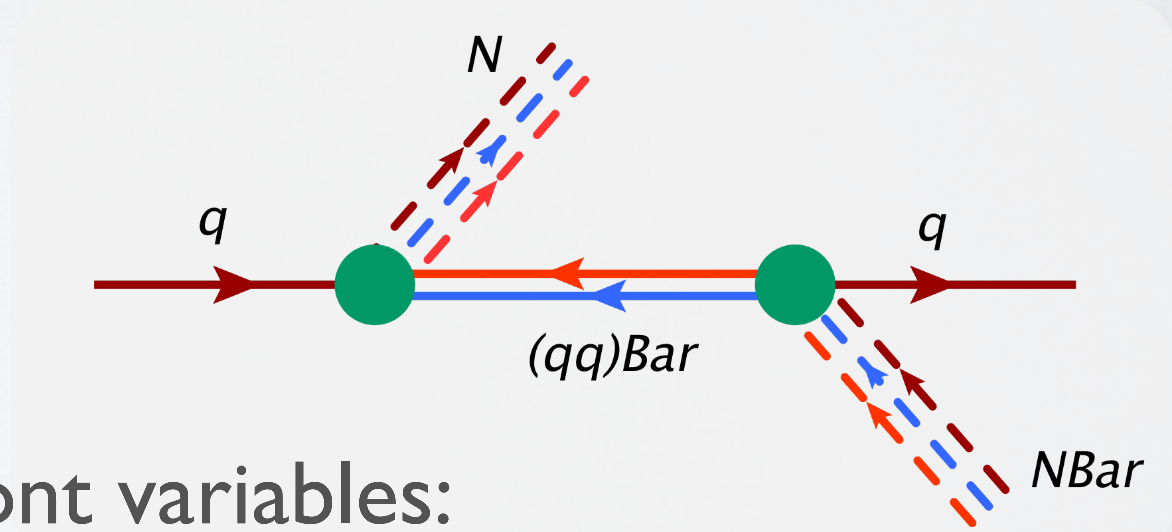
H.M., Thomas, Bentz, PRD. 83:074003, 2011

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}$$

- Add the decay of the resonances:

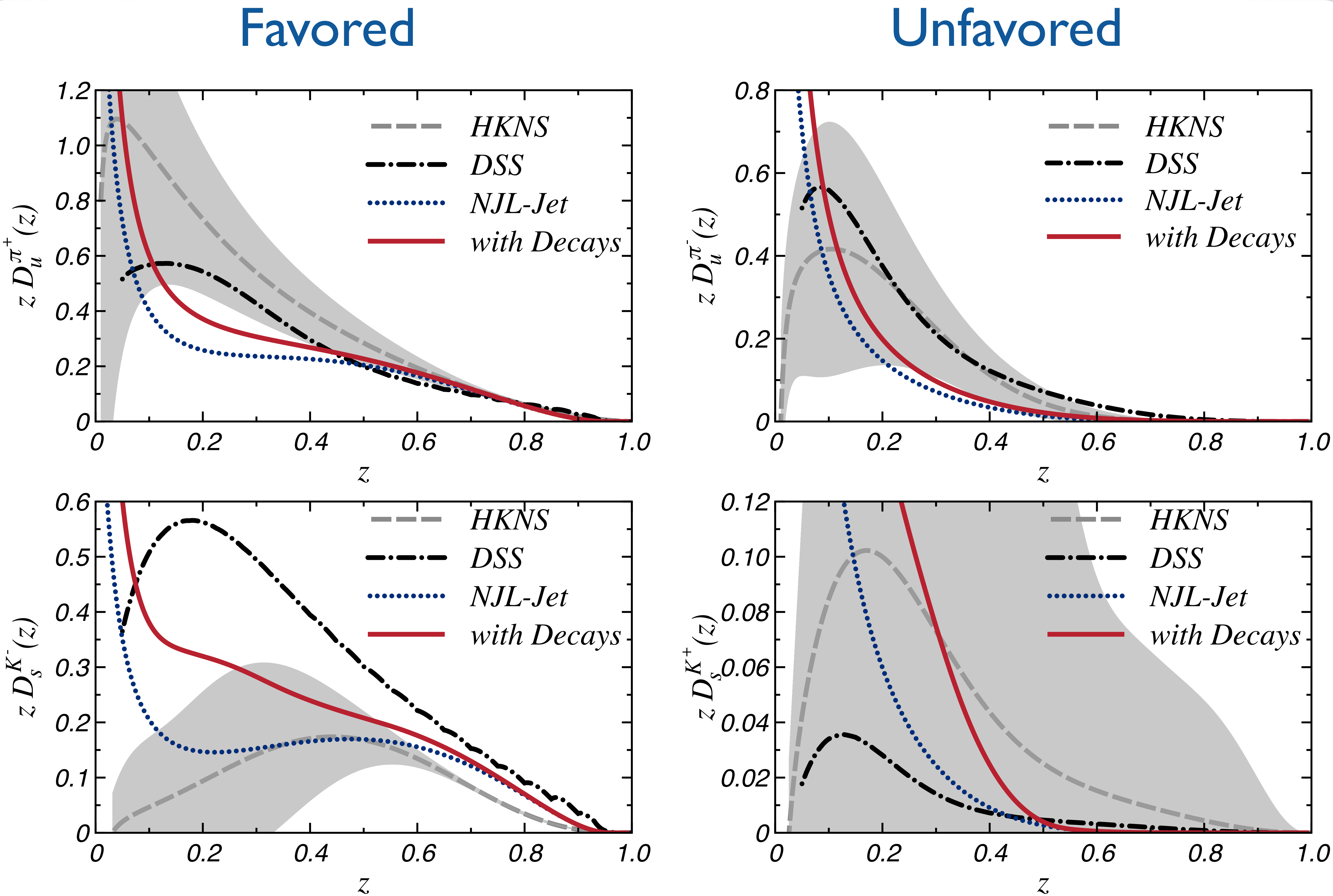


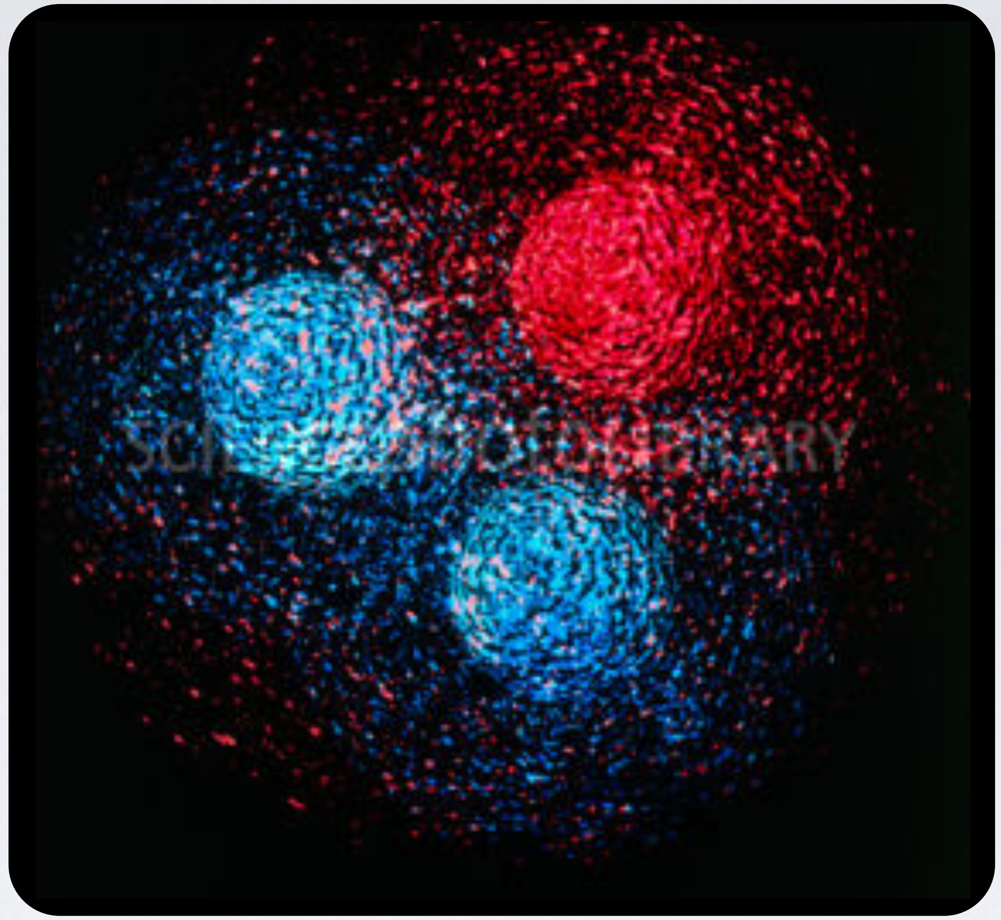


- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Results with VM decays: $Q^2 = 4\ \mathrm{GeV}^2$

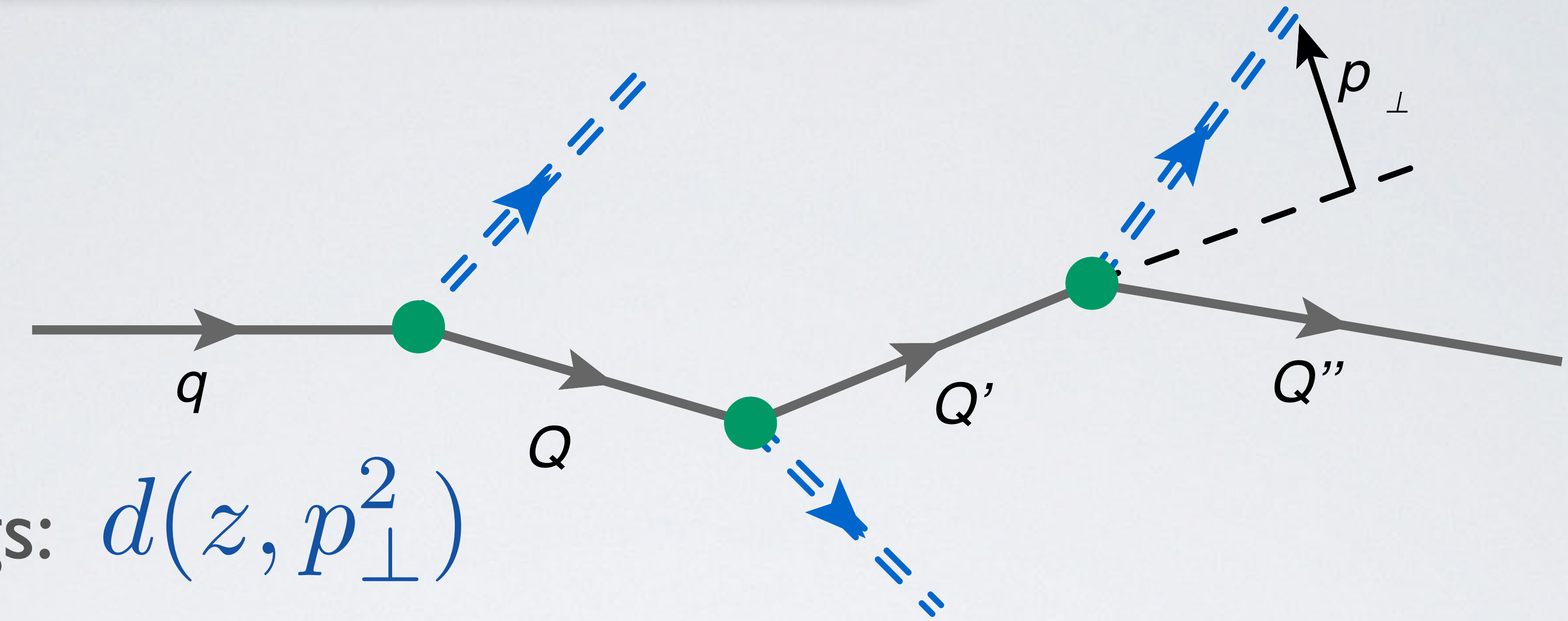




TRANSVERSE MOMENTUM DEPENDENCE

INCLUDING THE TRANSVERSE MOMENTUM

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

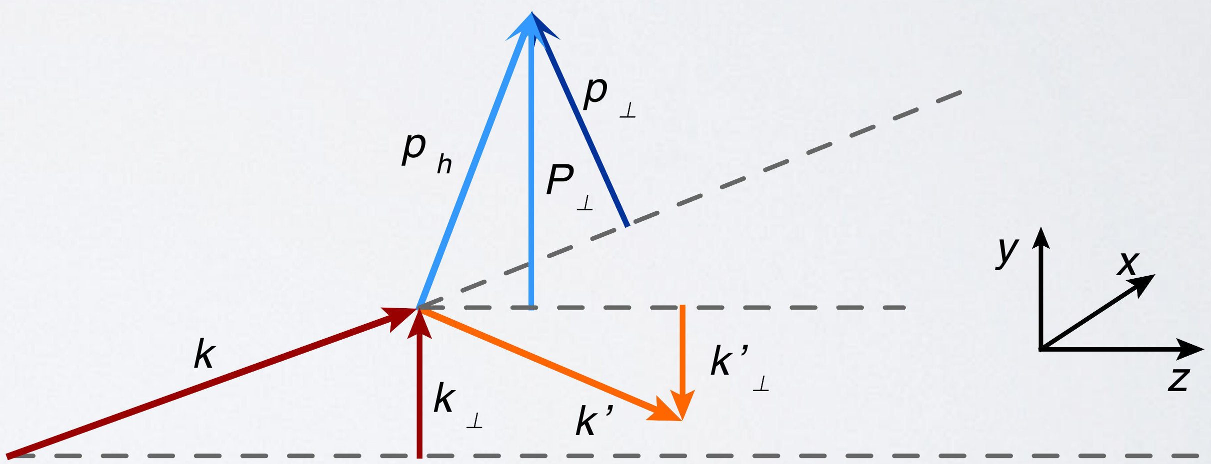


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\boxed{\mathbf{P}_\perp = \mathbf{p}_\perp + z\mathbf{k}_\perp}$$

$$\boxed{\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp}$$



- Calculate the Number Density

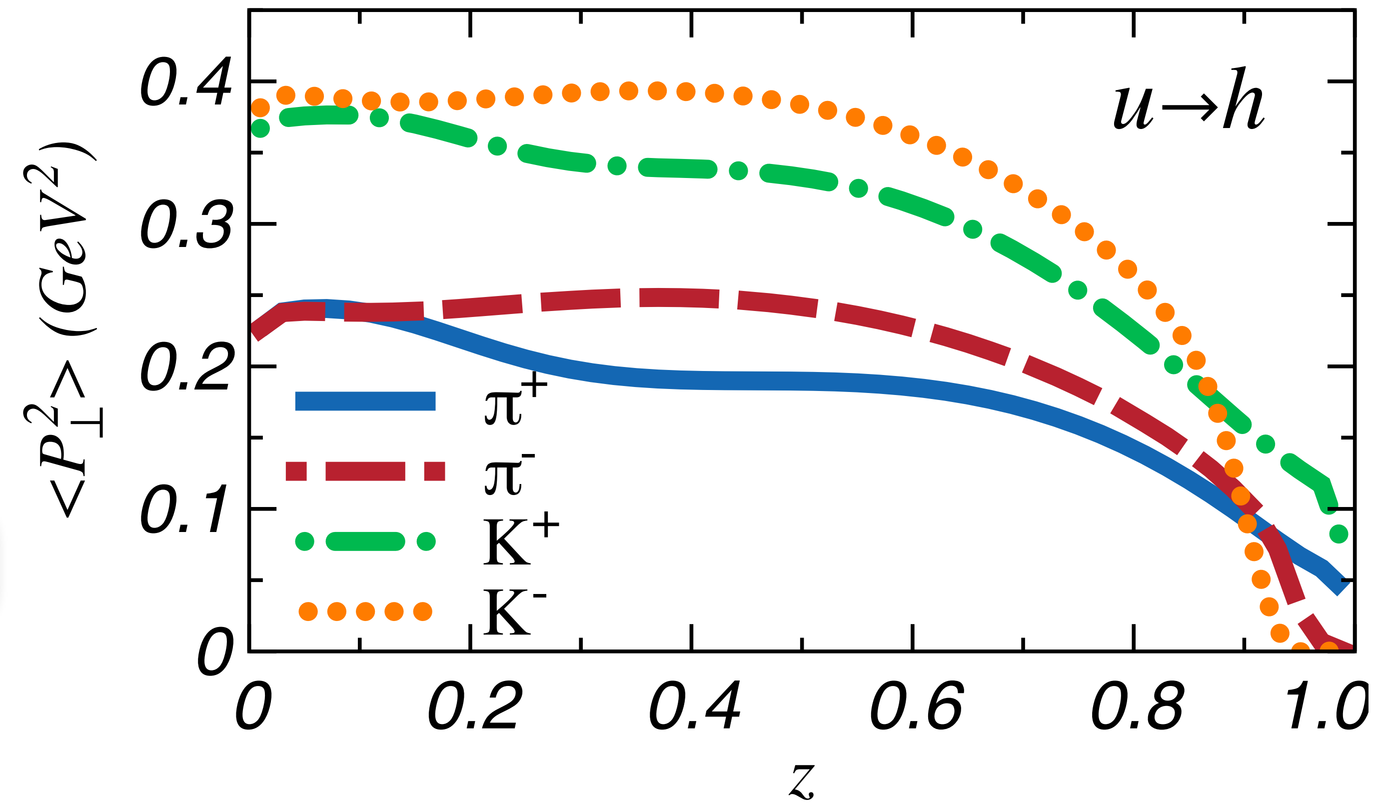
$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2 + \Delta P_\perp^2)}{N_{Sims}}.$$

AVERAGE Transverse Momenta vs z

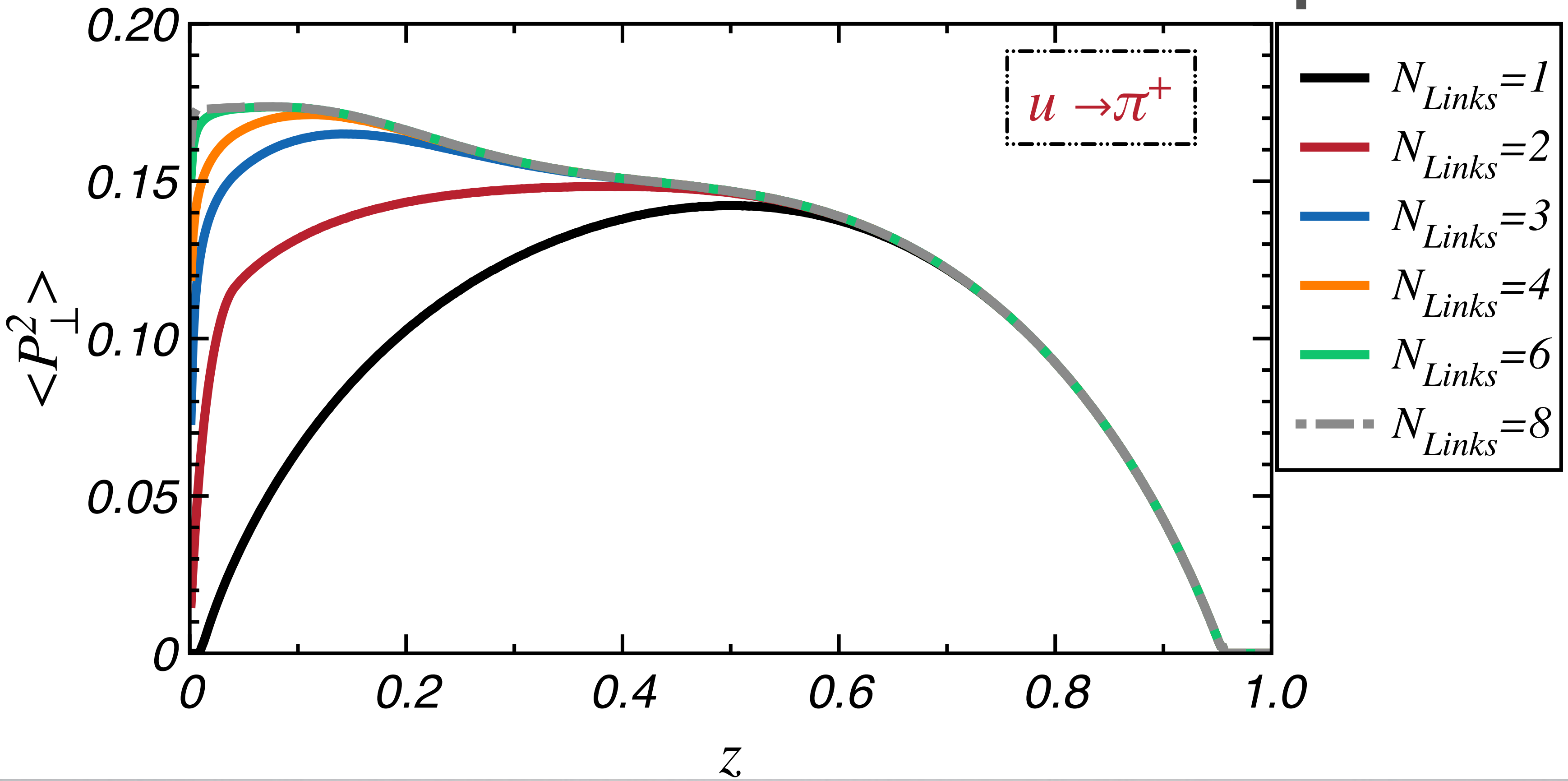
FRAGMENTATION

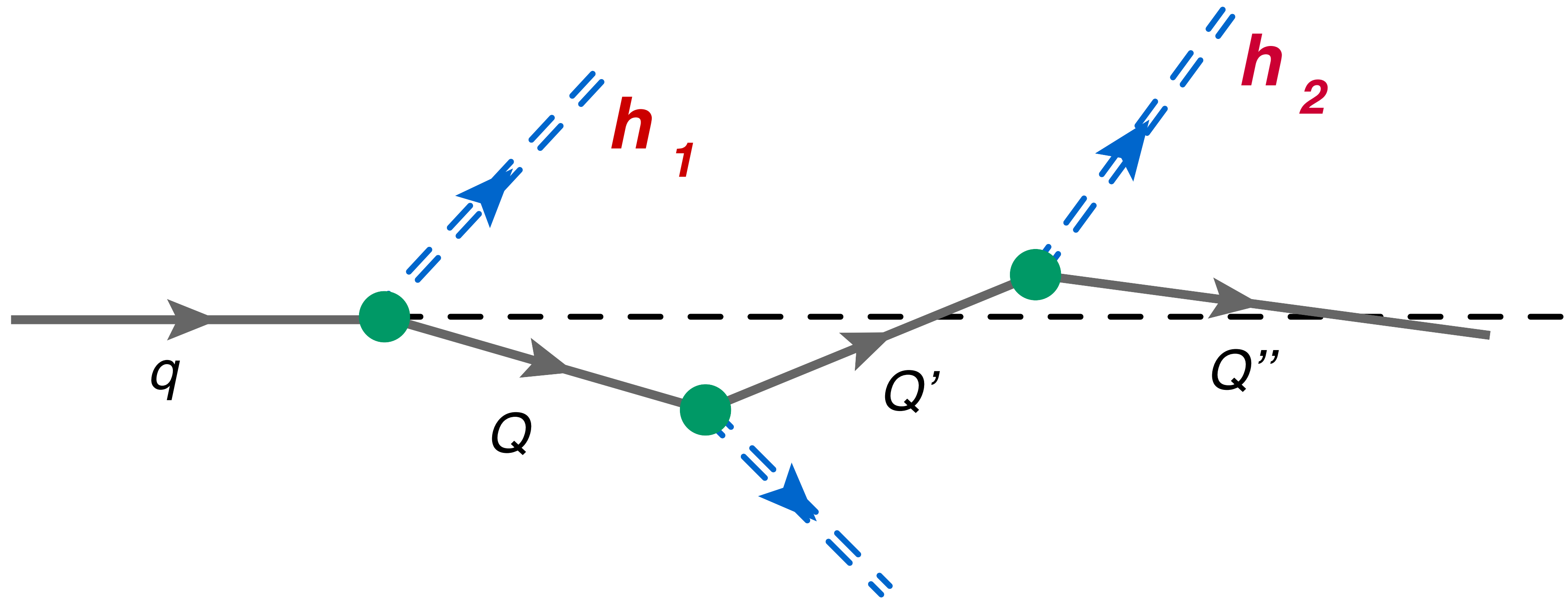
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦Indications from HERMES data: A. Signori, et al: JHEP 1311, 194 (2013)



✓Multiple hadron emissions: ***broaden*** the TM dependence at ***low z***!

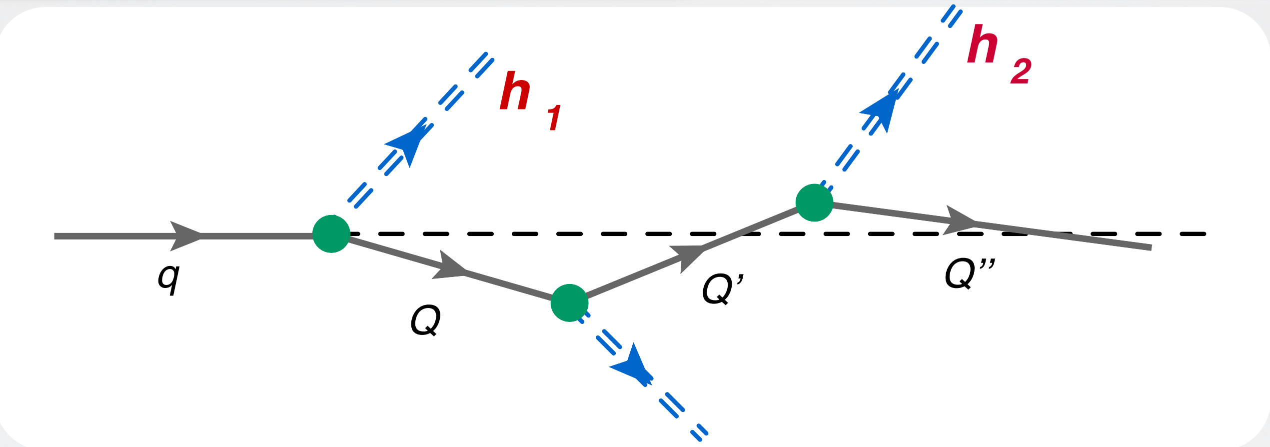




TWO HADRON CORRELATIONS:
DIHADRON FRAGMENTATION FUNCTIONS

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}),\ P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}),\ P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$\boxed{D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2)\rangle}$$

$$z = z_1 + z_2 \qquad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

$$\boxed{z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0}$$

- In MC simulations record all the pairs in every decay chain.

THE EFFECT OF VECTOR MESONS (VM)

- A naive assumption: VMs should have modest contribution due to relatively small production probability $P(\pi^+)/P(\rho^+) \approx 1.7$
- ***But***: Combinatorial factors enhance VM contribution significantly!
- Let's consider only two hadron emission

Direct: $u \to d + \pi^+ \to u + \pi^- + \pi^+$

VM: $u \to d + \pi^+ \to u + \underbrace{\rho^-}_{\to \pi^- \pi^0} + \pi^+$

...

$u \to u + \rho^0 \to u + \underbrace{\rho^0}_{\to \pi^+\pi^-} + \rho^0 \to \pi^+ \pi^-$

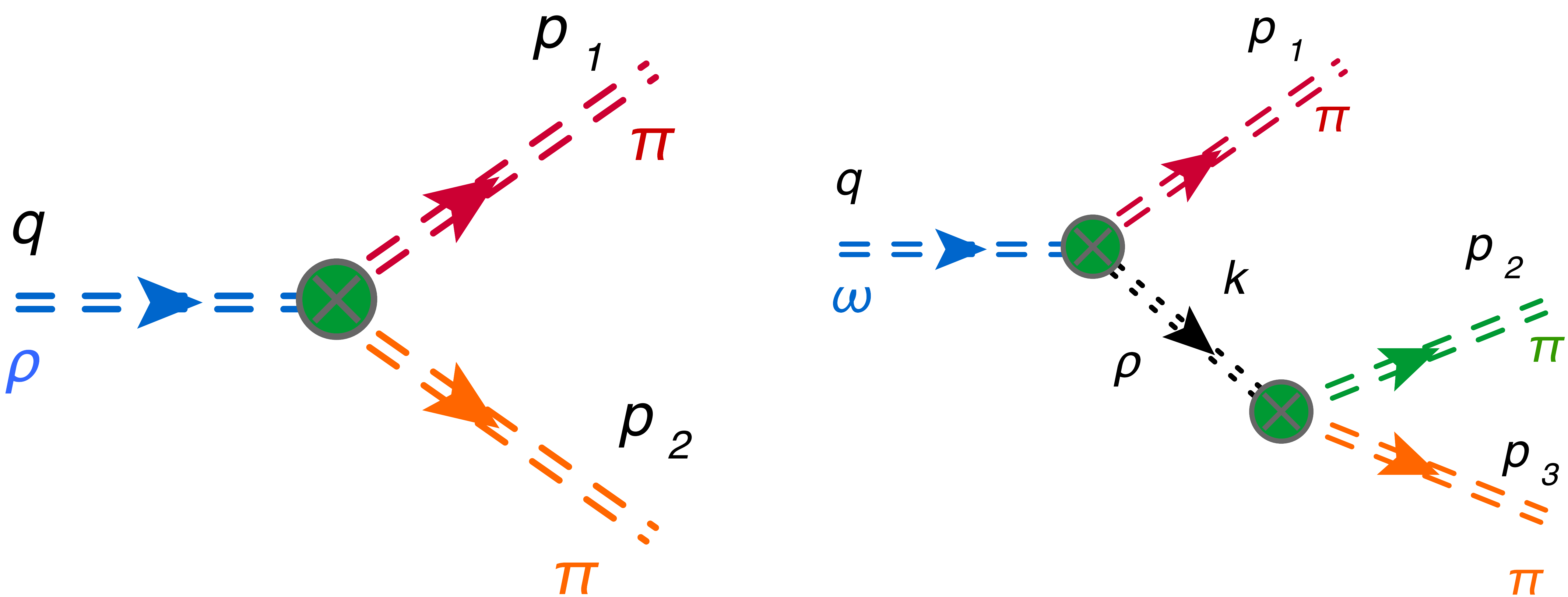
$$P_{Dir}(\pi^+\pi^-)/P_{VM}(\pi^+\pi^-) \approx \frac{1}{4}$$

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

Achasov et al. (SND), PRD 68, 052006, (2003).



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- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ.

Achasov et al. (SND), PRD 68, 052006, (2003).

- 2-body decay amplitude:

$$M(p_1, p_2) = \frac{g_V^{h_1 h_2} \epsilon^\mu (p_{2\mu} - p_{1\mu})}{D_V(q^2)}$$

- Resonance propagator:

$$D_V(s) = m_V^2 - s - i\sqrt{s}\Gamma_V(s)$$

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V \left(\frac{q(s)}{q(m_V^2)}\right)^3$$

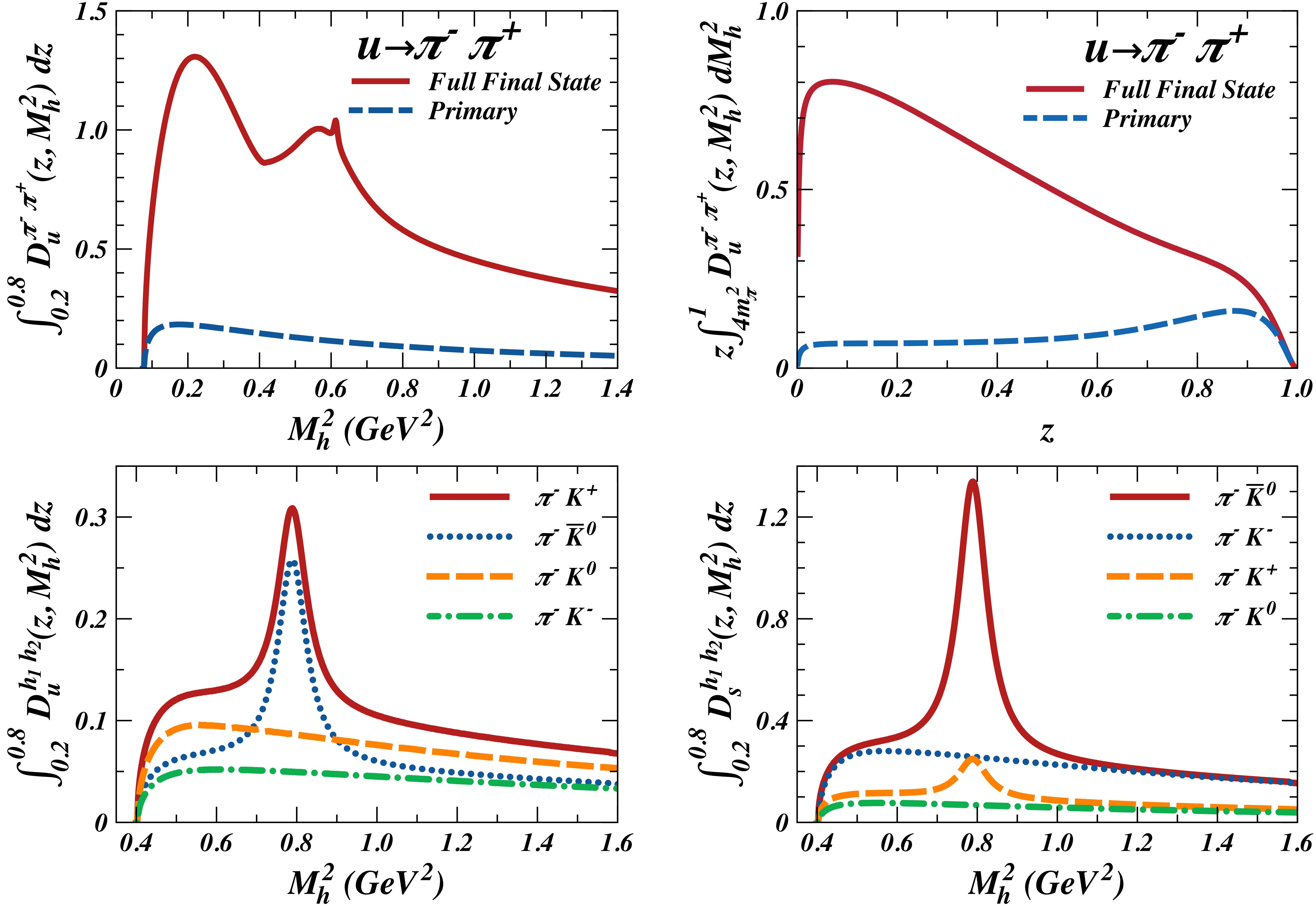
Relative Momentum of daughters in their CM frame.

- 3-body decay amplitude (ignore small width):

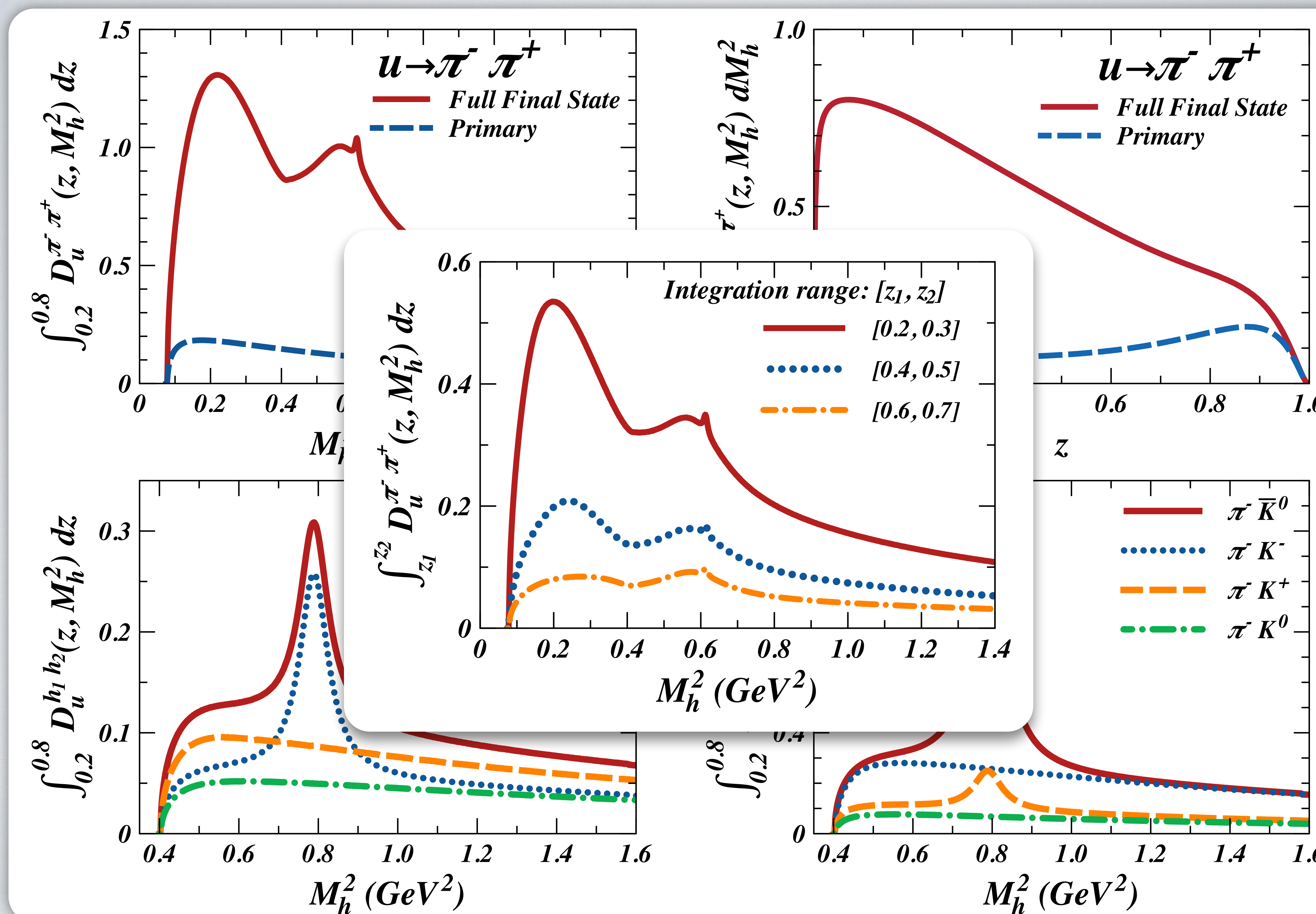
$$M(p_1, p_2, p_3) = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\mu p_1^\alpha p_2^\beta p_3^\gamma \sum_{i=0,\pm} \frac{g_{V\rho_i\pi}\ g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$

- Simulate 2- and 3-body phase space in LC.

RESULTS FOR DFFS $N_{Links} = 8$



RESULTS FOR DFFS $N_{Links} = 8$

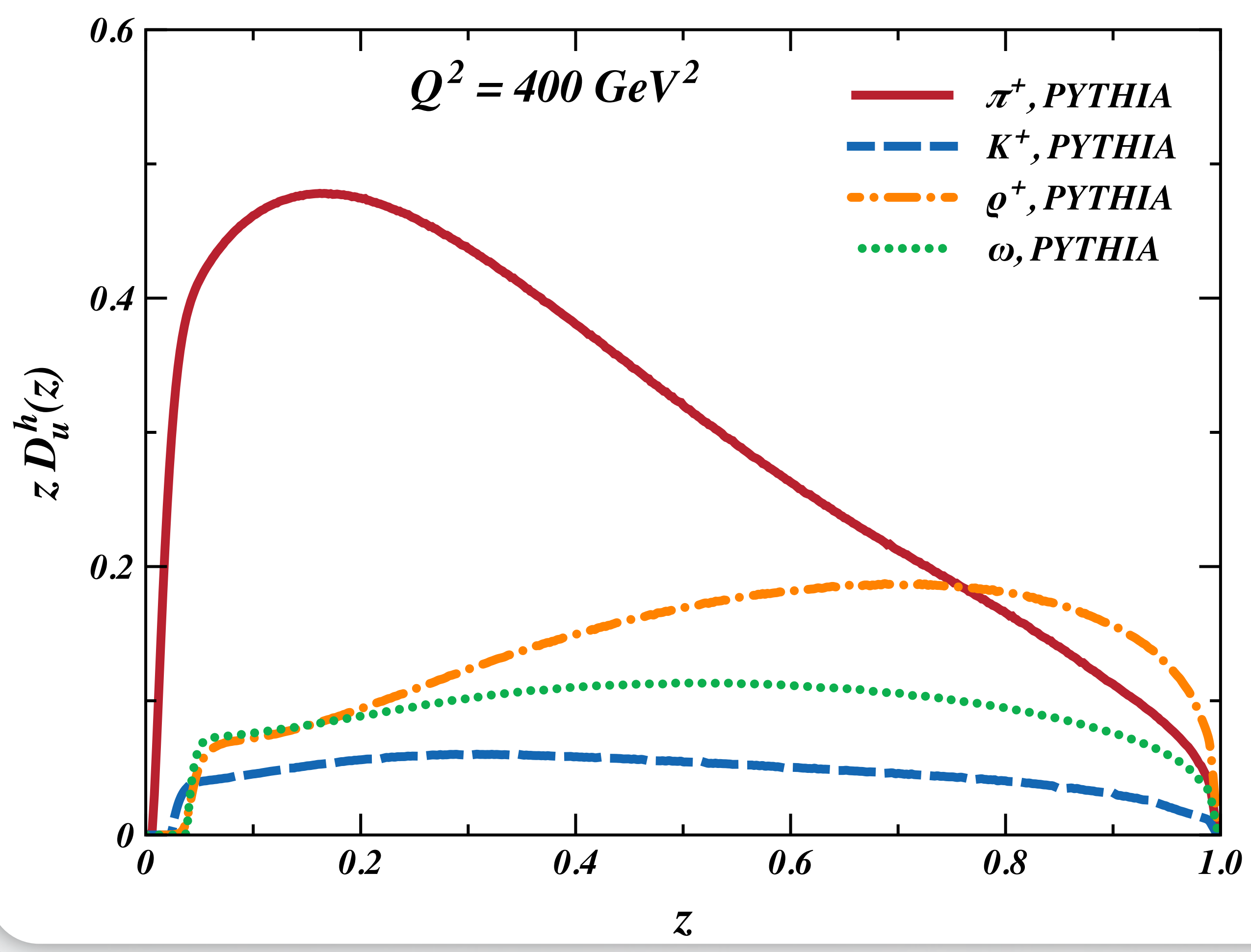


PYTHIA SIMULATIONS

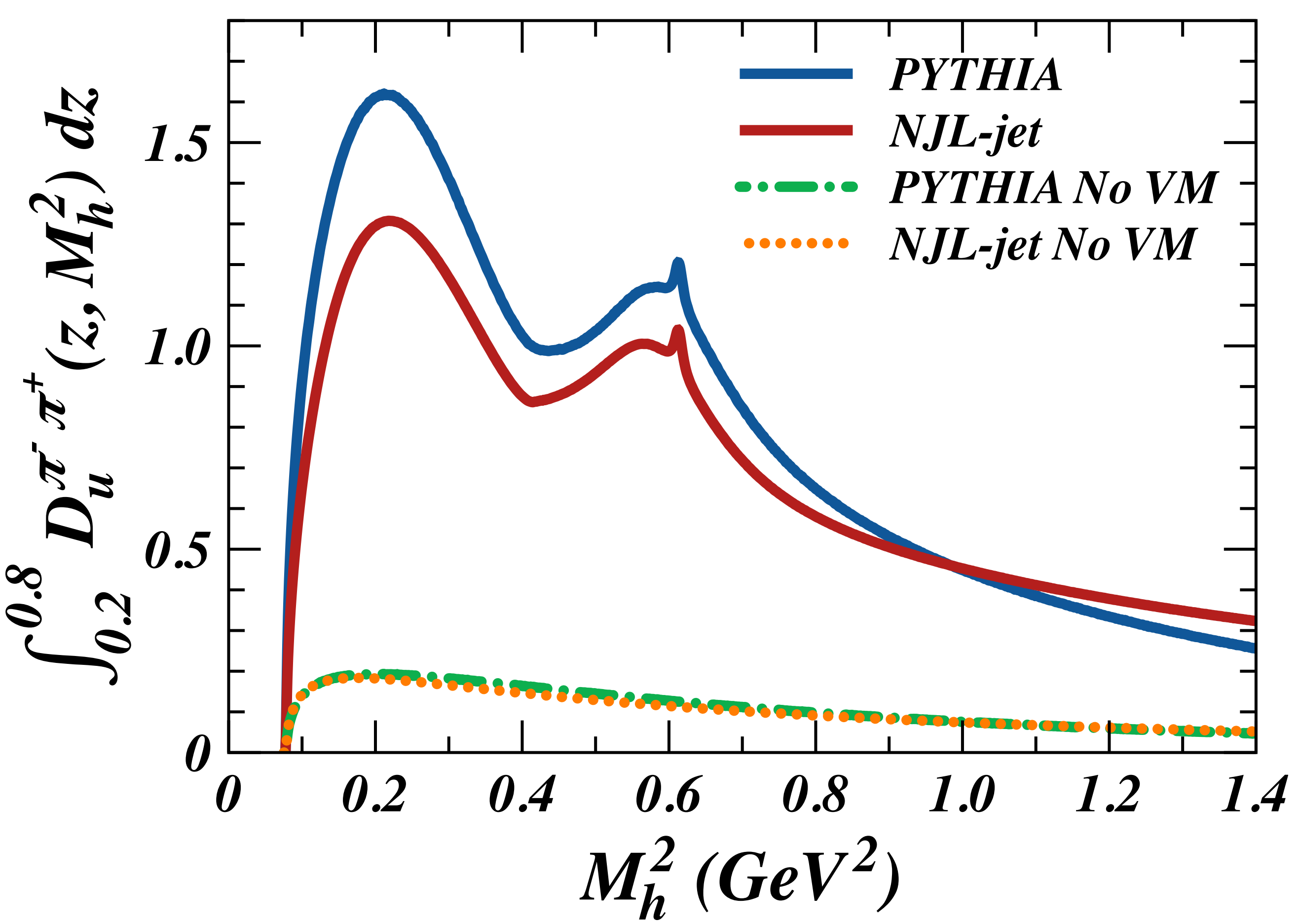
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- *Only Hadronize.* Allow the same resonance decays as NJL-jet.
- Assign hadrons with positive p_z to q fragmentation.

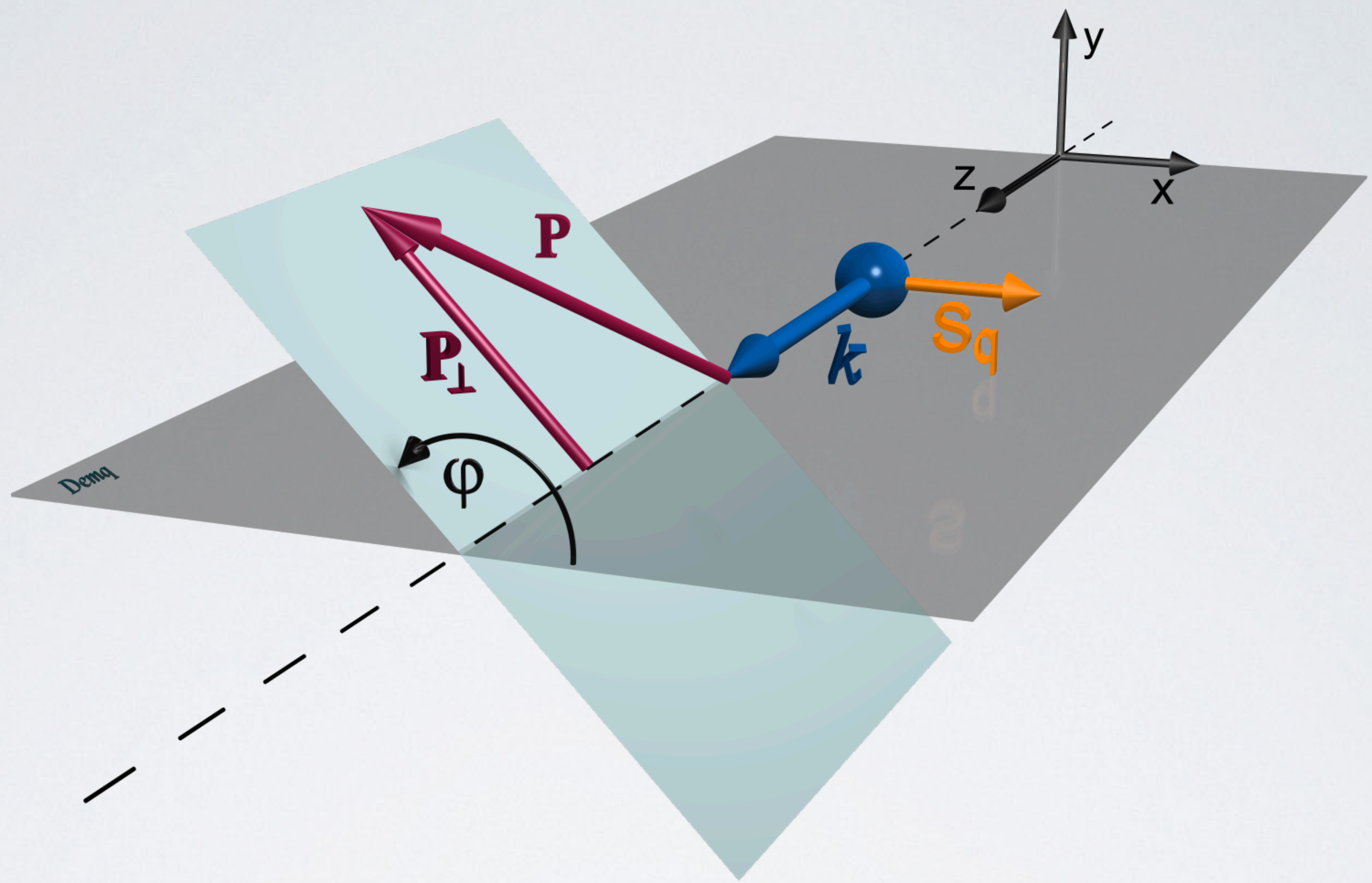
$$E_q = 10\ \text{GeV}$$

Single Hadron



Dihadron



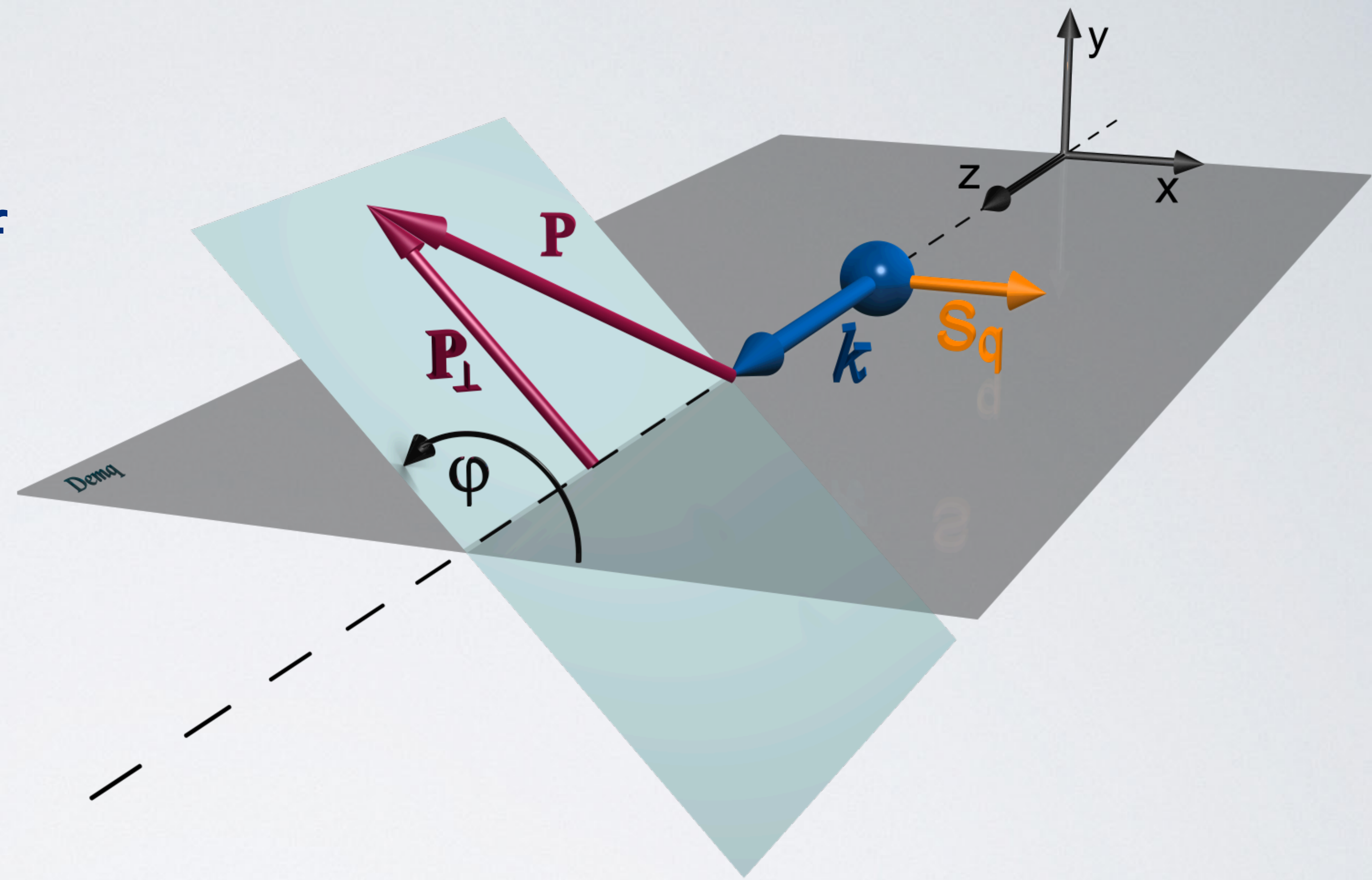


TRANSVERSELY POLARIZED QUARK FRAGMENTATION:
COLLINS EFFECT AND TWO-HADRON CORRELATIONS

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



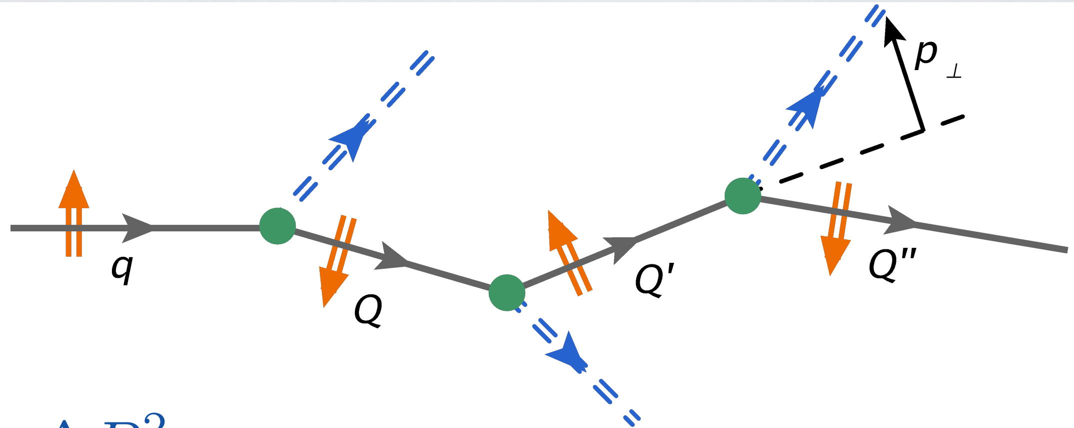
Unpolarized

$$D_{h/q^{\uparrow}}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

Collins

- **Chiral-ODD**: Needs to be coupled with another chiral-odd quantity to be observed.

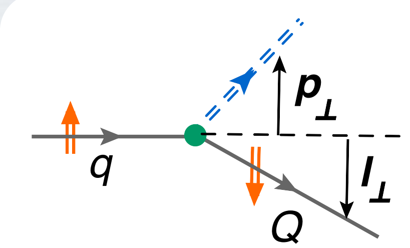
COLLINS EFFECT - NJL-jet MKII



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

MKII Model Assumptions: **H.M., Kotzinian, Thomas, PLB731 208-216 (2014).**

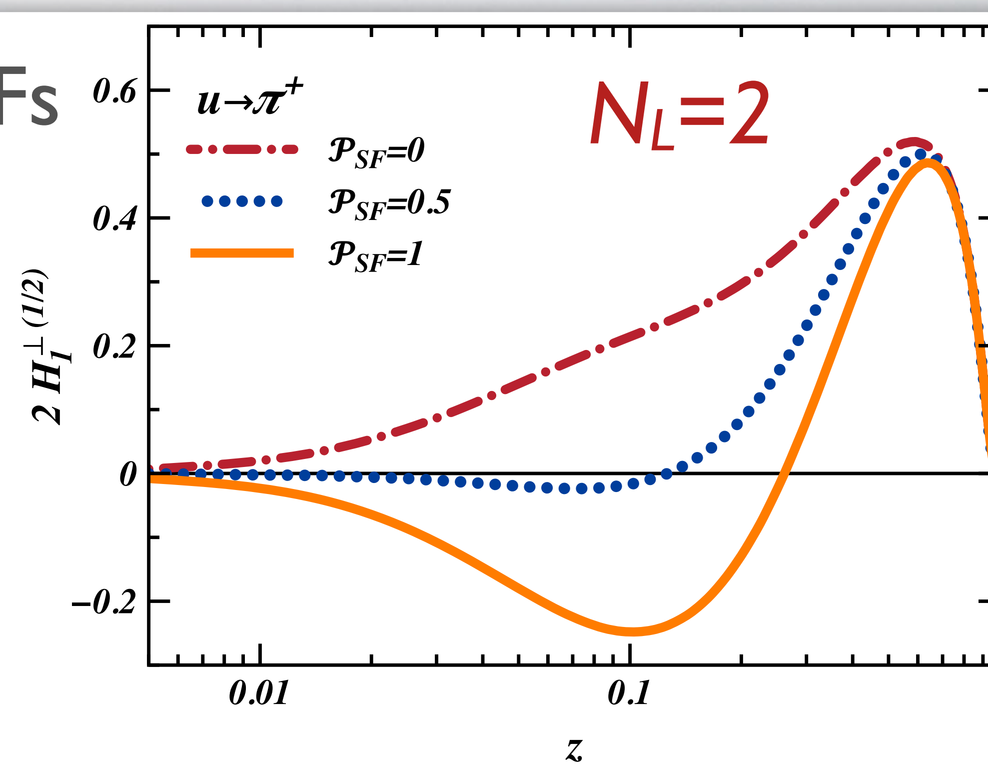
1. Allow for Collins Effect only in a SINGLE emission vertex (N_L^{-1} scaling of the resulting Collins function).
2. Use constant values for spin flip probability: $\mathcal{P}_{SF}$.



✦ Use fit form to extract unpol. and Collins FFs from $D_{h/q^\uparrow}$.

$$\boxed{F(c_0, c_1) = c_0 - c_1 \sin(\varphi)}$$

✦ The results for N_L=2, $\mathcal{P}_{SF} = 1$.



COLLINS EFFECT - MKII

MKII Model Assumptions:

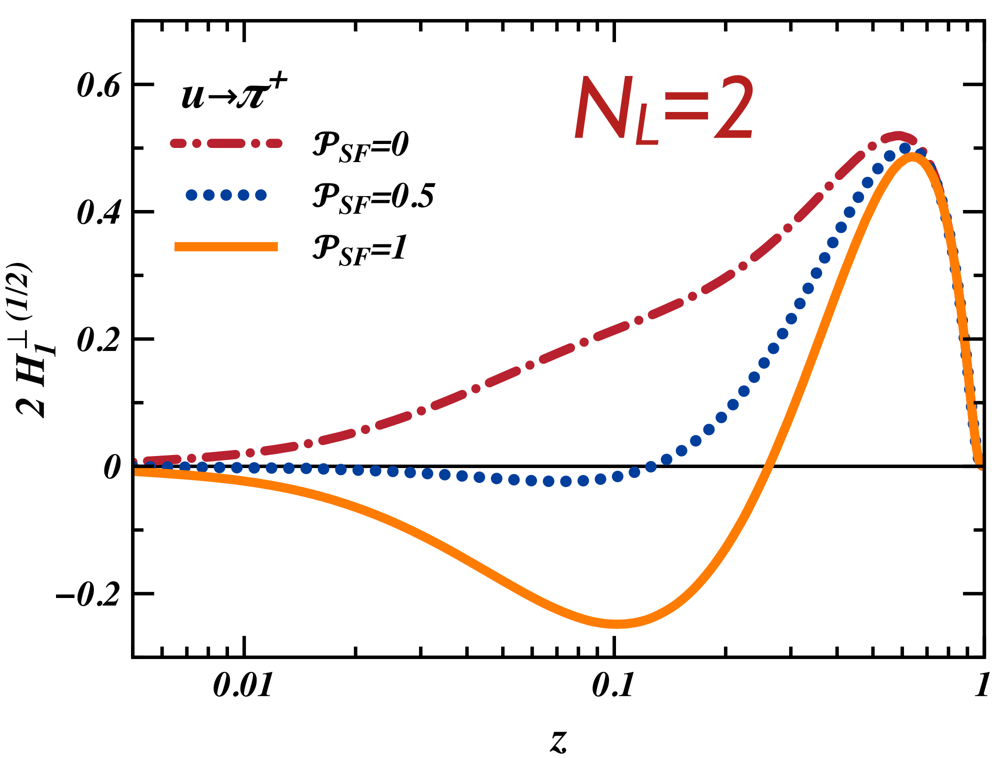
H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

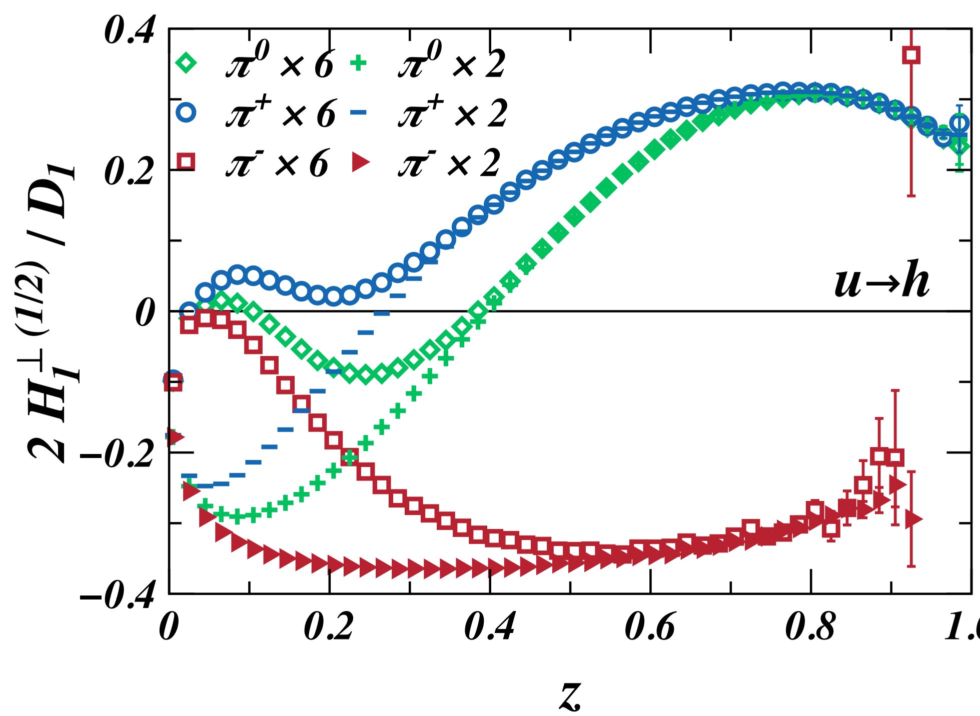
1. Allow for Collins Effect only in a SINGLE emission vertex - N_L^{-1} scaling of the resulting Collins function.
2. Use constant values for $\mathcal{P}_{SF}$.

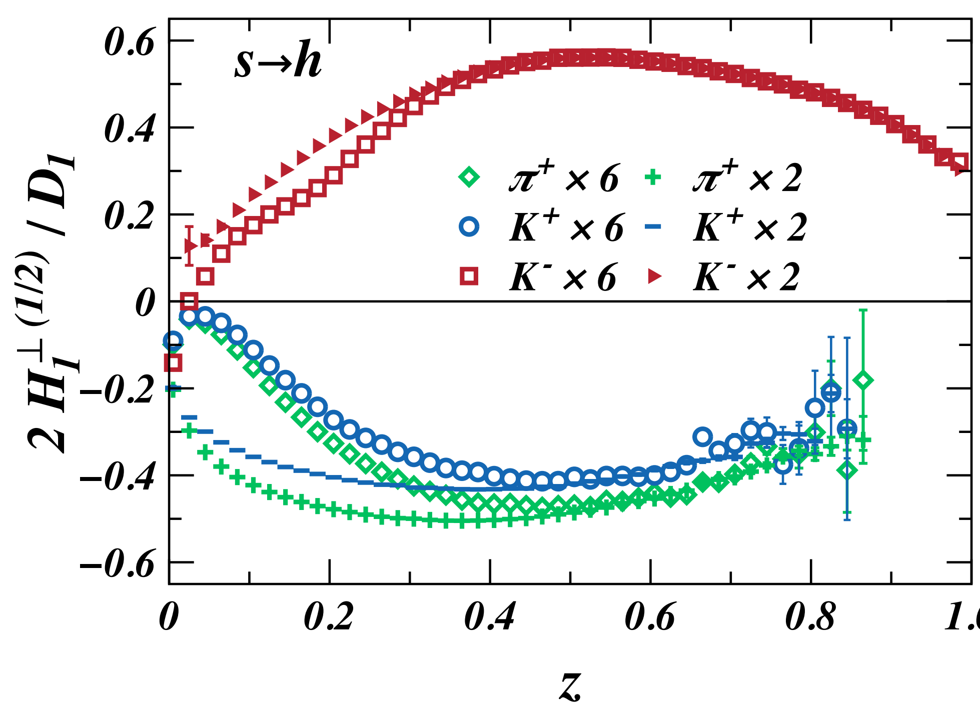
$$\mathcal{P}_{SF} = 1$$

✦ The results for N_L=2 and N_L=6, scaled up by a factor N_L.

$$F(c_0, c_1) = c_0 - c_1 \sin(\varphi)$$

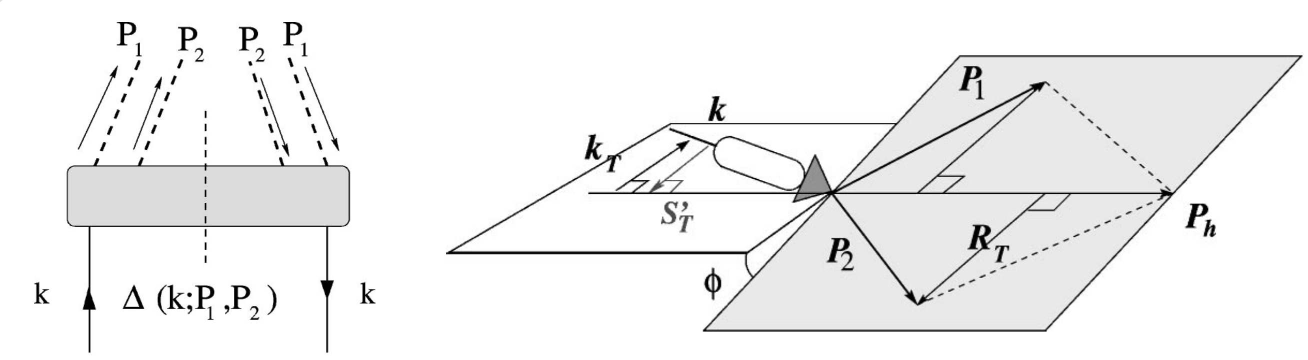






TWO-HADRON FRAGMENTATION

A. Bianconi, et al: PRD 62, 034008 (2000). M. Radici, et al: PRD 65, 074031 (2002).



- Kinematic Variables:

$$P_1 = \left[\xi P_h^- , \frac{M_1^2 + \vec{R}_T^2}{2\,\xi P_h^-}, \vec{R}_T \right], \qquad k = \left[\frac{P_h^-}{z}, z\frac{k^2 + \vec{k}_T^2}{2P_h^-}, \vec{k}_T \right] \qquad z \equiv z_h = z_1 + z_2$$

$$P_2 = \left[(1-\xi) P_h^- , \frac{M_2^2 + \vec{R}_T^2}{2(1-\xi) P_h^-}, -\vec{R}_T \right] \qquad \mathbf{R} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2} \qquad \xi = \frac{z_1}{z_1 + z_2}$$

- The relevant terms of the quark correlator at leading order for a **Transversely Polarized Quark:**

Unpolarized

$$\Delta^{[\gamma^-]} = D_1(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

Interference

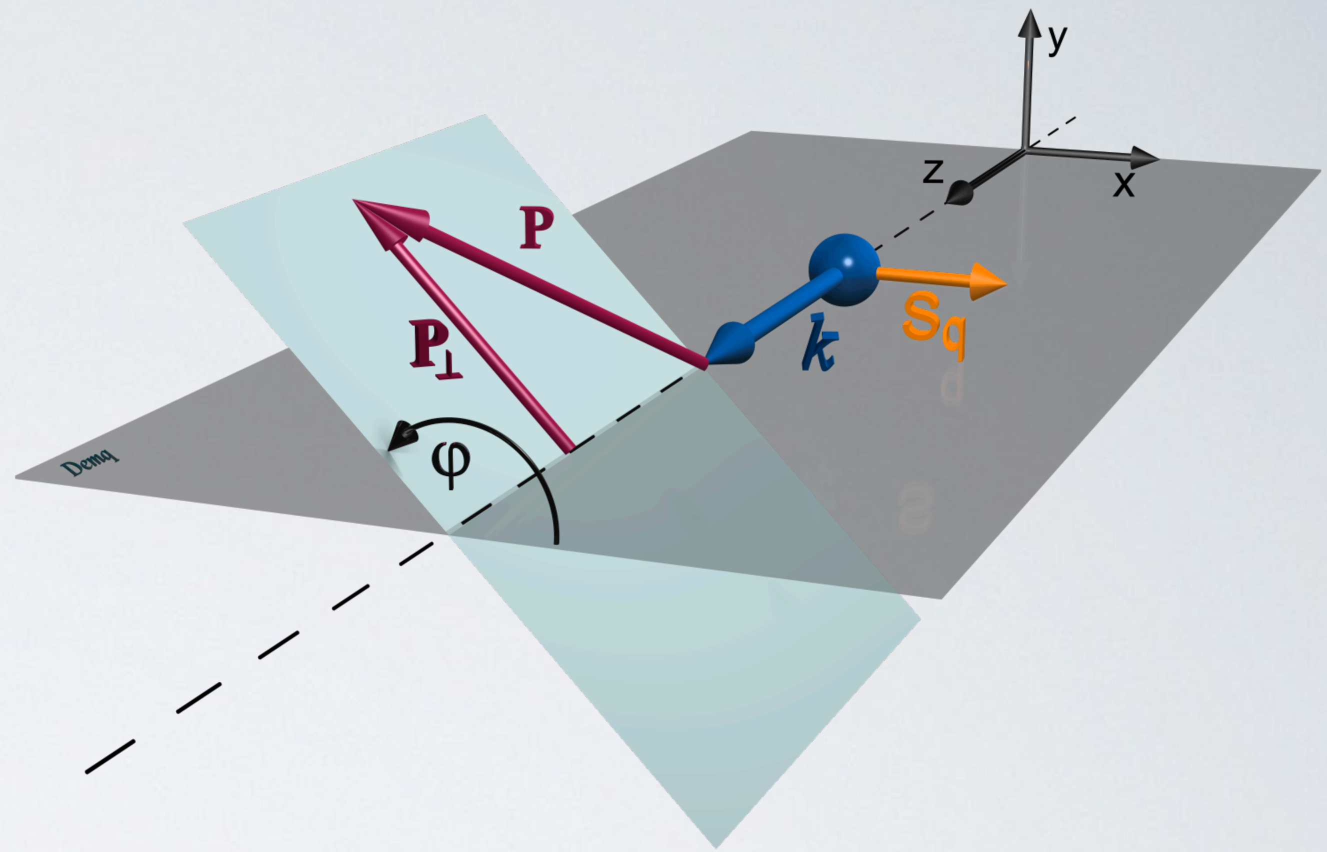
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T)$$

- **IFFS are Chiral-ODD**: Need to be coupled with another chiral-odd quantity to be observed (e.g. transversity).

TWO-HADRON FRAGMENTATION

✦ **Transformation to frame $\mathbf{k}_T = 0$**

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$



$$\mathbf{P}_T = \mathbf{P}^{\perp}_{h_1} + \mathbf{P}^{\perp}_{h_2}$$
$$\mathbf{R} = (\mathbf{P}^{\perp}_{h_1} - \mathbf{P}^{\perp}_{h_2})/2$$

✦ Integrate over one or other momentum:

$$D^{h_1 h_2}_{q^\uparrow}(\varphi_R) = D^{h_1 h_2}_{1,q} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D^{h_1 h_2}_{q^\uparrow}(\varphi_T) = D^{h_1 h_2}_{1,q} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

✦ The IFF surviving after $\mathbf{k}_T$ integration is redefined as

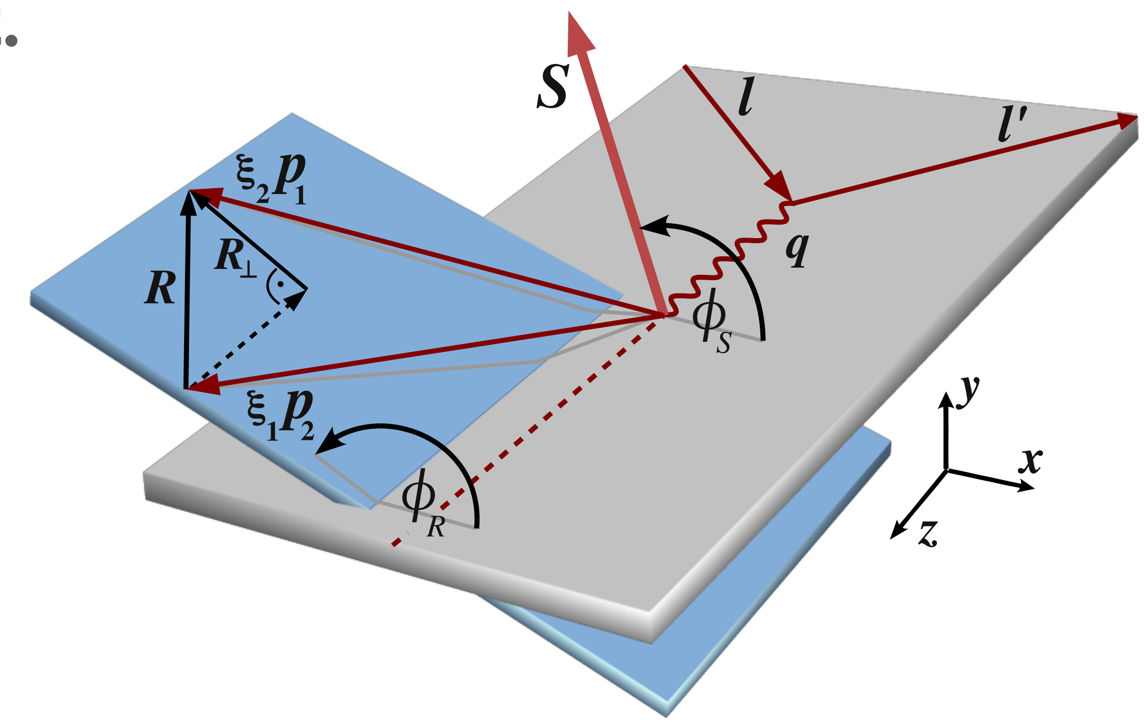
A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$

RECENT COMPASS RESULTS

COMPASS, PLB736, 124-131 (2014).

✦ SIDIS with transversely polarized target.



✦ Collins single spin asymmetry:

$$A_{Coll} = \frac{\sum_q e_q^2 \; h_1^q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \; f_1^q \otimes D_1^{h/q}}$$

✦ Two hadron single spin asymmetry:

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

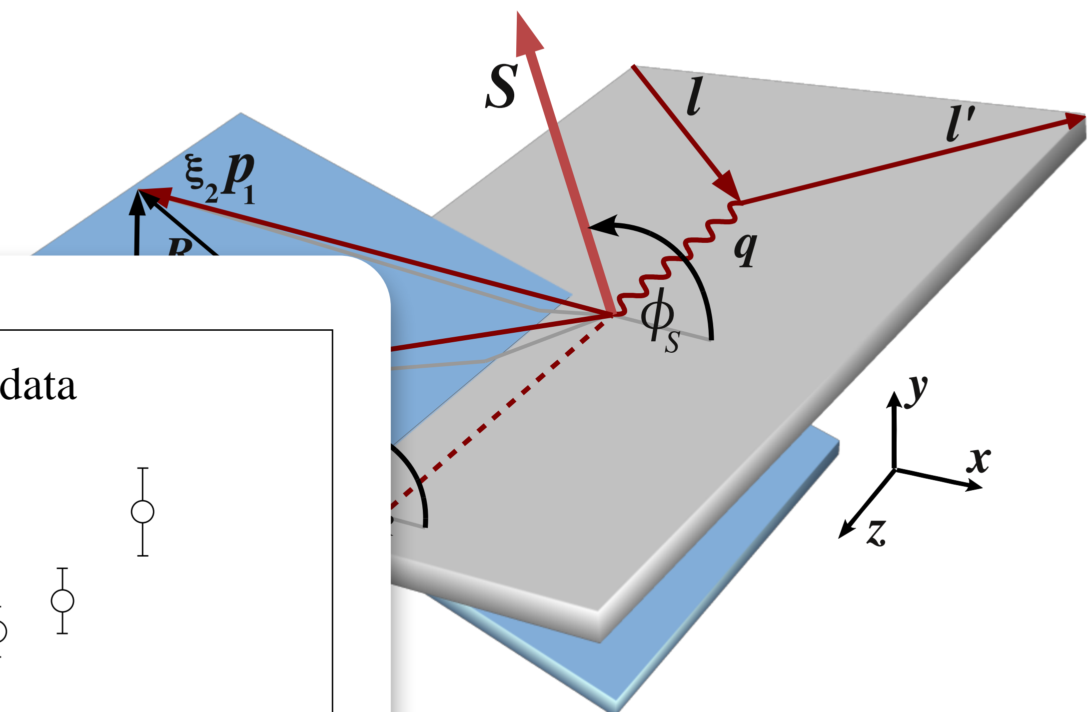
✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

RECENT COMPASS RESULTS

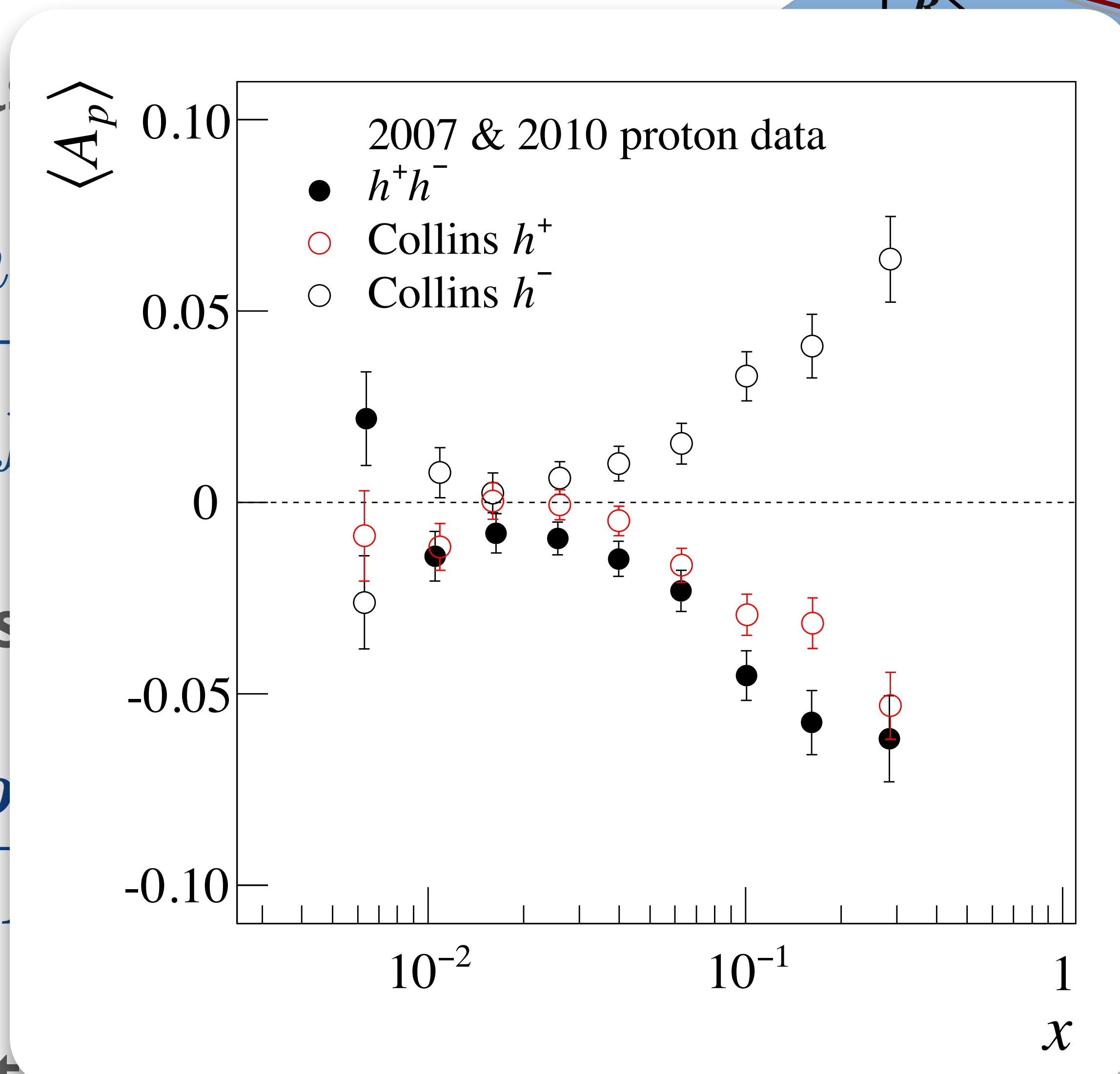
COMPASS, PLB736, 124-131 (2014).

✦ SIDIS with transversely polarized target.



✦ Collins single spin a

$$A_{Coll} = \frac{\sum_q e_q^2 \; h \ldots}{\sum_q e_q^2 \; \ldots}$$



✦ Two hadron single s

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p} \ldots \; M^2_{h^+h^-}, \cos\theta)}{2 \ldots \; M^2_{h^+h^-}, \cos\theta)}$$

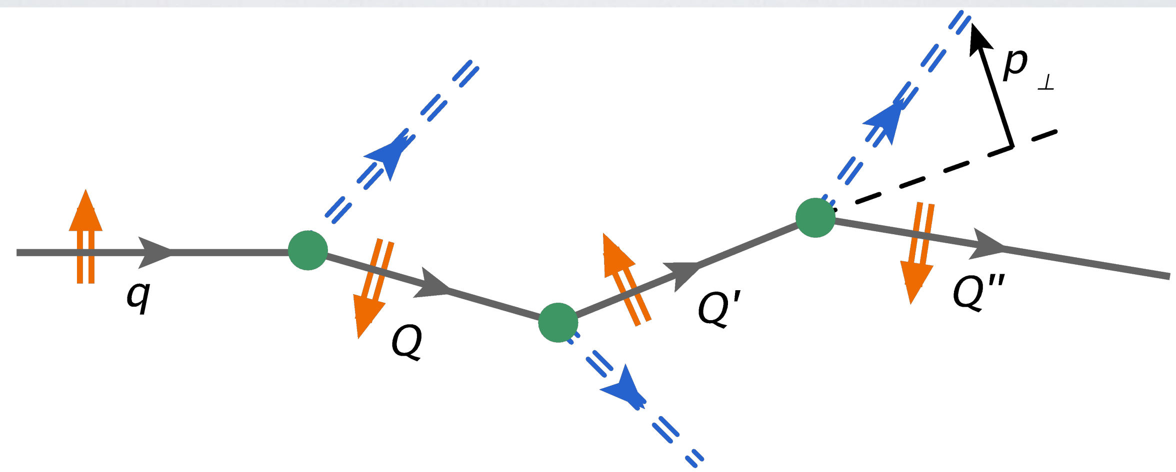
✦ Note the choice of the vector

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

- Use the NJL-jet Model including Collins effect (MKII) to study DiFFs.

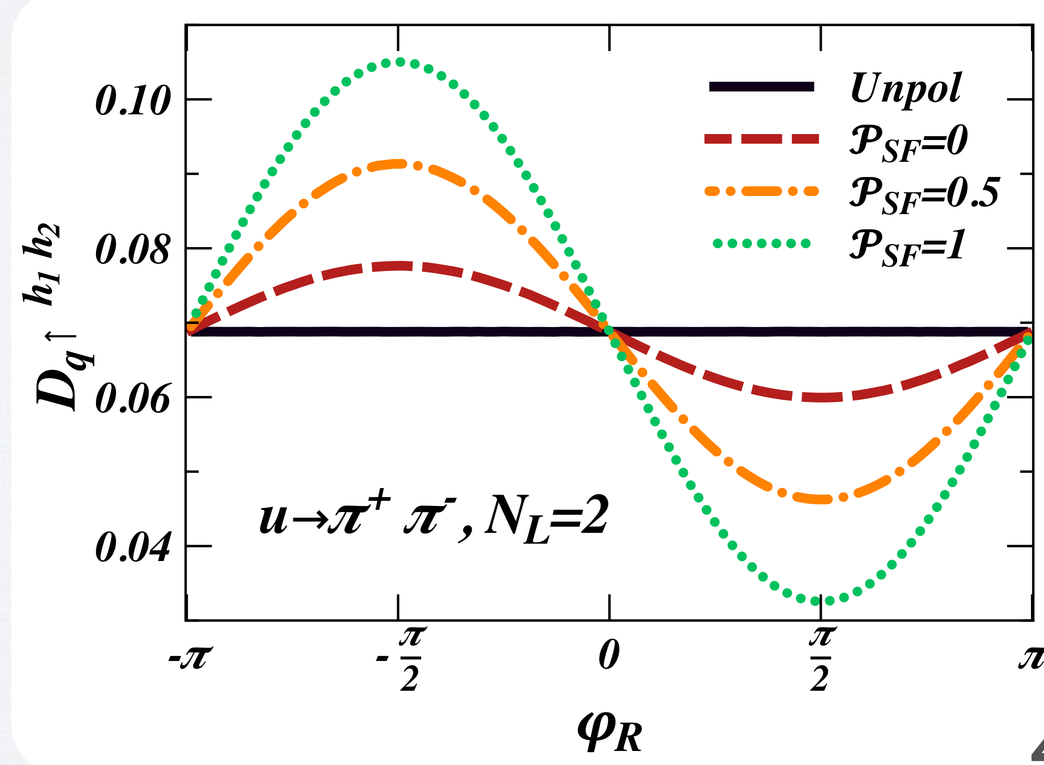


$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R)\right\rangle .$$

- *Choose a constant Spin flip probability:* $\mathcal{P}_{SF}$

- *Simple model to start with:*
Only pions and extreme ansatz for the Collins term in elementary function.

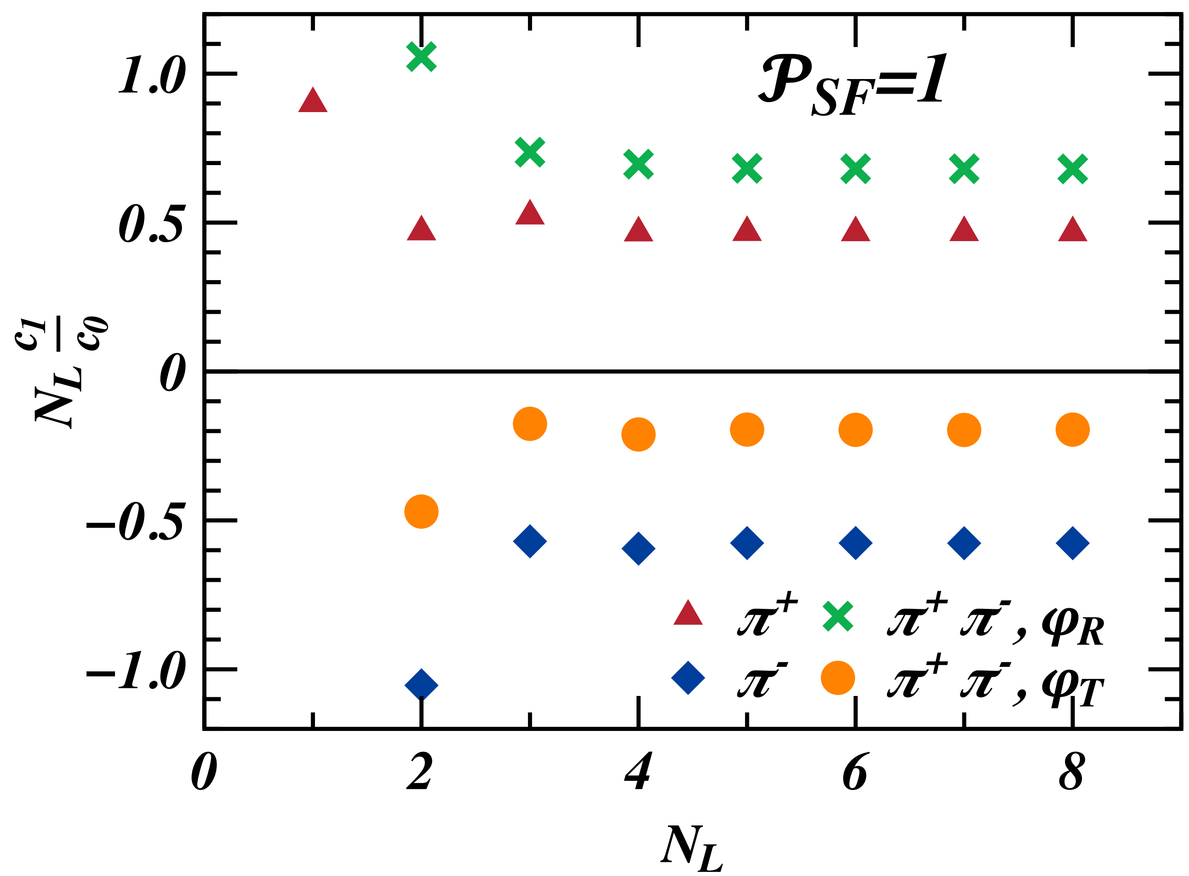
$$\boxed{d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)}$$

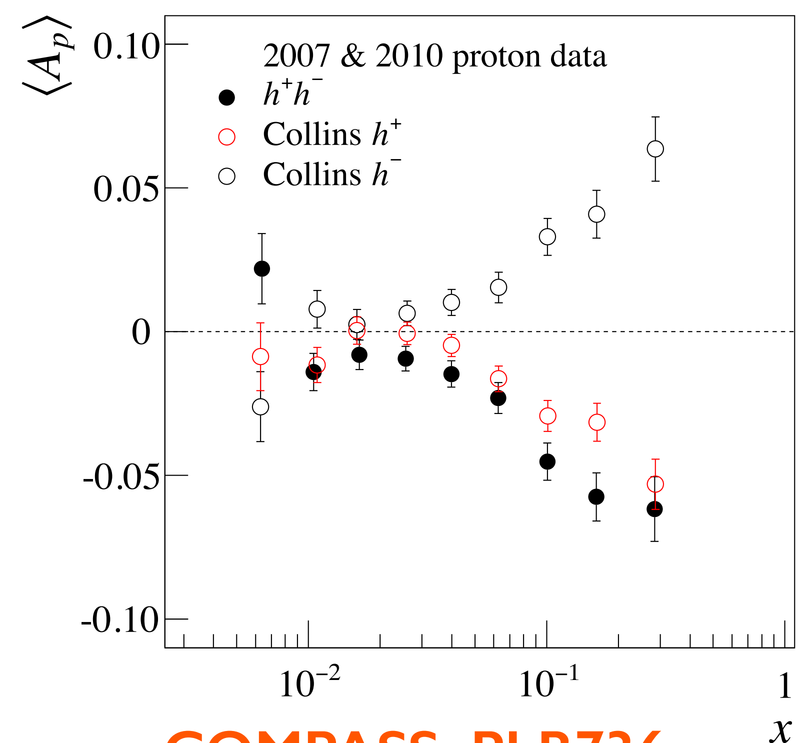


INTEGRATED ANALYZING POWERS

H.M., Kotzinian, Thomas, PLB731 208-216 (2014).

$z_{1,2} > 0.2, z > 0.2$





COMPASS: PLB736, 124-131 (2014).

✓ ***NJL-jet** model results are **consistent** with **COMPASS** measurements on interplay between one- and two- hadron SSAs.*

INTEGRATED ANALYZING POWERS

Single Hadron

$$\boxed{N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)}$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2\, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\, q \otimes D_1^{h/q}}$$

DiHadron

$$\boxed{N_{h^+h^-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)}$$

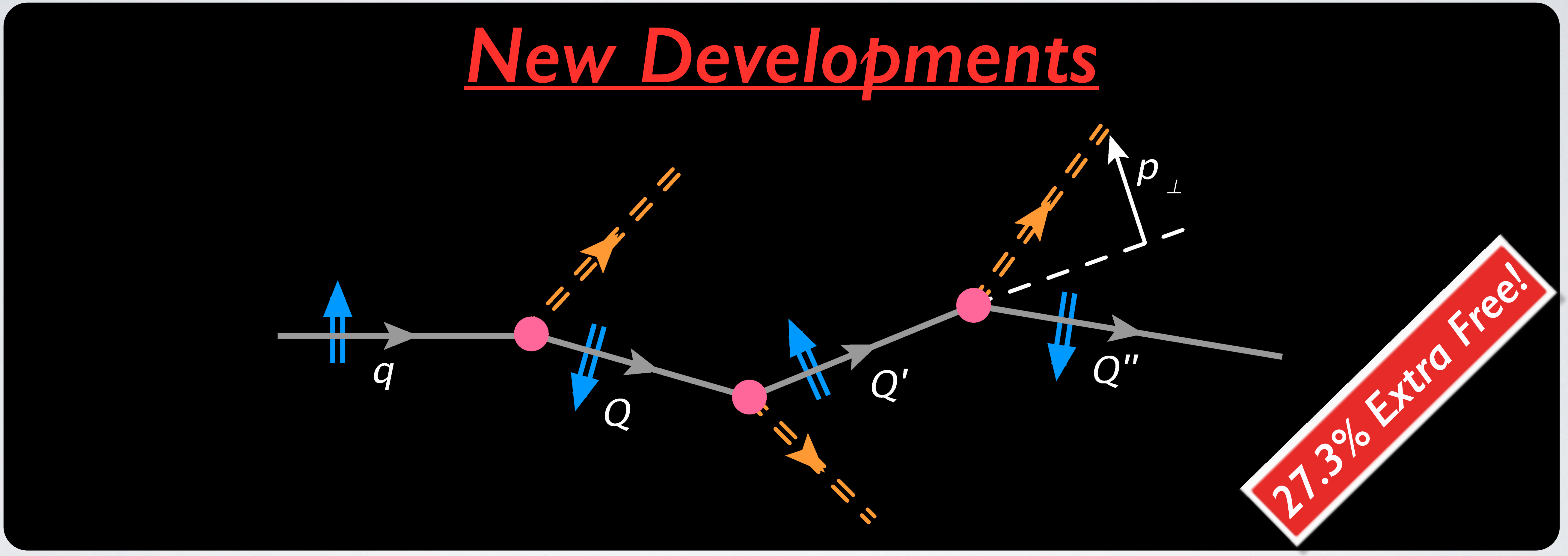
$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

$$\boxed{D_{q^\uparrow} = c_0 - \sin(\phi) c_1}$$

NJL-jet MKIII



What's Inside the Green BoX?

✦ TMD Polarized Fragmentation Functions at LO.

- *Only two for unpolarised final state hadrons.*

PDFs

N/q	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1\, h_{1T}^\perp$

Fragment. Functions

h/q	U	L	T
U	D_1		$H_1^\perp$

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N/q	U	L	T
U	f_1		$h_1^\perp$
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T	$f_{1T}^\perp$	$g_{1T}^\perp$	$h_1 \, h_{1T}^\perp$

Fragment. Functions

h/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

- *8 for spin 1/2 final state (including quark). Similar to TMD PDFs.*

Field-Theoretical Definitions

- ***We can use the same "spectator" type calculations as for pion.***

$$\Delta^{[\Gamma]}(z,\vec{p}_T) \equiv \frac{1}{4}\int \frac{dp^+}{(2\pi)^4} Tr[\Delta\Gamma]|_{p^-=zk^-}$$

$$= \frac{1}{4z}\sum_X \int \frac{d\xi^+ d^2\vec{\xi}_T}{2(2\pi)^3} e^{i(p^-\xi^+/z-\vec{\xi}_T\cdot\vec{p}_T)} \langle 0|\psi(\xi^+,0,\vec{\xi}_T)|p,S_h,X\rangle\langle p,S_h,X|\bar{\psi}(0)\Gamma|0\rangle$$

- ***The definitions of FFs from the quark correlator***

$$\Delta^{[\gamma^+]} = D(z,p_\perp^2) - \frac{1}{M}\epsilon^{ij}k_{Ti}S_{Tj}D_T^\perp(z,p_\perp^2)$$

$$\Delta^{[\gamma^+\gamma_5]} = S_L G_L(z,p_\perp^2) + \frac{\boldsymbol{k}_T\cdot S_T}{M} G_T(z,p_\perp^2)$$

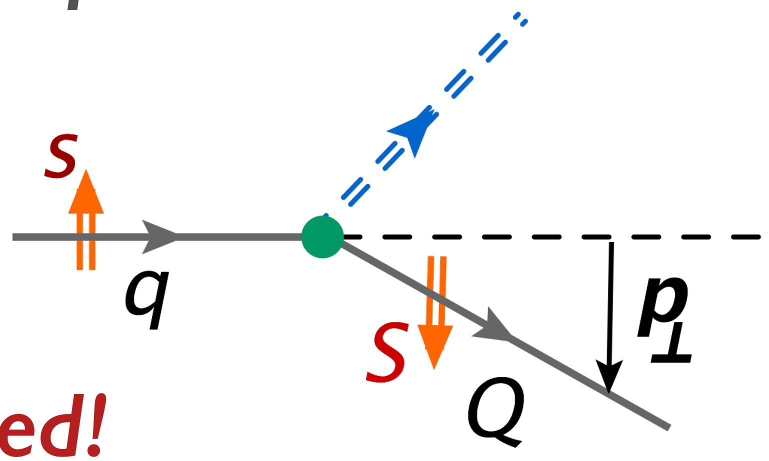
$$\Delta^{[i\sigma^{i+}\gamma_5]} = S_T^i H_T(z,p_\perp^2) + \frac{S_L}{M}k_T^i H_L^\perp(z,p_\perp^2) + \frac{k_T^i(\boldsymbol{k}_T\cdot S_T)}{M^2}H_T^\perp(z,p_\perp^2) - \frac{\epsilon^{ij}k_{Tj}}{M}H^\perp(z,p_\perp^2)$$

Spin Transfer in quark-jet Framework.

✦NJL-jet MKIII:

- *The probability for the process* $q \to Q$, *initial spin* $\mathbf{s}$ *to* $\mathbf{S}$

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{S}) = \alpha_s + \beta_{\mathbf{s}}\cdot\mathbf{S}$$

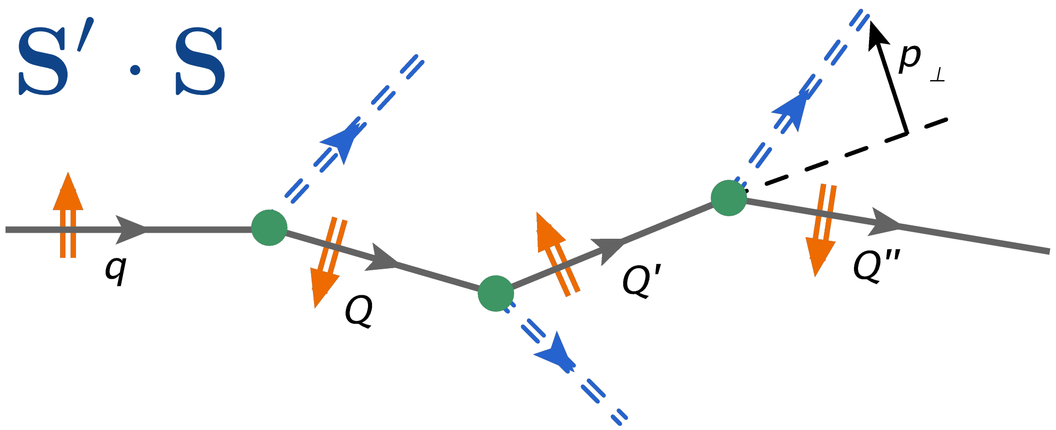


- ***Intermediate** quarks in quark-jet are **unobserved!***

Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii: **QUANTUM ELECTRODYNAMICS (1982).**

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{S}) \sim \mathrm{Tr}[\rho^{\mathbf{S}'}\rho^{\mathbf{S}}] \sim 1 + \mathbf{S}'\cdot\mathbf{S}$$

$$\mathbf{S}' = \frac{\beta_{\mathbf{s}}}{\alpha_{\mathbf{s}}}$$



- *Remnant quark's* S' ***uniquely determined by*** $z,\mathbf{p}_\perp$ ***and*** $\mathbf{s}$ ***!***

- *Process probability is **the same** as transition to **unpolarized state.***

$$F^{q\to Q}(z,\mathbf{p}_\perp;\mathbf{s},\mathbf{0}) = \alpha_s$$

Example: Pion production.

Fragment. Functions

$$F^{q\to Q}(z, \boldsymbol{p}_\perp; \boldsymbol{s}, \boldsymbol{S})$$

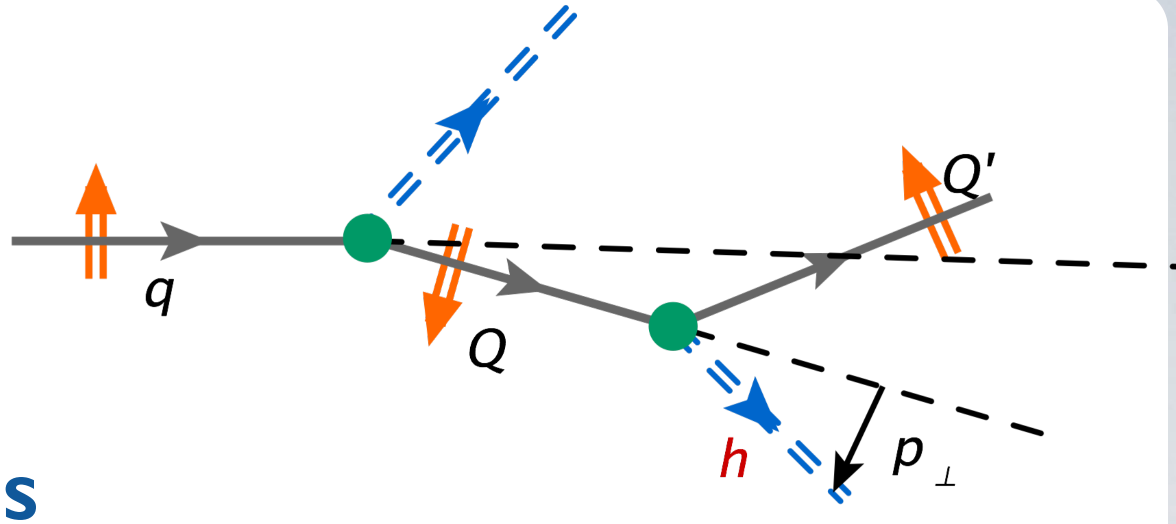
Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T} H_{1T}^\perp$

$$F^{q\to\pi}(z, \boldsymbol{p}_\perp; \boldsymbol{s})$$

π/q	U	L	T
U	D_1		$H_1^\perp$

Example: Pion prod. up to Rank 2

✦ **Only consider pion produced in the first two emission steps!**



✦ **Then the polarised number density is**

$$F^{(2)q\to\pi} = \underbrace{\boxed{f^{q\to\pi}}}_{\text{1st rank}} + \underbrace{\boxed{f^{q\to Q} \otimes f^{Q\to\pi}}}_{\text{2nd rank}}$$

✦ **"Elementary" number densities:** *only favoured types are non-zero.*

$$\boxed{f^{q\to\pi} = d^{q\to\pi} - \frac{p_\perp}{zM_h} s_T h_1^{\perp q\to\pi}} \qquad \boxed{f^{u\to\pi^-} = 0}$$

✦ *It is shown <u>analytically</u> that only Collins modulations appear!*

$$\boxed{F^{(2)q\to\pi}(z, p_\perp^2, \varphi_C) = F_0^{(2)}(z, p_\perp^2) - \sin(\varphi_C) F_1^{(2)}(z, p_\perp^2)}$$

Example: Pion prod. up to Rank 2

✦*It is shown <u>analytically</u> that only Collins modulations appear!*

$$F^{(2)q\to\pi}(z,p_\perp^2,\varphi_C) = F_0^{(2)}(z,p_\perp^2) - \sin(\varphi_C)F_1^{(2)}(z,p_\perp^2)$$

✦*Up to **<u>unspecified</u>** coefficients, using.*

<u>Unpolarised term:</u>

From TM-induced Spin of intermediate quark

$$F_0^{(2)q\to\pi} = d^{q\to\pi} + (d^{q\to Q}\otimes d^{Q\to\pi} + d_T^{\perp q\to Q}\otimes h^{\perp Q\to\pi})$$

<u>Collins term:</u>

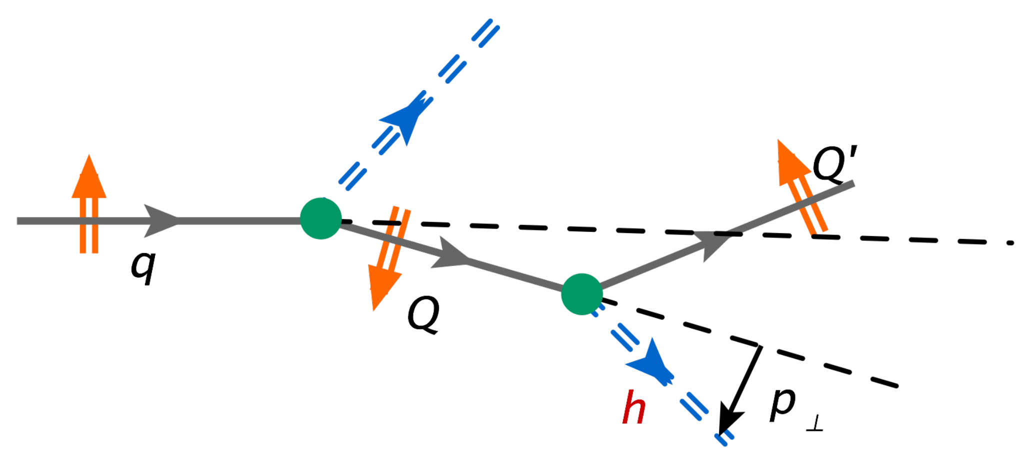
"Recoil" TM contribution

$$F_1^{(2)q\to\pi} \sim h^{\perp q\to\pi} + [h^{\perp q\to Q}\otimes d^{Q\to\pi} + (h_T^{q\to Q} + h_T^{\perp q\to Q})\otimes h^{\perp Q\to\pi}]$$

Transferred Spin of intermediate quark

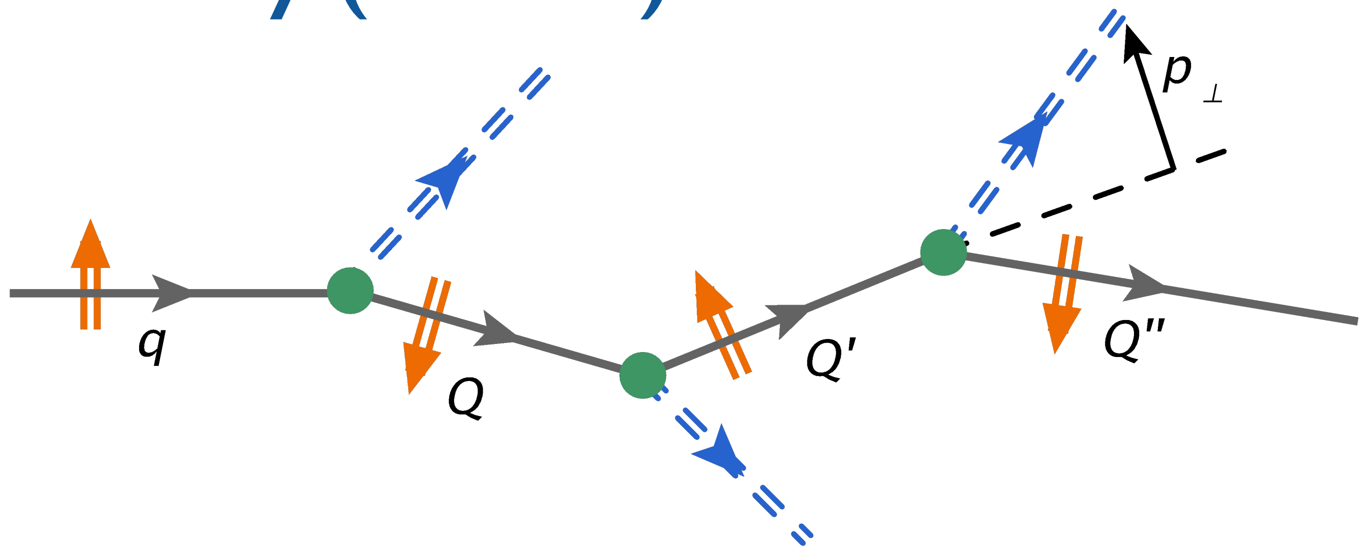
✦ *Reminder*

Q/q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$D_{1T}^\perp$	G_{1T}	$H_{1T}H_{1T}^\perp$



Full Hadronization

- ✦ We can consider many (infinite) number of emissions.



- ✦ Then the polarised number density is

$$\boxed{F^{q\to h} = f^{q\to h} + f^{q\to Q} \otimes F^{Q\to h}}$$

- ✦ *Again, only Collins modulations appear for unpolarised h!*

$$F^{q\to h}(z, p_\perp^2, \varphi_C) = F_0(z, p_\perp^2) - \sin(\varphi_C) F_1(z, p_\perp^2)$$

- ✦ Also applicable for *polarised* hadron production.
- ✦ We can also employ *MC approach*.
- ✦ We only need the "elementary" splittings.

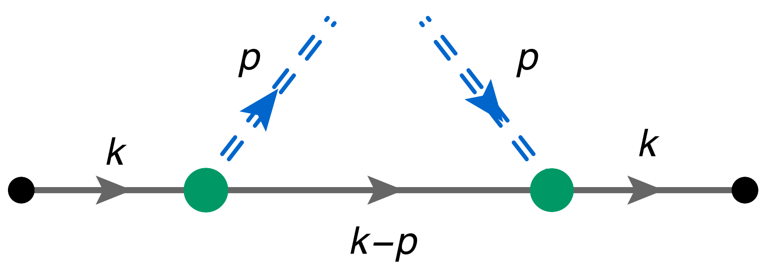
$$\boxed{f^{q\to h}} \qquad \boxed{f^{q\to Q}}$$

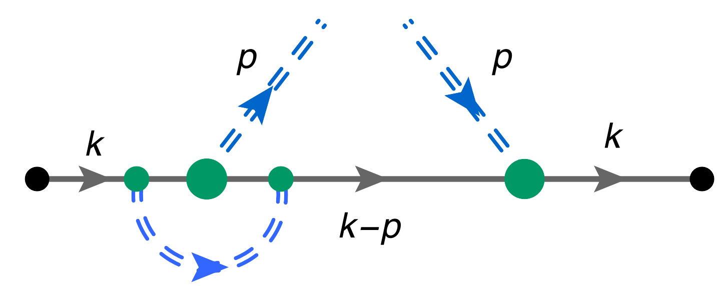
Model Calculations of $q \to Q$ Splittings

✦We can use the same “spectator” type calculations as for pion.

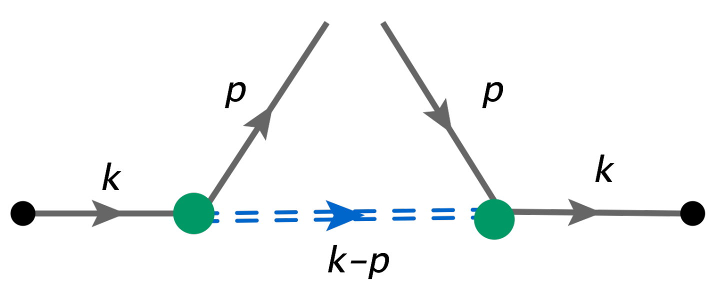
T-even ***T-odd***

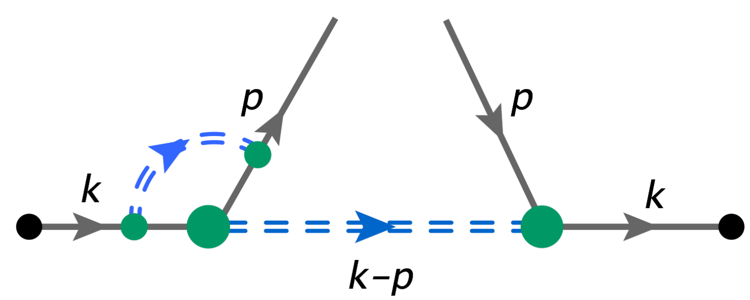
$$q \to h$$





$$q \to Q$$





Conclusions

- ❖ Polarised TMD FFs provide a wealth of information about the *spin-spin* and *spin-momentum* correlations in hadronisation.

- ❖ Modelling Polarised Quark Hadronization is needed for both calculations of polarised FFs and phenomenological studies of various correlations (Collins and IFF, etc).

- ❖ Incorporating polarised parton hadronisation into *MC generators* is needed for *supporting future experiments* in mapping out the 3D structure of nucleon (*JLab12, BELLE II, EIC*).

- ❖ The *NJL-jet* model provides a robust and extendable framework for *microscopic* description of various fragmentation phenomena using MC simulations: *TMD, Collins, DiHadron*.

- ❖ The extension of the underlying *quark-jet* mechanism to include polarisation can be readily incorporated in other MC frameworks.

Happy Birthday Tony!