Uses and misuses of the NJL model

— Heavy-light mesons, magnetic fields, dense matter

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New Directions in Subatomic Physics

Workshop to celebrate the 60th birthday of Tony Williams

The University of Adelaide
7 - 10 March 2016

Happy Birthday Tony!

Tony is an open-minded guy:

— Soliton model

— Cloudy bag model

— Ansatz approach to DCSB

— QMC model, Walecka model

— Chiral quark model

— Lattice QCD

— BSM, SUSY, etc

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— Lattice QCD

— BSM, SUSY, etc

BUT not NJL model, why?!

Does not have confinement,

does not have asymptotic freedom,

needs regularization, too much freedom,

cutoff spoils symmetries, …

Why Contact Interaction Models?

Why Contact Interaction Models?

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi[1], S. Durr[1,2], Z. Fodor[1,2,3,*], C. Hoelbling[1], S. D. Katz[3,4], S. Krieg[1,2], L. Lellouch[5], T. Lippert[1,2], A. Portelli[5,6], K. K. Szabo[1,2], B. C. Toth[1]

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Science AAAS

ABSTRACT EDITOR'S SUMMARY

Weighing the neutron against the proton

Elementary science textbooks often state that protons have the same mass as neutrons. This is not far from the truth—the neutron is about 0.14% heavier (and less stable) than the proton. The precise value is important, because if the mass difference were bigger or smaller, the world as we know it would likely not exist. Borsanyi *et al.* calculated the mass difference to high precision using a sophisticated approach that took into account the various forces that exist within a nucleon. The calculations reveal how finely tuned our universe needs to be.

Science, this issue p. 1452

DCSB and the neutron-proton mass difference

VOLUME 62, NUMBER 22 PHYSICAL REVIEW LETTERS 29 MAY 1989

Nambu–Jona-Lasinio Model and Charge Independence

E. M. Henley and G. Krein[a]

Institute for Nuclear Theory, Department of Physics, FM-15, University of Washington, Seattle, Washington 98195

(Received 28 September 1988)

For different up and down current quark masses, charge independence follows in the Nambu–Jona-Lasinio model from chiral-symmetry breaking; however, chiral symmetry is restored at high densities. The dependence of the constituent quark masses and the neutron-proton mass difference on the density are examined. The effect of the up-down-quark mass difference on the neutron-proton mass difference is large and in the right direction to explain the Nolen-Schiffer anomaly.

PACS numbers: 11.30.Hv, 12.40.Aa, 21.10.Sf

In medium: $M_n \to M_p$

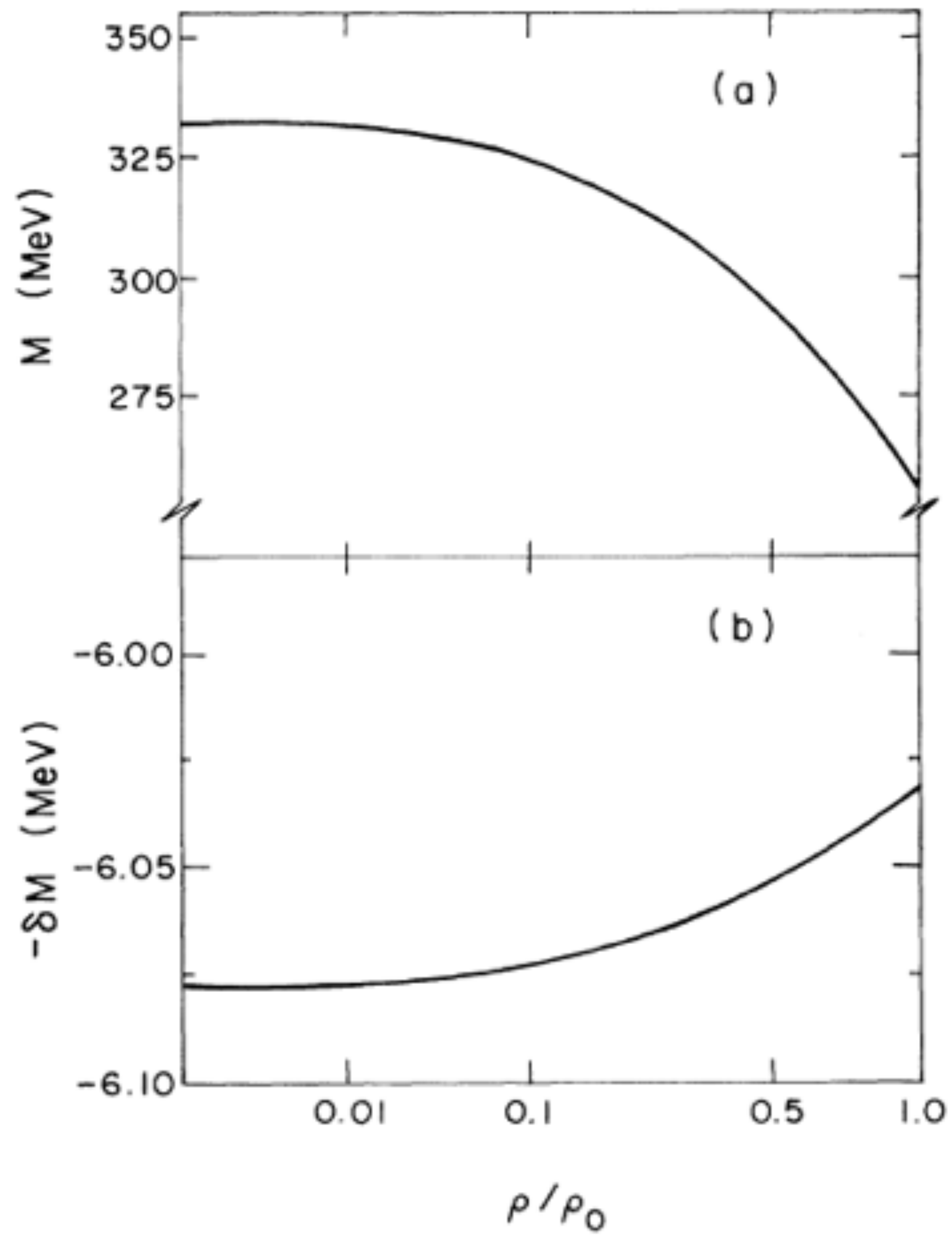

FIG. 1. (a) The average quark mass $M=\frac{1}{2}(M_n+M_d)$ and (b) the mass difference $-\delta M=M_u-M_d$ as functions of the ratio of the average nuclear density to that of nuclear matter, $\rho_0=0.17\ \mathrm{fm}^{-3}$.

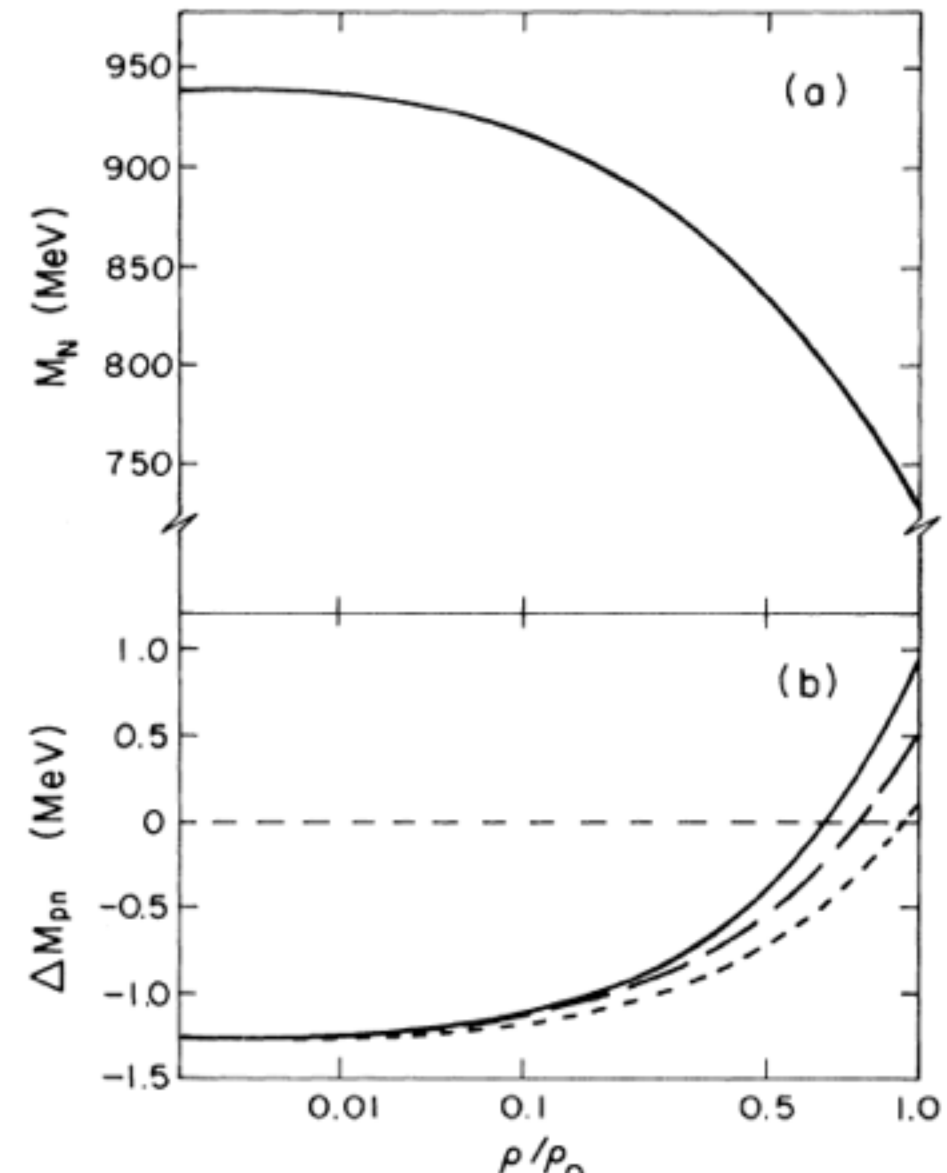

FIG. 2. (a) The average nucleon mass $M_N=\frac{1}{2}(M_p+M_u)$ and (b) the mass difference $\Delta M_{pn}=M_p-M_n$ as functions of the ratio of the average nuclear density to that of nuclear matter, $\rho_0=0.17\ \mathrm{fm}^{-3}$. The long-dashed curve shows the effect of a linear variation of α_s with density, and the short-dashed curve is for $\alpha_s \propto M^{9/4}$ (see text).

Right direction to explain the Nolen-Schiffer anomaly

$$\Delta M_{\mathrm{mirror}} = \Delta M_{pn} + \Delta M_{\mathrm{MB}}$$

TABLE I. Discrepancies and ΔM_{pn} for nuclei with masses in the regions of $A=16$ and $A=40$. ρ_{av} is the average density defined in the text.

Nuclei	Discrepancy with $\Delta M_{pn}=-1.3$ MeV	ΔM_{pn} at ρ_{av}	Discrepancy with NJL correction
$A=16$, $\rho_{\mathrm{av}}\simeq 0.09\ \mathrm{fm}^{-3}$	0.3	-0.4	-0.6
$A=40$, $\rho_{\mathrm{av}}\simeq 0.10\ \mathrm{fm}^{-3}$	0.7	-0.12	-0.5

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Part of the NS anomaly used to be explained by rho-omega mixing component in the NN interaction

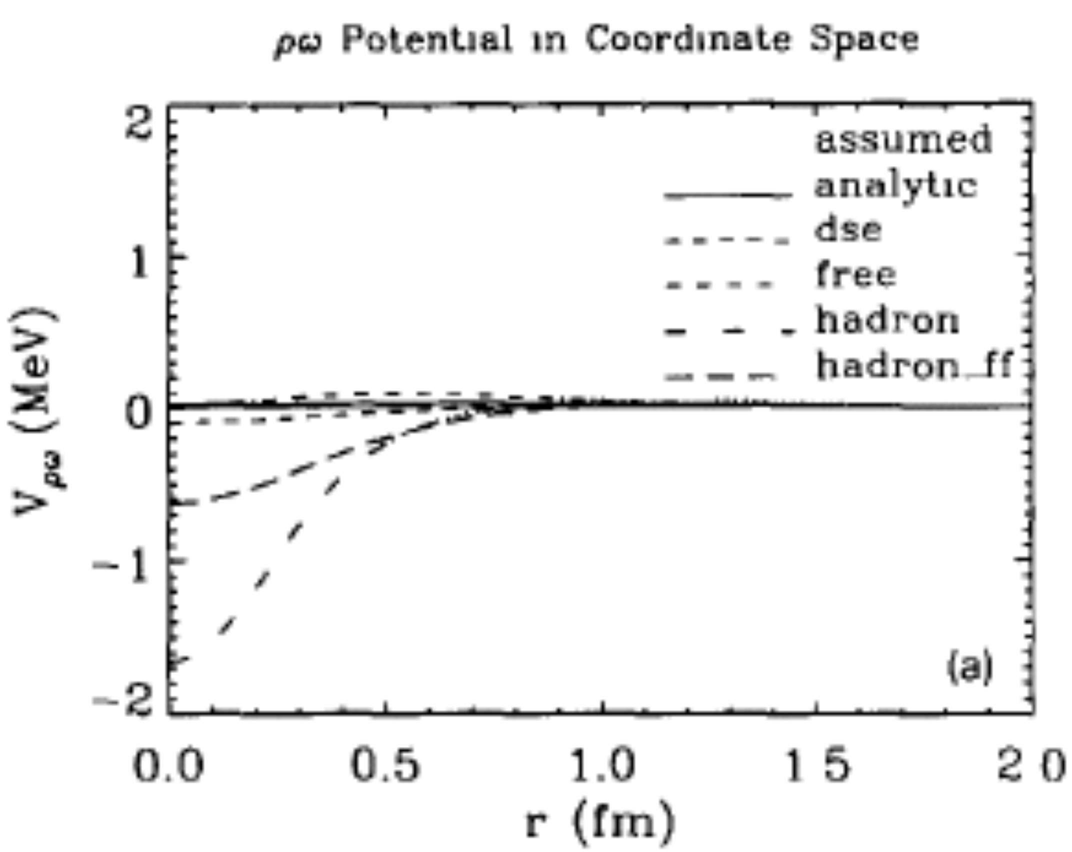
Physics Letters B 317 (1993) 293–299
North-Holland

PHYSICS LETTERS B

Charge-symmetry breaking, rho–omega mixing, and the quark propagator

G. Krein [a,1], A.W. Thomas [b,2] and A.G. Williams [b,c,3]

[a] *Instituto de Física Téorica, UNESP, Rua Pamplona 145, 01405 São Paulo, SP, Brazil*
[b] *Department of Physics and Mathematical Physics, University of Adelaide, GPO Box 498, Adelaide, S Aust 5001, Australia*
[c] *Department of Physics and the Supercomputer Computations Research Institute, Florida State University, Tallahassee, FL 32306, USA*



No rho-omega mixing!

Heavy-light mesons

— Light quark moving in a (static) background of a heavy color source

Laboratory for learning about confinement and dynamical chiral symmetry breaking

Neat measurable* decays: light flavor changing decay

$$D^+ \to D^0\, e^+\, \nu$$
$$D_s^+ \to D^0\, e^+\, \nu$$

$$B_s^0 \to B^-\, e^+\, \nu$$
$$B_s^0 \to B^{*-}\, e^+\, \nu$$

* Small phase space

Charm in matter

— Understanding of the nuclear force, role of glue

— J/Ψ, η_c, ⋯ : color polarizability, van der Waals force

— D-mesons in medium: chiral-symmetry restoration

— Charm hypernuclei

— Quark-gluon plasma

Experiments ongoing and underway:

LHC, RHIC, JLab @ 12 GeV, JPARC, Fair, NICA

Need crucial input

— DN interaction

Need crucial input

— DN interaction

P̄ANDA @ FAIR

Need crucial input

— DN interaction

PANDA @ FAIR

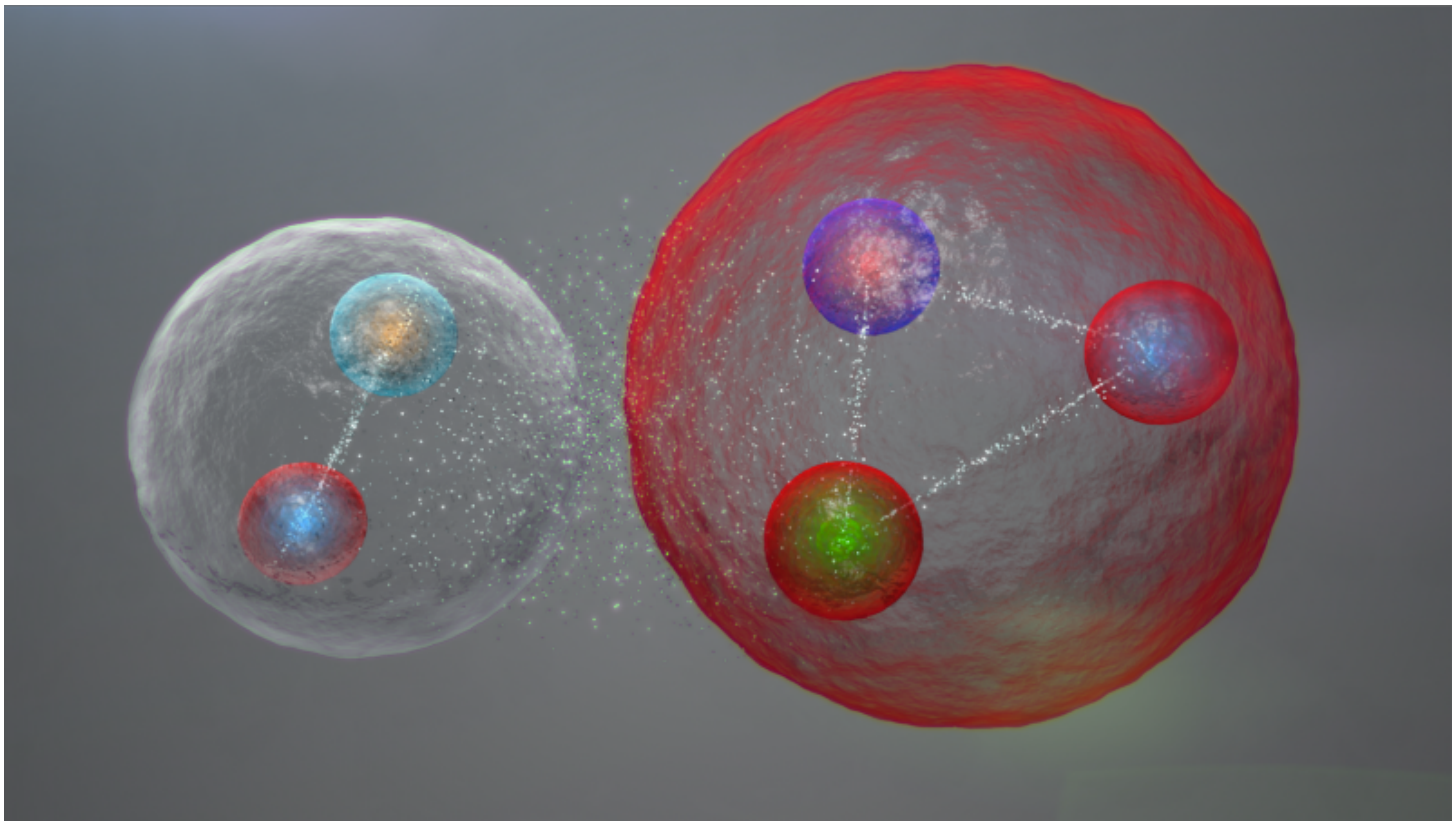
J/Ψ binding to nuclei

Generically: two independent mechanisms

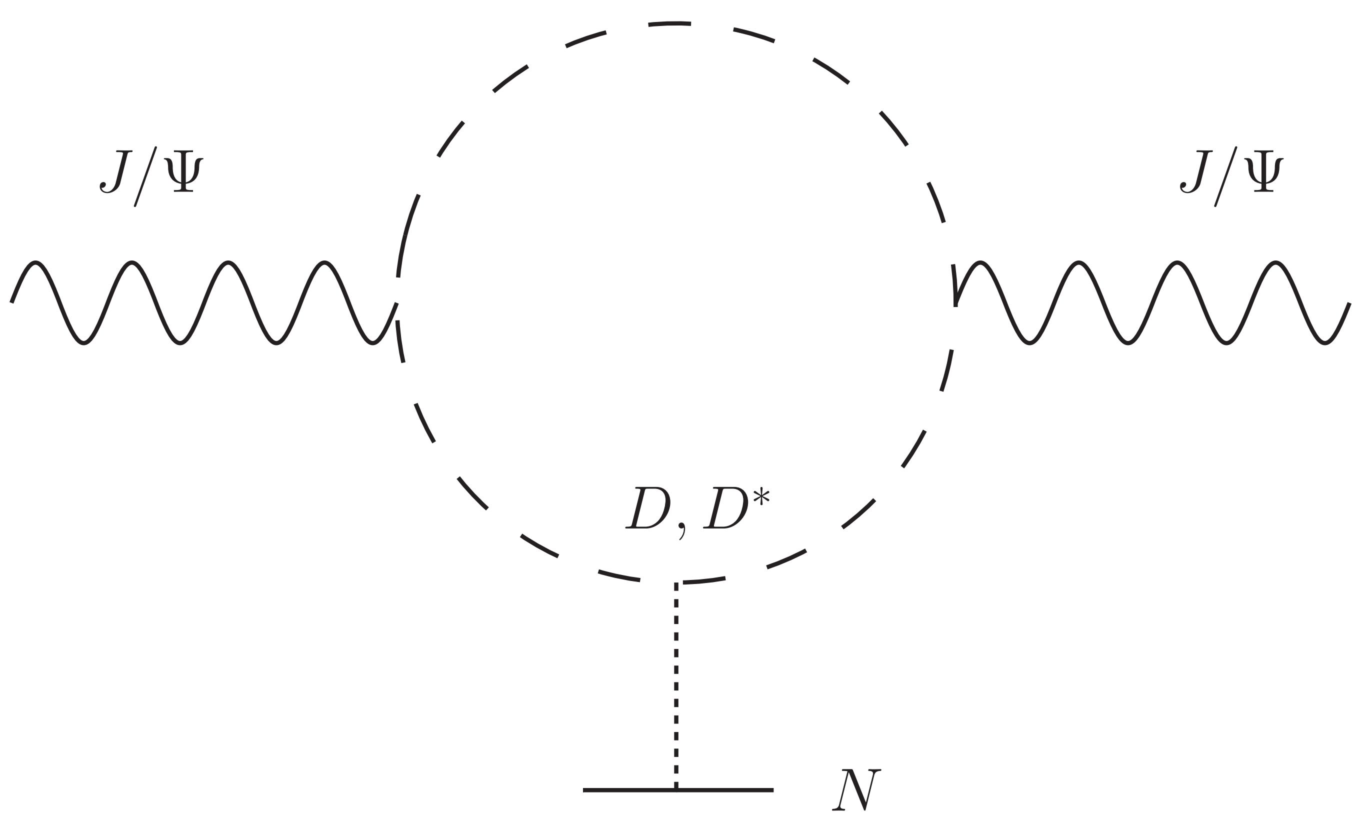
— Second order stark effect – octet intermediate state
van der Waals force

— D,D* meson-loop – color singlet intermediate state
D mesons interact with the nuclear mean fields

van der Waals type of force



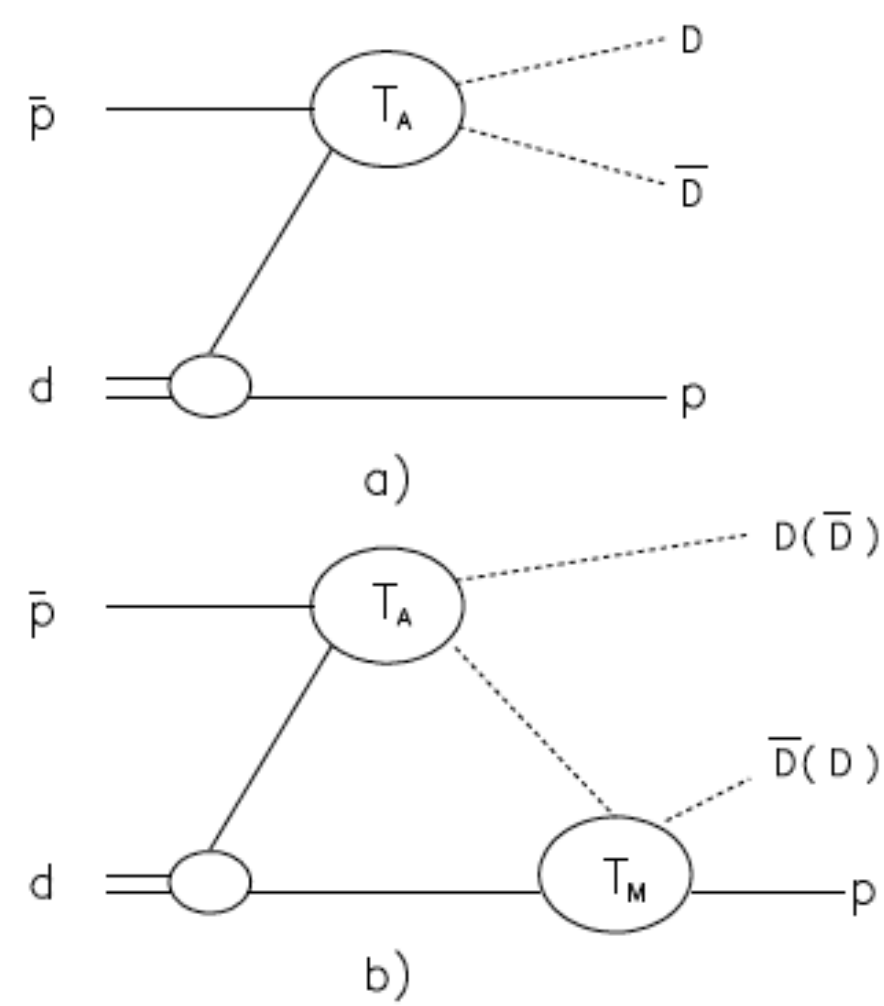
D,D*-meson loops



DN Experiment

- antiproton annihilation on the deuteron*

P̄anda @ FAIR



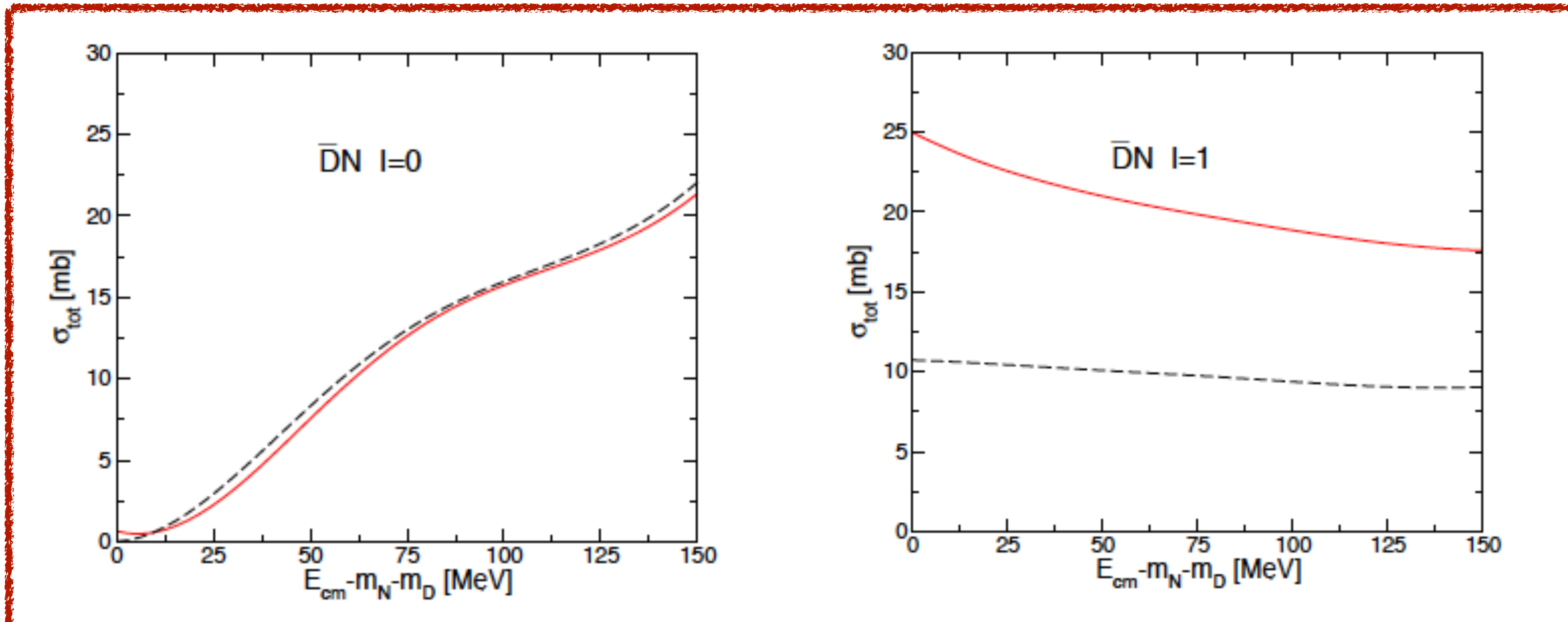
* J. Haidenbauer, GK, U.-G. Meissner, A. Sibirtsev

1) Eur. Phys. J. A 33, 107 (2007)

2) Eur. Phys. J. A 37, 55 (2008)

Predictions for the PANDA measurement

Use SU(4) symmetry for couplings:



Similar magnitude to KN

VOLUME 64, NUMBER 9 PHYSICAL REVIEW LETTERS 26 FEBRUARY 1990

Nuclear-Bound Quarkonium

Stanley J. Brodsky and Ivan Schmidt[(a)]
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Guy F. de Téramond
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(Received 25 September 1989)

We show that the QCD van der Waals interaction due to multiple-gluon exchange provides a new kind of attractive nuclear force capable of binding heavy quarkonia to nuclei. The parameters of the potential are estimated by identifying multigluon exchange with the Pomeron contributions to elastic meson-nucleon scattering. The gluonic potential is then used to study the properties of $c\bar{c}$ nuclear-bound states. In particular, we predict bound states of the η_c with ^{3}He and heavier nuclei. Production modes and rates are also discussed.

PACS numbers: 21.90.+f, 12.40.−y, 21.30.+y, 25.10.+s

Variational calculation of binding energy

$$V_{(c\bar{c})N}(r) = -\frac{\alpha}{r}e^{-\mu r}$$

TABLE I. Binding energies $\epsilon = |\langle H \rangle|$ of the η_c to various nuclei, for unscreened and partially screened potentials; all masses are given in GeV. The data for $\langle R_A^2 \rangle^{1/2}$ (in GeV^{-1}) are from Ref. 10.

				Unscreened			Partially screened		
A	$\langle R_A^2 \rangle^{1/2}$	μ	$m_{\rm red}$	α	γ	$\langle H \rangle$	α	γ	$\langle H \rangle$
1	3.9	0.60	0.715	0.59		>0	0.42		>0
2	10.7	0.23	1.15	0.172		>0	0.122		>0
3	9.5	0.26	1.45	0.327	0.40	−0.019	0.231	0.24	−0.003
4	8.2	0.30	1.66	0.585	0.92	−0.143	0.414	0.62	−0.051
6	11.2	0.22	1.95	0.470	0.89	−0.128	0.332	0.61	−0.050
9	11.2	0.22	2.20	0.705	1.53	−0.407	0.499	1.07	−0.179

VOLUME 67, NUMBER 16 PHYSICAL REVIEW LETTERS 14 OCTOBER 1991

Comment on "Nuclear-Bound Quarkonium"

David A. Wasson
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The Ohio State University
Columbus, Ohio 43210

Smearing of the interaction over the nuclear volume

$$V_{(c\bar{c})A}(r) = \int d^3r'\, V_{(c\bar{c})N}(r - r')\rho_A(r')$$

$$\int d^3r'\, \rho_A(r') = A$$

A	ϵ [MeV]	$\langle r^2\rangle^{1/2}$ [fm]
3	- 0.8	3.5
4	- 5.0	1.9
12	- 13	1.7
40	- 19	2.0
200	- 27	3.1

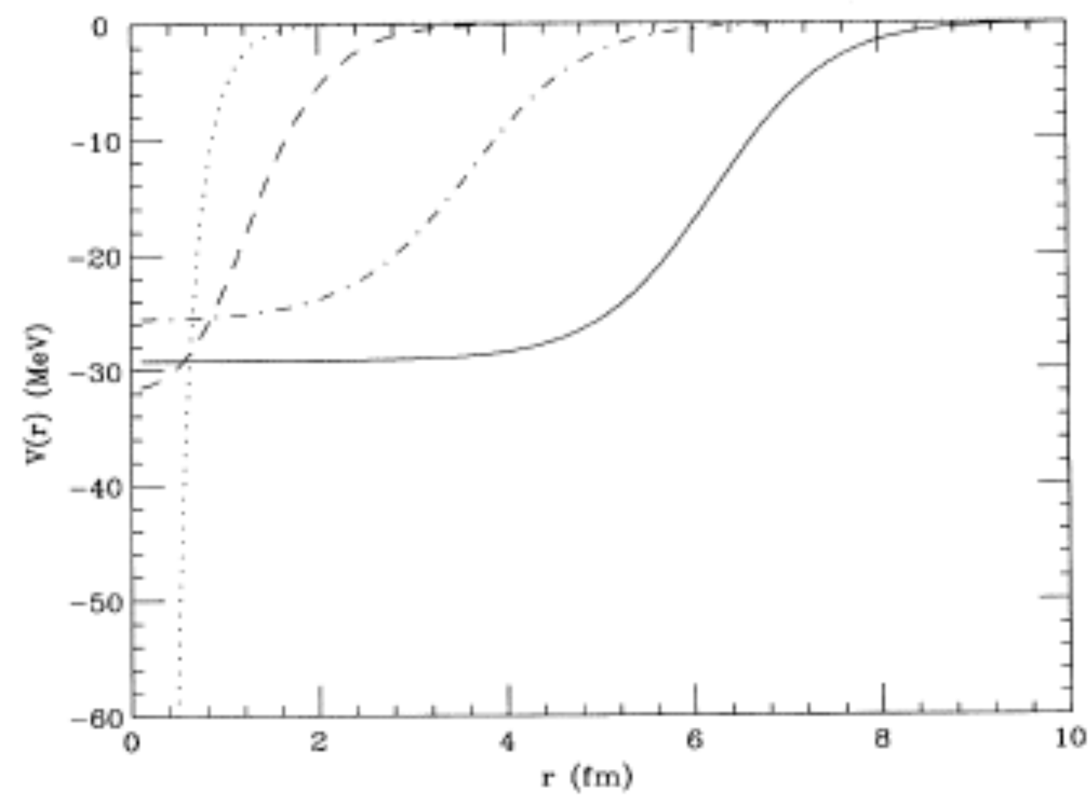


FIG. 1. Effective charmonium-nucleus potentials for various A. The dotted line is the $A=1$ result given by Eq. (1) and the dashed line is the $A=4$, the dash-dotted line is the $A=40$, and the solid line is the $A=200$ results predicted by Eq. (2).

Physics Letters B 288 (1992) 355–359
North-Holland

PHYSICS LETTERS B

A QCD calculation of the interaction of quarkonium with nuclei

Michael Luke, Aneesh V. Manohar and Martin J. Savage
Department of Physics 0319, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0319, USA

Binding to nuclear matter

$$\Upsilon, \eta_c \quad : \quad \epsilon \simeq -2 \ \text{ to } \ -4\,\mathrm{MeV}$$

$$\left.\begin{array}{lll} J/\Psi & : & \epsilon \simeq -8 \ \text{ to } \ -11\,\mathrm{MeV} \\ & & \\ \psi'(2s) & : & \epsilon \simeq 700\,\mathrm{MeV} \end{array}\right\} \text{less reliable}$$

Bhanot-Peskin theory

— van der Waals force (second order Stark effect)

J/Ψ as a small color electric dipole

$$\Delta m_\psi(\rho_B) = -\frac{1}{9}\int dk^2 \left|\frac{\partial\psi(k)}{\partial k}\right|^2 \frac{k}{k^2/m_c+\epsilon}\langle\alpha_s E^2/\pi\rangle_N \frac{\rho_B}{2m_N}$$

$\psi(k)$: charmonium wave function

ρ_B : nuclear density

m_c, m_N : masses charm quark and nucleon

$\epsilon = 2m_c - m_\psi$: energy shift octet – charmonium

Note: intermediate state is noninteracting

Numerical results

$$\langle \alpha_s E^2/\pi \rangle_N = 0.5\,\mathrm{GeV}^2$$

ψ : Gaussian

$\langle r^2 \rangle$: from Cornell potential model

$$\boxed{\Delta m_\psi = -8\,\mathrm{MeV}}$$

at normal nuclear matter density

Sibirtsev & Voloshin, PRD 71, 076005 (2005)

J/Ψ N cross section > 17 mb

$$\boxed{\Delta m_\psi = -21\,\mathrm{MeV}}$$

23 October 1997

ELSEVIER

PHYSICS LETTERS B

Physics Letters B 412 (1997) 125–130

Is J/ψ-nucleon scattering dominated by the gluonic van der Waals interaction? [1]

Stanley J. Brodsky [a], Gerald A. Miller [a,2,b]

[a] *Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, USA*

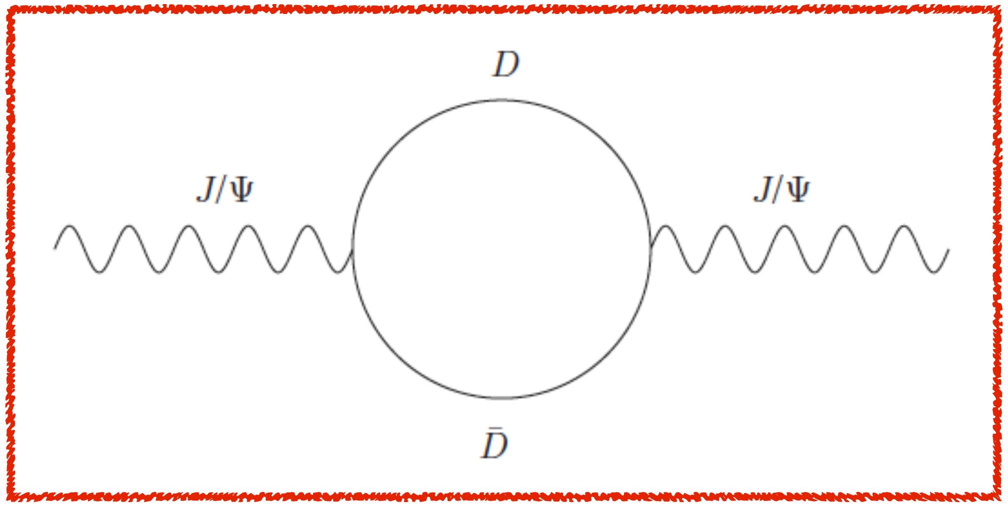
[b] *National Institute for Nuclear Theory, Box 35150, University of Washington, Seattle, WA 98195-1560, USA*

Received 17 July 1997
Editor: M. Dine

Abstract

The gluon-exchange contribution to J/ψ-nucleon scattering is shown to yield a sizeable scattering length of about -0.25 fm, which is consistent with the sparse available data. Hadronic corrections to gluon exchange which are generated by $\rho\pi$ and $D\bar{D}$ intermediate states of the J/ψ are shown to be negligible. We also propose a new method to study J/ψ-nucleon elastic scattering in the reaction $\pi^+ d \to J/\psi\, pp$.

D,D*-meson loops



Calculate loop with effective Lagrangians

– need coupling constants & form factors

– need medium dependence of D masses

D,D* in medium

Light quarks in D mesons

— quark condensate changes for nonzero temperature and baryon density

— quark-model: masses of the D mesons change

Quark condensate at finite T
— model independent result*

For low temperatures:

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle} = 1 - \frac{\Sigma_\pi}{m_\pi^2 f_\pi^2}\,\rho_s^\pi(T)$$

$$\simeq 1 - \frac{T^2}{8 f_\pi^2}$$

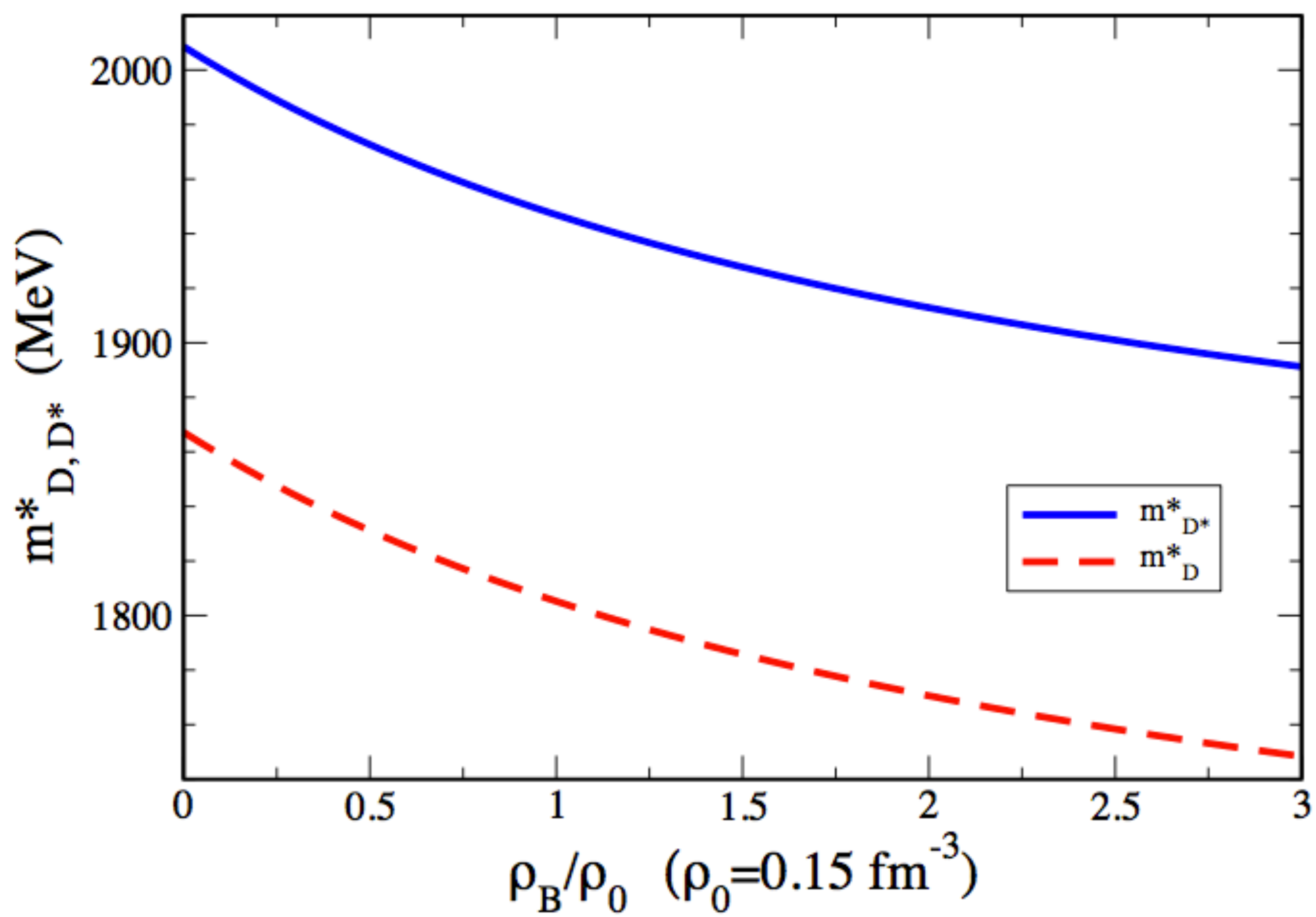
$$\Sigma_\pi = m_q \frac{dm_\pi}{dm_q}$$

$\rho_s^\pi(T)$: scalar density of pions in medium

* Gerber & Leutwyler (1989)

D,D* in medium

— QMC model

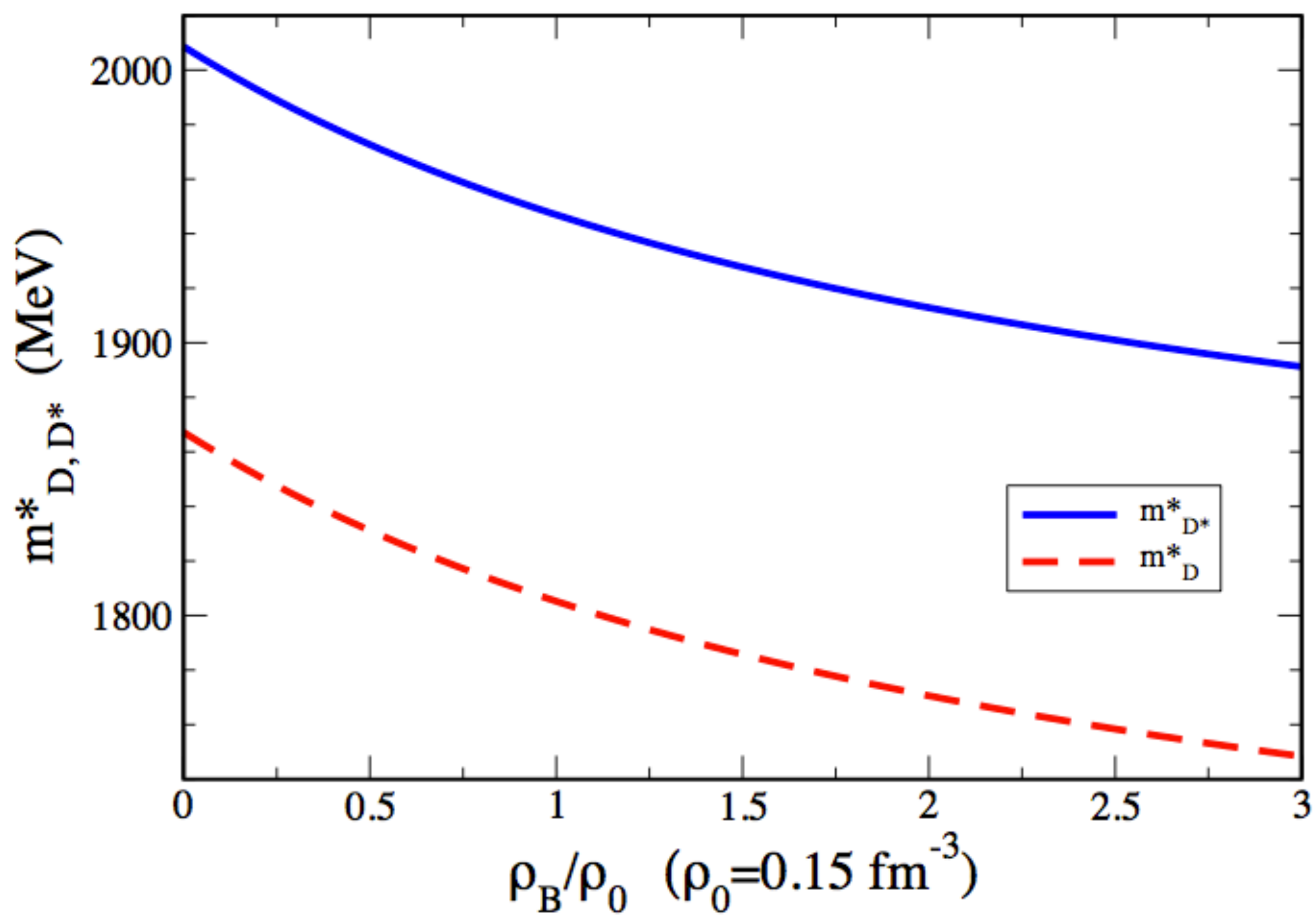


Low temperature and density:

K. Saito, K. Tsushima & A.W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007)

D,D* in medium

— QMC model

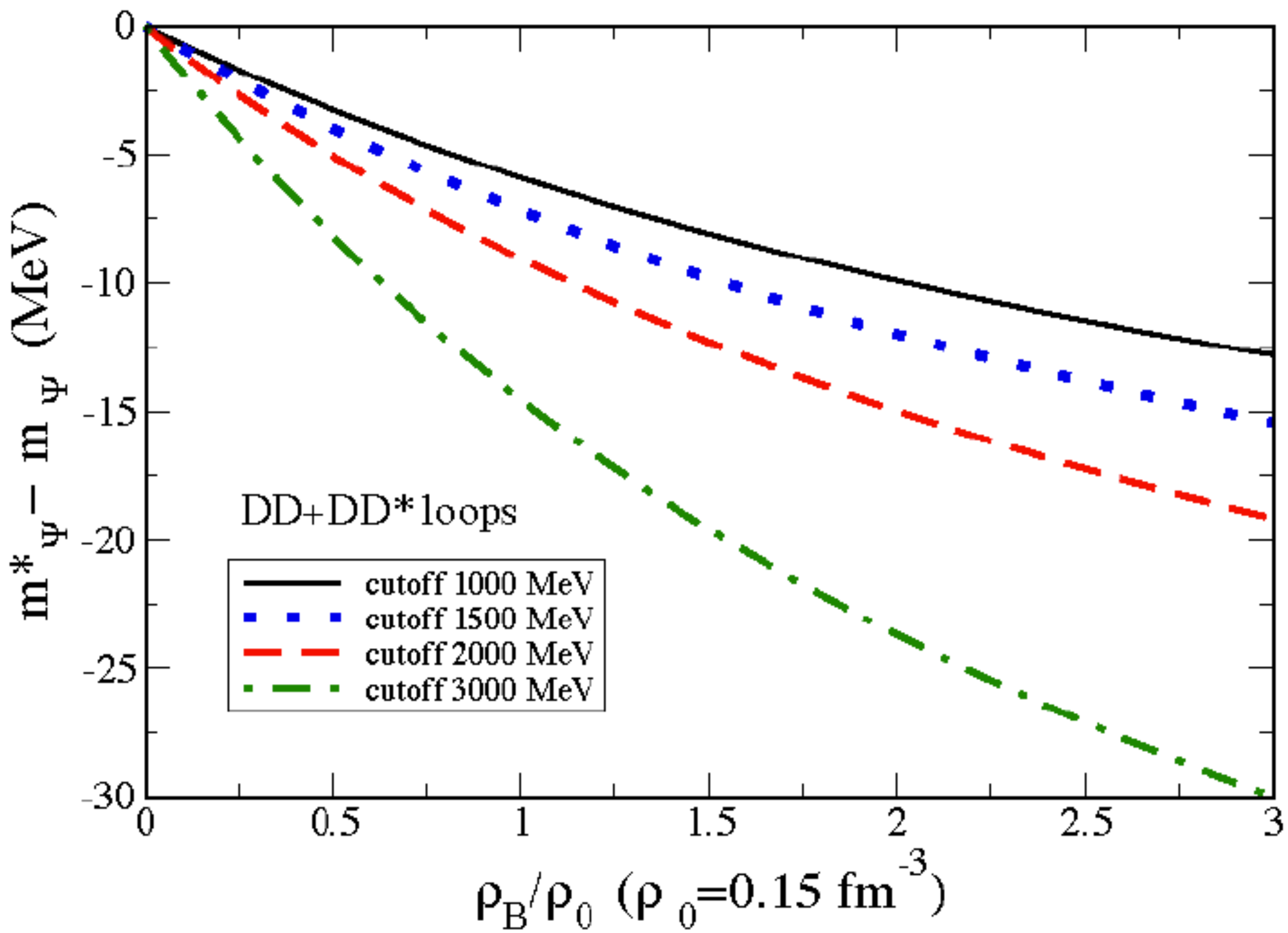


Low temperature and density:

$$\langle \bar{q}q \rangle \simeq \left(1 - \frac{T^2}{8f_\pi^2} - 0.3\frac{\rho}{\rho_0}\right) \langle \bar{q}q \rangle_{\rm vac}$$

K. Saito, K. Tsushima & A.W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007)

J/Ψ in nuclear matter



GK, A. W. Thomas & K. Tsushima PLB 697, 136 (2011)

J/Ψ single-particle energies in nuclei

— from Klein-Gordon equation

		$\Lambda_{D,D^*} = 1500$ MeV	$\Lambda_{D,D^*} = 2000$ MeV
		E (MeV)	E (MeV)
$^{4}_{\Psi}$He	1s	−4.19	−5.74
$^{12}_{\Psi}$C	1s	−9.33	−11.21
	1p	−2.58	−3.94
$^{16}_{\Psi}$O	1s	−11.23	−13.26
	1p	−5.11	−6.81
$^{40}_{\Psi}$Ca	1s	−14.96	−17.24
	1p	−10.81	−12.92
	1d	−6.29	−8.21
	2s	−5.63	−7.48
$^{90}_{\Psi}$Zr	1s	−16.38	−18.69
	1p	−13.84	−16.07
	1d	−10.92	−13.06
	2s	−10.11	−12.22
$^{208}_{\Psi}$Pb	1s	−16.83	−19.10
	1p	−15.36	−17.59
	1d	−13.61	−15.81
	2s	−13.07	−15.26

K. Tsushima, D. Lu, GK & A. W. Thomas PRC 83, 065208 (2011)

PHYSICAL REVIEW D **91,** 114503 (2015)

Quarkonium-nucleus bound states from lattice QCD

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[2]*Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1560, USA*
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[5]*Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[6]*Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, Barcelona, 08028, Spain*

(Received 9 November 2014; published 11 June 2015)

Quarkonium-nucleus systems are composed of two interacting hadronic states without common valence quarks, which interact primarily through multigluon exchanges, realizing a color van der Waals force. We present lattice QCD calculations of the interactions of strange and charm quarkonia with light nuclei. Both the strangeonium-nucleus and charmonium-nucleus systems are found to be relatively deeply bound when the masses of the three light quarks are set equal to that of the physical strange quark. Extrapolation of these results to the physical light-quark masses suggests that the binding energy of charmonium to nuclear matter is $B_{\mathrm{phys}}^{\mathrm{NM}} \lesssim 40$ MeV.

DOI: 10.1103/PhysRevD.91.114503 PACS numbers: 11.15.Ha, 12.38.Gc, 13.40.Gp

Models

TABLE I. Estimates for the binding energies of charmonium to light nuclei and nuclear matter (in MeV) from selected models. A "∗" indicates the system is predicted to be unbound, while entries with center dots indicate that the system was not addressed.

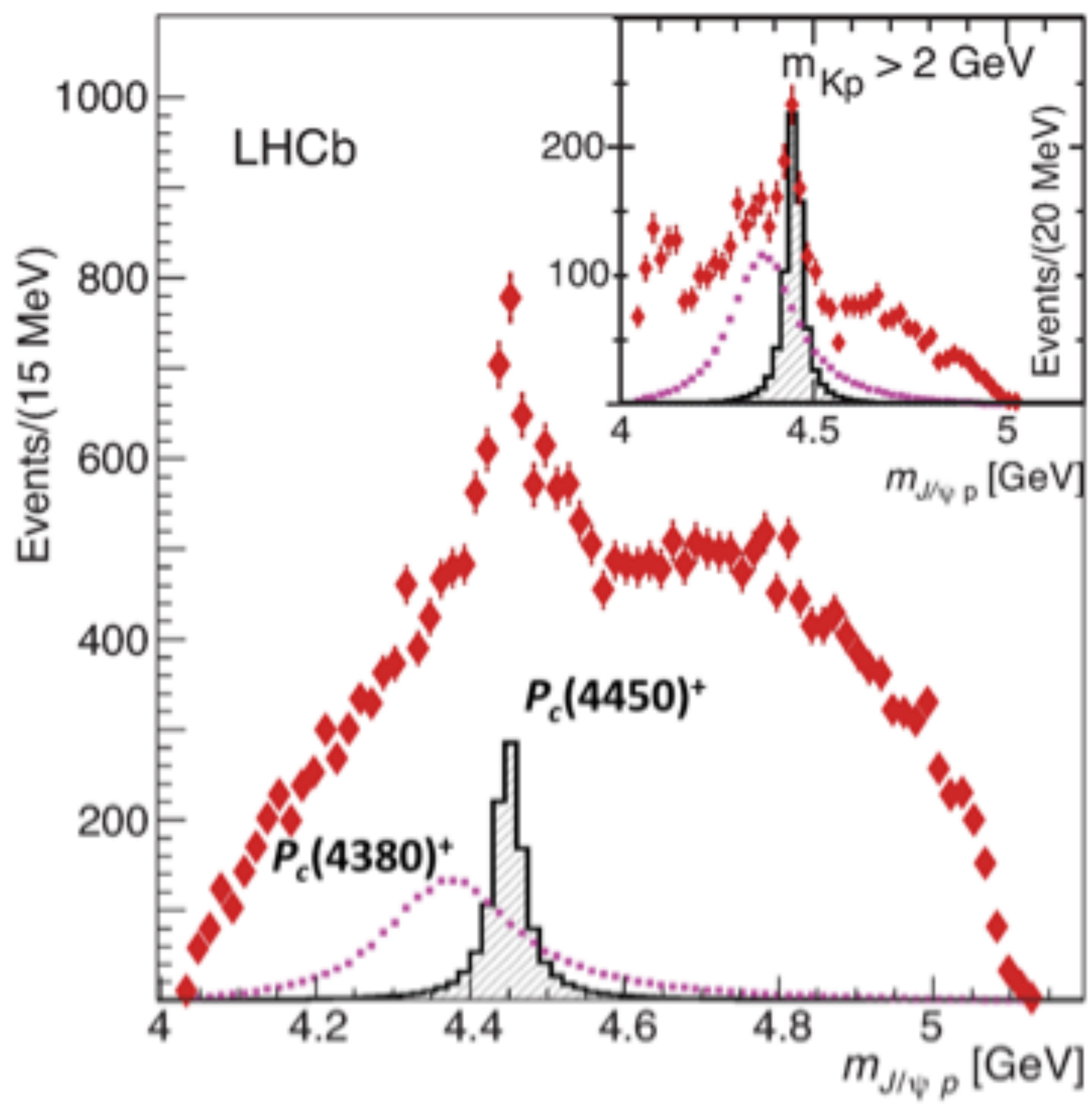
	Binding energy (MeV)			Binding energy (MeV)	
Ref.	^{3}He η_c	^{4}He η_c	NM η_c	^{4}He J/ψ	NM J/ψ
[1]	19	140	∗		
[2]	0.8	5	27		
[3]			10		10
[5]	∗	∗	9		
[6]					5
[7]				5	18
[8]				15.7	

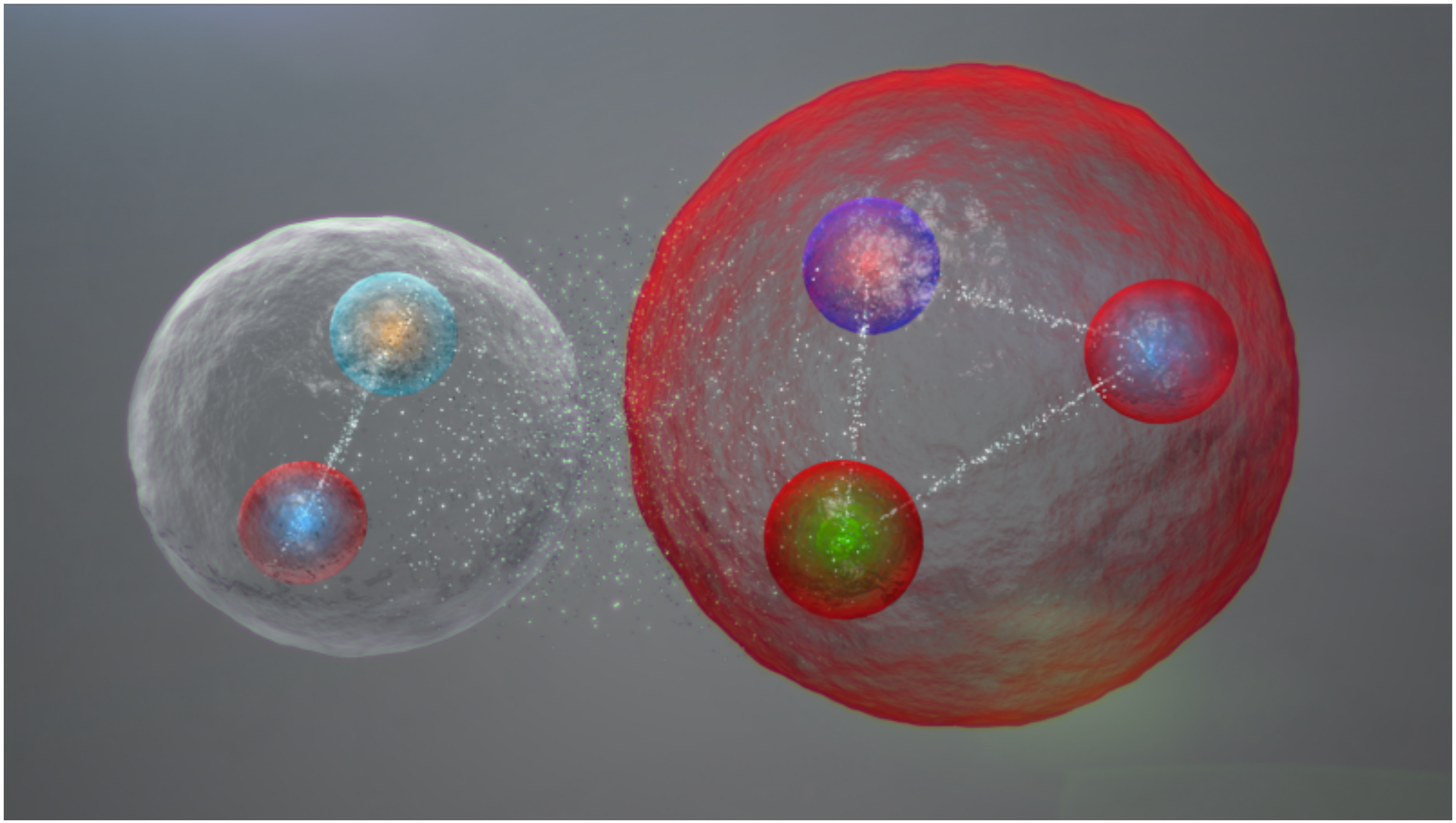
TABLE V. The binding energies (in MeV) of charmonium-nucleus systems calculated on the $L = 24$ and 32 ensembles. The rightmost column shows the infinite-volume estimate, which, without results on the $L = 48$ ensemble, is taken to be the binding calculated on the $L = 32$ ensemble. The first and second sets of parentheses shows the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

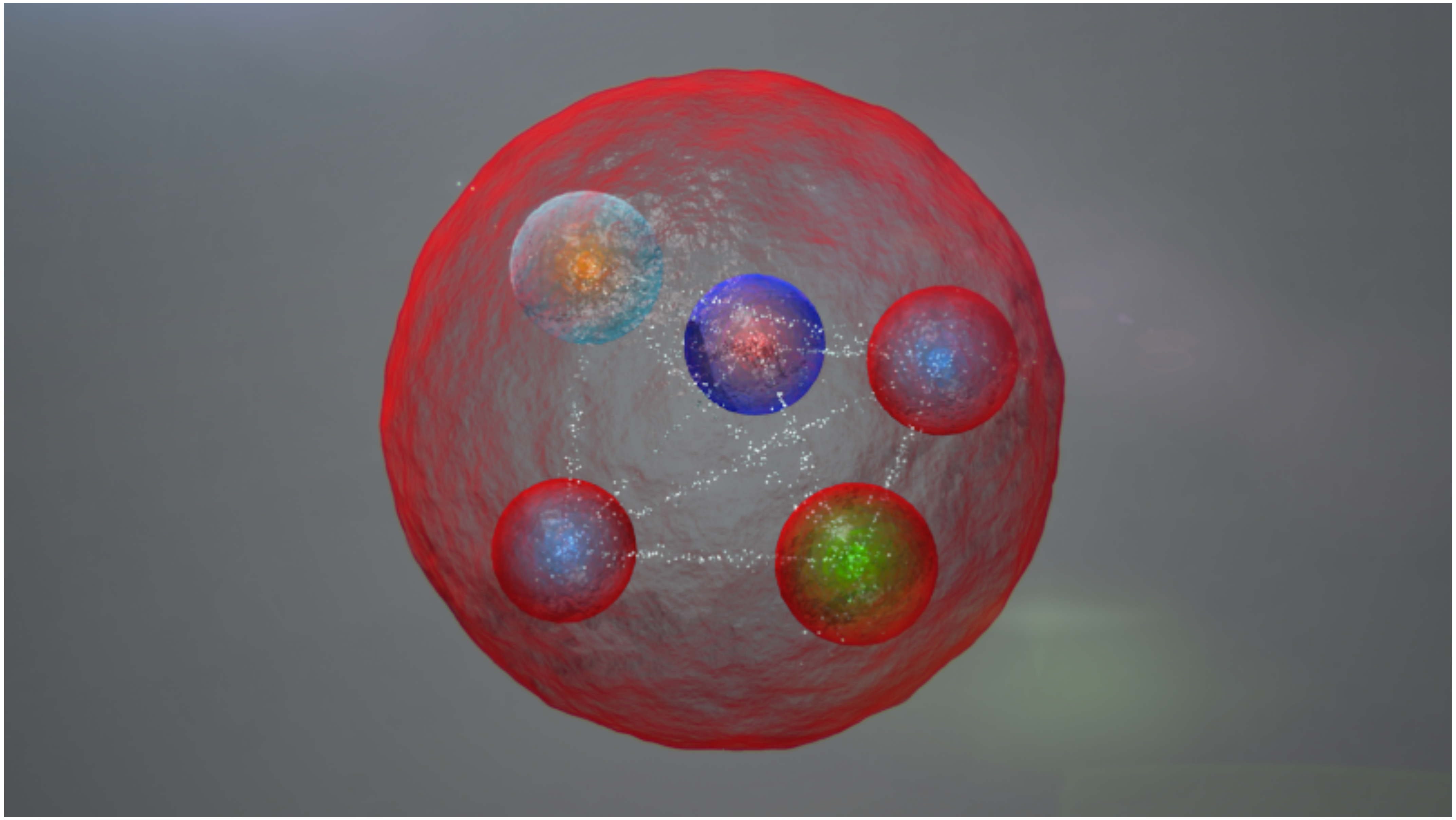
NPLQCD

System	$24^3 \times 64$	$32^3 \times 64$	$L = \infty$
$N\eta_c$	17.9(0.4)(1.5)	19.8(0.7)(2.6)	19.8(2.6)
$d\eta_c$	39.3(1.3)(4.8)	42.4(1.1)(7.9)	42.4(7.9)
$pp\eta_c$	37.8(1.1)(4.5)	41.5(1.0)(7.5)	41.5(7.6)
^{3}Heη_c	57.2(1.3)(8.3)	56.7(2.0)(9.4)	56.7(9.6)
^{4}Heη_c	70(02)(13)	56(06)(17)	56(18)
^{4}HeJ/ψ	75.7(1.9)(9.4)	53(07)(18)	53(19)

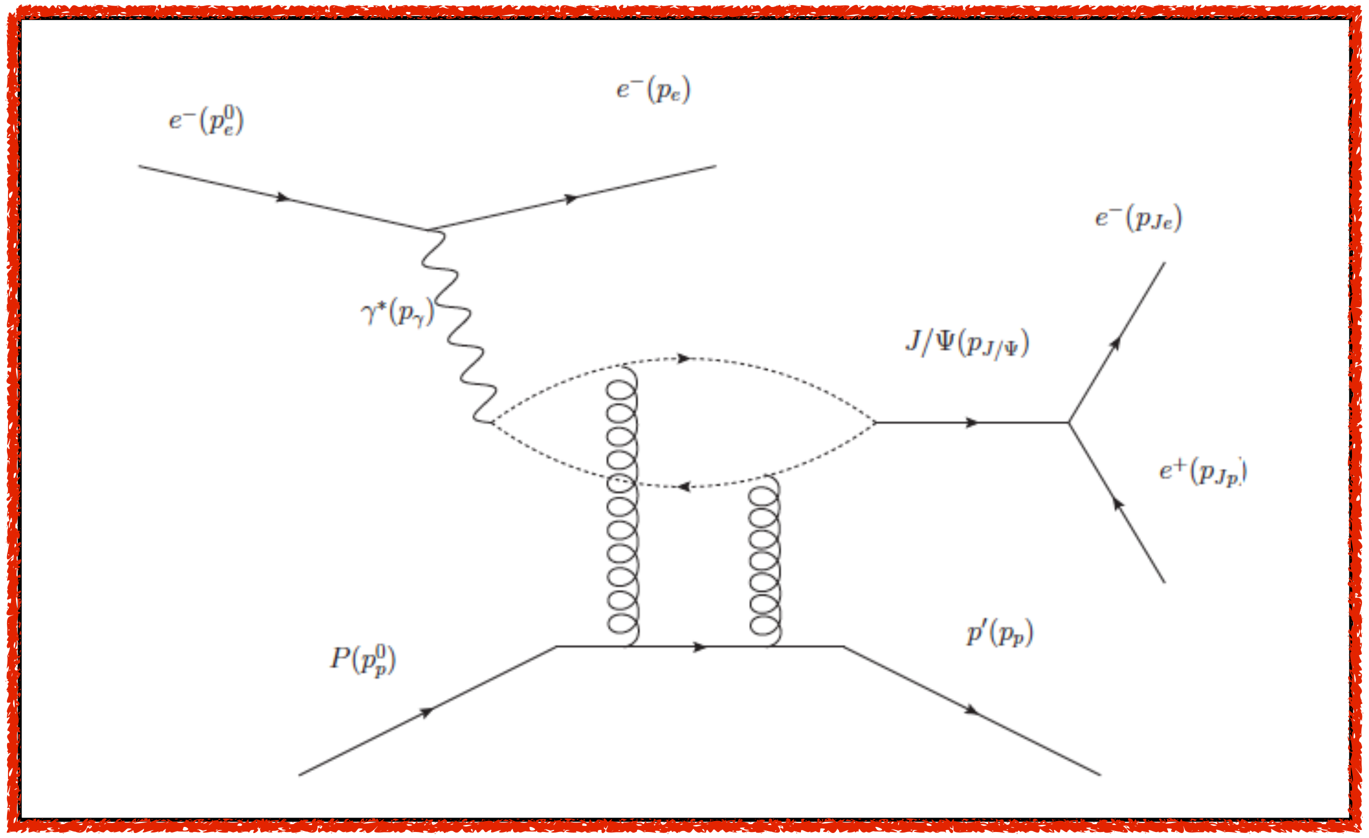
J/Ψ binding to proton?







ATHENNA* collaboration JLab @ 12 GeV



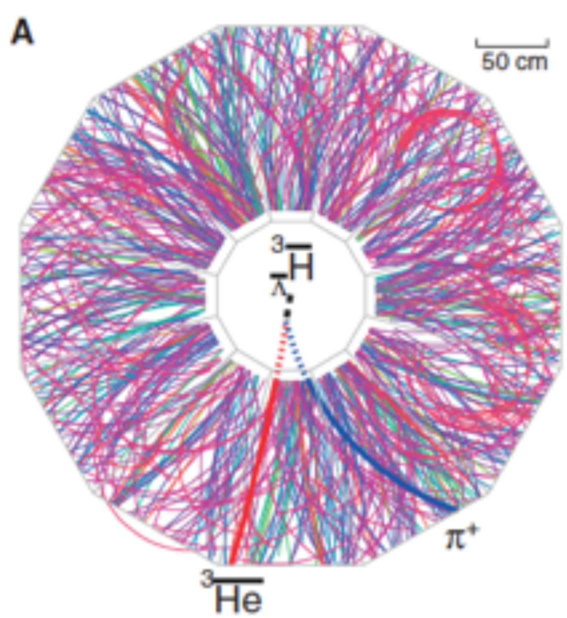
Z.-E. Meziani (Co-spokesperson/Contact)
N. Sparveris (Co-spokesperson)
Z. W. Zhao (Co-spokesperson)

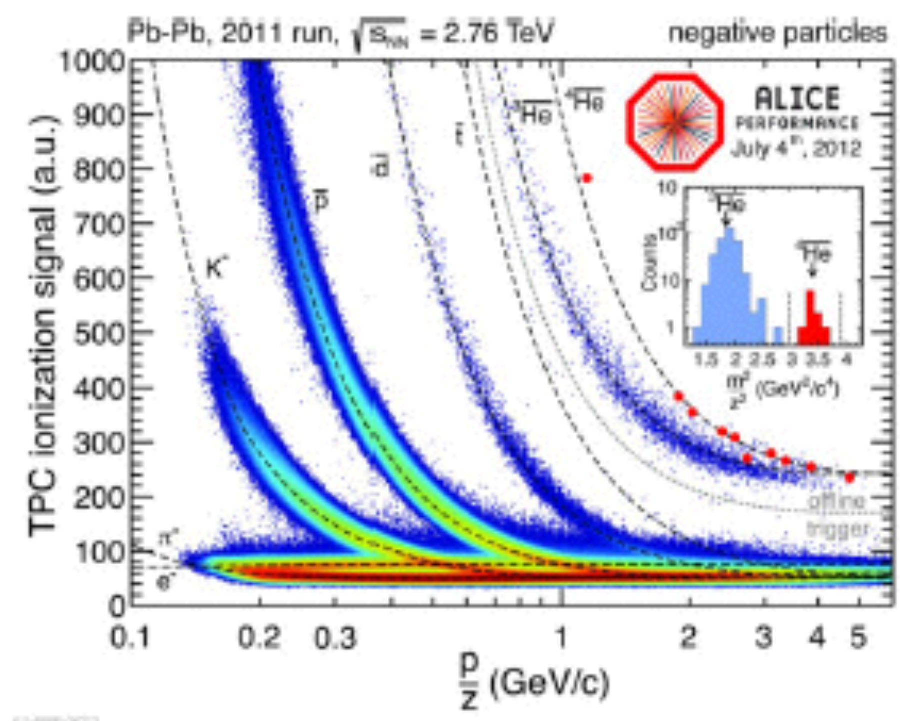
*A J/Ψ THreshold Electroproduction on the Nucleon and Nuclei Analysis

How About coalescence at the LHC?

— Chances of a charmed hadron meeting one or two nucleons not smaller than of two antinucleons and one antihyperon meeting to form an antihypernucleus

Observation of an Antimatter Hypernucleus
The STAR Collaboration
Science **328**, 58 (2010);
DOI: 10.1126/science.1183980



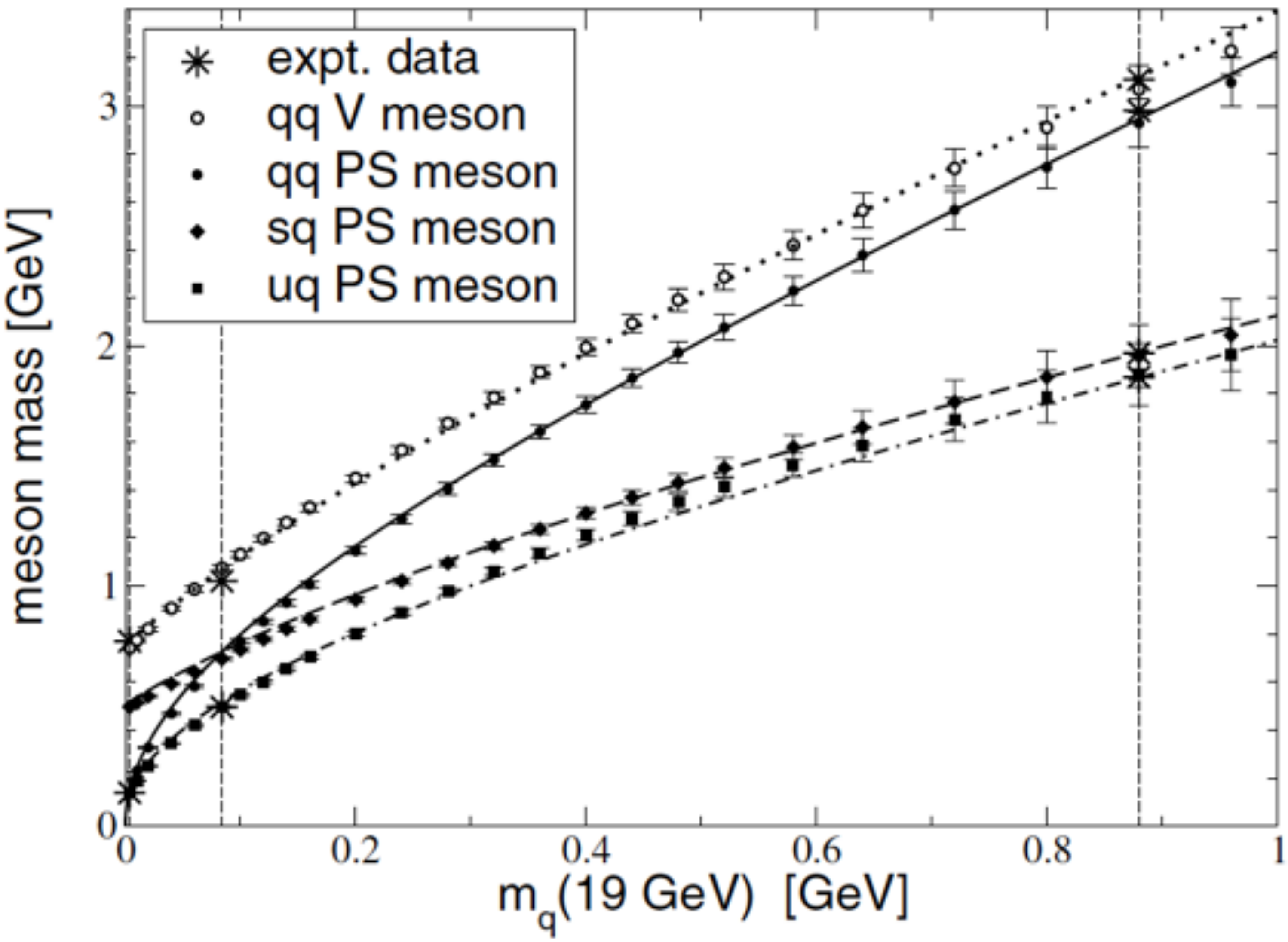


Need to detect in coincidence the decay products

Theory on
Heavy-Light Mesons

Dyson-Schwinger + Bethe-Salpeter
— Rainbow-Ladder

Mass spectrum: OK



P. Maris and P. C. Tandy, NP Suppl. 161, 136 (2006)

Dyson-Schwinger + Bethe-Salpeter
— Rainbow-Ladder

Leptonic decay constants: NOT OK

TABLE IV. Pseudoscalar and vector mesons decay constants with constituent mass approximation for the c or b quark propagators in the BSE and 1rp or 3ccp in f and N integrals. Experimental data are from [52].

Meson	Exp. (GeV)	$f^{1\mathrm{rp}}$ (GeV)	$f^{3\mathrm{ccp}}$ (GeV)
$D(uc)$	0.222	0.154	0.255 ± 0.010
$D^*(uc)$		0.164	0.288 ± 0.030
$D_s(sc)$	0.294	0.197	0.255 ± 0.005
$D_s^*(sc)$		0.180	0.326 ± 0.040
$B(ub)$	0.176	0.105	0.193 ± 0.005
$B^*(ub)$		0.182	0.549 ± 0.083
$B_s(sb)$		0.144	0.212 ± 0.002
$B_s^*(sb)$		0.20	0.425 ± 0.041
$\eta_c(c\bar{c})$	0.340	0.239	0.326
$J/\psi(c\bar{c})$	0.416	0.198	0.330
$B_c(cb))$		0.210	0.324 ± 0.004
$B_c^*(cb))$		0.18	0.328 ± 0.008
$\eta_b(b\bar{b})$		0.244	0.414
$\Upsilon(b\bar{b})$	0.700	0.210	0.381

N. Souchlas, PRD 81, 11419 (2010)

NJL model

D. BLASCHKE, P. COSTA, AND YU. L. KALINOVSKY PHYSICAL REVIEW D **85**, 034005 (2012)

TABLE I. Results for pseudoscalar meson properties in the light, strange, charm and bottom sectors for the corresponding current quark masses and dynamically generated quark masses of the model in the vacuum at $T = \mu = 0$.

flavor	P	M_P [MeV]	f_P [MeV]	m_f [MeV]	M_f [MeV]
u, d	π	135.0	92.4	5.5	368.0
s	K	497.7	95.4	140.7	587.4
c	D	1869.3	79.6	1279.9	1828.8
b	B	5279.4		4634.8	

Exp

93
113
146

$$M_h \to \infty : \quad f_{PS} \sim \frac{1}{\sqrt{M_h}}$$

Bethe-Salpeter equations

$$[\Gamma_{\mathcal{M}}(k;P)]_{AB} \quad = \quad \mathcal{M}_{AB} + \int_q [K(k,q;P)]_{AC,DB}\, [S(q_+)\Gamma_{\mathcal{M}}(q;P)S(q_-)]_{CD}$$

$q_\pm = q \pm \eta_\pm P$ $\qquad$ η_+, η_- : arbitrary

BUT, translational invariance requires: $\boxed{\eta_+ + \eta_- = 1}$

<u>Approximations & truncations:</u>

- independence of choices is not guarantee

Translational invariance:

η_+, η_- : arbitrary

BUT, translational invariance requires: $\boxed{\eta_+ + \eta_- = 1}$

<u>Approximations & truncations:</u>

- independence of choices is not guaranteed
- integral is divergent, change of variable is problematic

Ward-Takahashi identity

$$P_\mu \Gamma^{lh}_{5\mu}(k;P) \quad = \quad S_l^{-1}(k_+) i\gamma_5 + i\gamma_5 S_h^{-1}(k_-)$$
$$- i\,(m_l + m_h)\,\Gamma^{lh}_{\mathrm{PS}}(k;P)$$

$\boxed{\eta_+ + \eta_- = 1}$ — a different dependence on η_+, η_-

also violates this WT identity

Regularization & Symmetry preservation

Next:

- How to make sure in your approximation/truncation/regularization scheme symmetries are preserved

1) Isolate divergences
2) Isolate symmetry-offending terms
- without performing change of variables

Contact interaction

DSE

$$S_f(k)^{-1} = i\gamma \cdot k + m_f + \int_q g^2 D_{\mu\nu}(k-q)\,\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,k)$$

BSE

$$\left[\Gamma^{lh}_{\mathrm{PS}}(k;P)\right]_{AB} = \int_q [K(k,q;P)]_{AC,DB}\left[S_l(q_+)\Gamma^{lh}_{\mathrm{PS}}(q;P)S_h(q_-)\right]_{CD}$$

WTI

$$P_\mu\Gamma^{lh}_{5\mu}(k;P) = S_l^{-1}(k_+)i\gamma_5 + i\gamma_5 S_h^{-1}(k_-) - i\,(m_l+m_h)\,\Gamma^{lh}_{\mathrm{PS}}(k;P)$$

Contact interaction

$$g^2 D_{\mu\nu}(k-q) = \frac{4\pi\alpha_{IR}}{m_G^2}\delta_{\mu\nu} \qquad \text{and} \qquad \Gamma^a_\mu \to \frac{\lambda^a}{2}\gamma_\mu$$

$$[K(k,q;P)]_{AC,DB} = -\frac{4\pi\alpha_{IR}}{m_G^2}\left(\frac{\lambda^a}{2}\gamma_\mu\right)_{AC}\left(\frac{\lambda^a}{2}\gamma_\mu\right)_{DB}$$

$$\boxed{\begin{aligned} S_f(k)^{-1} &= i\gamma\cdot k + M_f \\ M_f &= m_f + \frac{64\pi\alpha_{IR}}{3m_G^2}\int_q \frac{M_f}{q^2+M_f^2} \end{aligned}}$$

$$\Gamma^{lh}_{\rm PS}(P) = \gamma_5 \left[i E^{lh}_{\rm PS}(P) + \frac{1}{2M_{lh}} \gamma \cdot P\, F^{lh}_{\rm PS}(P) \right] \qquad M_{lh} = M_l M_h/(M_l + M_h)$$

$$\boxed{\left[\begin{array}{c} E^{lh}_{\rm PS}(P) \\ F^{lh}_{\rm PS}(P) \end{array}\right] = \frac{4\pi\alpha_{IR}}{3m_G^2} \left[\begin{array}{cc} \mathcal{K}^{EE}_{lh} & \mathcal{K}^{EF}_{lh} \\ \mathcal{K}^{FE}_{lh} & \mathcal{K}^{FF}_{lh} \end{array}\right] \left[\begin{array}{c} E^{lh}_{\rm PS}(P) \\ F^{lh}_{\rm PS}(P) \end{array}\right]}$$

$$\mathcal{K}^{EE}_{lh} = -\int_q {\rm Tr}[\gamma_5\, \gamma_\mu\, S_l(q_+)\, \gamma_5\, S_h(q_-)\, \gamma_\mu]$$

$$\mathcal{K}^{EF}_{lh} = \frac{i}{2M_{lh}} \int_q {\rm Tr}[\gamma_5\, \gamma_\mu\, S_l(q_+)\, \gamma_5\, \gamma \cdot P\, S_h(q_-)\, \gamma_\mu] \qquad \mathcal{K}^{FE}_{lh} = \frac{2M_{lh}}{P^2} \mathcal{K}^{EF}_{lh}$$

$$\mathcal{K}^{FF}_{lh} = \frac{1}{P^2} \int_q {\rm Tr}[\gamma_5\, \gamma \cdot P\, \gamma_\mu\, S_l(q_+) \gamma_5\, \gamma \cdot P\, S_h(q_-)\, \gamma_\mu]$$

Must not depend on $\longrightarrow$ $\boxed{\eta_+ , \eta_-}$

Regularization

1) Assume a regularization:

$$\int \frac{d^4q}{(2\pi)^4} f(q) \to \int_q^\Lambda f(q)$$

2) Make subtractions of the kind:

$$\frac{1}{q_\pm^2 + M^2} = \left(\frac{1}{q_\pm^2 + M^2} - \frac{1}{q^2 + M^2} \right) + \frac{1}{q^2 + M^2}$$

$$= \frac{1}{q^2 + M^2} - \frac{q_\pm^2 - q^2}{(q^2 + M^2)} \frac{1}{(q_\pm^2 + M^2)}$$

3) Make as many subtractions as necessary to obtain enough powers in the denominator to obtain a finite amplitude

Subtraction lowers the degree of diverge

Performing the subtractions*:

BSE

Example $$\mathcal{K}_{lh}^{EE} = 16\int_q^{\Lambda}\frac{q_+\cdot q_- + M_l M_h}{(q_+^2+M_l^2)(q_-^2+M_h^2)}$$

WTI

$$0 = (M_l - m_l) + (M_h - m_h)$$
$$-\frac{64\pi\alpha_{IR}}{3m_G^2}\int_q^{\Lambda}\left(\frac{M_l}{q_+^2+M_l^2}+\frac{M_h}{q_-^2+M_h^2}\right)$$

$$0 = \int_q^{\Lambda}\left(\frac{q_+\cdot P}{q_+^2+M_l^2}-\frac{q_-\cdot P}{q_-^2+M_h^2}\right)$$

*F.E. Serna, M.A. Brito, GK

Results of the subtractions:

BSE

$$\mathcal{K}_{\rm PS}^{EE} = 8\Big\{\left(\eta_+^2+\eta_-^2\right) A_{\mu\nu}(M^2) P_\mu P_\nu$$
$$+ I_{\rm quad}(M_l^2) + I_{\rm quad}(M_h^2) - \left[P^2 + (M_h - M_l)^2\right]$$
$$\times \left[I_{\rm log}(M^2) - Z_0(M_l^2, M_h^2, P^2; M^2)\right]\Big\}$$

$$A_{\mu\nu}(M^2) = \int_q^\Lambda \frac{4q_\mu q_\nu - (q^2+M^2)\delta_{\mu\nu}}{(q^2+M^2)^3}$$

$$I_{\rm quad}(M^2) = \int_q^\Lambda \frac{1}{q^2+M^2} \qquad I_{\rm log}(M^2) = \int_q^\Lambda \frac{1}{(q^2+M^2)^2}$$

$$Z_k[\lambda_1^2, \lambda_2^2, q^2; \lambda^2] = \int_0^1 dz\, z^k \ln\left[\frac{q^2 z(1-z) - (\lambda_1^2 - \lambda_2^2)z + \lambda_1^2}{\lambda^2}\right]$$

Result of the subtractions:

WTI

$$
\begin{aligned}
0 \quad = \quad & (M_l - m_l) + (M_h - m_h) \\
& - \frac{64\pi\alpha_{IR}}{3m_G^2} \Big\{ M_l I_{\rm quad}(M_l^2) + M_h I_{\rm quad}(M_h^2) \\
& + \left(M_l\, \eta_+^2 + M_h\, \eta_- \right) A_{\mu\nu}(M^2) P_\mu P_\nu \Big\}
\end{aligned}
$$

$$
\begin{aligned}
0 \quad = \quad & \int_q^\Lambda \left(\frac{q_+ \cdot P}{q_+^2 + M^2} - \frac{q_- \cdot P}{q_-^2 + M^2} \right) \\
\sim \quad & \text{terms proportional to } \eta_\pm \left(A_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu\rho\sigma} \right)
\end{aligned}
$$

Where

$$A_{\mu\nu}(M^2) = \int_q^{\Lambda} \frac{4q_\mu q_\nu - (q^2+M^2)\delta_{\mu\nu}}{(q^2+M^2)^3}$$

$$B_{\mu\nu}(M^2) = \int_q^{\Lambda} \frac{2q_\mu q_\nu - (q^2+M^2)\delta_{\mu\nu}}{(q^2+M^2)^2}$$

$$C_{\mu\nu\rho\sigma}(M^2) = \int_q^{\Lambda} \frac{c_{\mu\nu\rho\sigma}(q, M^2)}{(q^2+M^2)^4}$$

$$c_{\mu\nu\rho\sigma}(q^2, M^2) = 24q_\mu q_\nu q_\rho q_\sigma - 4(q^2+M^2)$$

$$\times(\delta_{\mu\nu} q_\rho q_\sigma + \text{perm. } \nu\sigma\rho)$$

What to do with the divergent symmetry-offending terms?

Ideally, regularization scheme should take care of them

— Dimensional regularization does that:

$$A_{\mu\nu} = 0, \qquad B_{\mu\nu} = 0, \qquad C_{\mu\nu\rho\sigma} = 0$$

— Cutoff: $A_{\mu\nu} \neq 0, \qquad B_{\mu\nu} \neq 0, \qquad C_{\mu\nu\rho\sigma} \neq 0$

Truncations & approximations

Criterion for elimination of symmetry-offending terms

is integral part of the

truncation/approximation scheme

IMPOSE $\boxed{A_{\mu\nu} = 0, \qquad B_{\mu\nu} = 0, \qquad C_{\mu\nu\rho\sigma} = 0}$

$$A_{\mu\nu} = 0, \qquad B_{\mu\nu} = 0, \qquad C_{\mu\nu\rho\sigma} = 0$$

WTI

$$\begin{aligned} 0 &= (M_l - m_l) + (M_h - m_h) \\ &\quad - \frac{64\pi\alpha_{IR}}{3m_G^2}\Big\{ M_l I_{\text{quad}}(M_l^2) + M_h I_{\text{quad}}(M_h^2) \\ &\quad + \cancel{\left(M_l\,\eta_+^2 + M_h\,\eta_-^2\right) A_{\mu\nu}(M^2) P_\mu P_\nu} \Big\} \end{aligned}$$

$$\begin{aligned} 0 &= \int_q^\Lambda \left(\cancel{\frac{q_+ \cdot P}{q_+^2 + M^2}} - \frac{q_- \cdot P}{q_-^2 + M^2} \right) \\ &\sim \text{terms proportional to } \eta_\pm\,(A_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu\rho\sigma}) \end{aligned}$$

$$A_{\mu\nu} = 0, \qquad B_{\mu\nu} = 0, \qquad C_{\mu\nu\rho\sigma} = 0$$

WTI

$$\begin{aligned} 0 &= (M_l - m_l) + (M_h - m_h) \\ &\quad - \frac{64\pi\alpha_{IR}}{3m_G^2}\Big\{ M_l I_{\text{quad}}(M_l^2) + M_h I_{\text{quad}}(M_h^2) \\ &\quad + \cancel{\left(M_l\,\eta_+^2 + M_h\,\eta_-^2\right)} A_{\mu\nu}(M^2) P_\mu P_\nu \Big\} \end{aligned}$$

$$\begin{aligned} 0 &= \int_q^\Lambda \left(\cancel{\frac{q_+ \cdot P}{q_+^2 + M^2}} - \frac{q_- \cdot P}{q_-^2 + M^2} \right) \\ &\sim \text{ terms proportional to } \eta_\pm \left(A_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu\rho\sigma} \right) \end{aligned}$$

$$A_{\mu\nu} = 0, \qquad B_{\mu\nu} = 0, \qquad C_{\mu\nu\rho\sigma} = 0$$

Gap equation

WTI

$$\begin{aligned} 0 &= (M_l - m_l) + (M_h - m_h) \\ &\quad - \frac{64\pi\alpha_{IR}}{3m_G^2}\Big\{ M_l I_{\rm quad}(M_l^2) + M_h I_{\rm quad}(M_h^2) \\ &\quad + \cancel{\left(M_l\,\eta_+^2 + M_h\,\eta_-^2\right)} A_{\mu\nu}(M^2) P_\mu P_\nu \Big\} \end{aligned}$$

$$\begin{aligned} 0 &= \int_q^\Lambda \left(\cancel{\frac{q_+ \cdot P}{q_+^2 + M^2}} - \frac{q_- \cdot P}{q_-^2 + M^2} \right) \\ &\sim \ \text{terms proportional to } \eta_\pm \left(A_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu\rho\sigma}\right) \end{aligned}$$

Contact with: L. X. Gutiérrez-Guerrero, A. Bashir, I. C. Clöet and C. D. Roberts, Phys. Rev. C **81**, 065202 (2010).

They work in the chiral limit:

— $M_l = M_h \equiv M$

— Only term with $B_{\mu\nu}(M^2)$ appears in the WTI

— They choose $\eta_+ = 1, \quad \eta_- = 0$

$$B_{\mu\nu}(M^2) = 0 = \delta_{\mu\nu} \int_\Lambda \frac{d^4q}{(2\pi)^4} \frac{\frac{1}{2}q^2 + M^2}{(q^2+M^2)^2}$$

Contact with: L. X. Gutiérrez-Guerrero, A. Bashir, I. C. Clöet and C. D. Roberts, Phys. Rev. C **81**, 065202 (2010).

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Eq. (17) of

Confinement

— no quark-antiquark thresholds

$$\begin{aligned}
\mathcal{K}^{EE}_{\mathrm{PS}} &= 8\Big\{\left(\eta_+^2+\eta_-^2\right) A_{\mu\nu}(M^2) P_\mu P_\nu \\
&\quad + I_{\mathrm{quad}}(M_l^2) + I_{\mathrm{quad}}(M_h^2) - \left[P^2+(M_h-M_l)^2\right] \\
&\quad \times \left[I_{\log}(M^2) - Z_0(M_l^2, M_h^2, P^2; M^2)\right]\Big\}
\end{aligned}$$

$$Z_0(M_l^2, M_h^2, P^2; M^2) = \int_0^1 dz\, \ln\left[\frac{z(1-z)P^2 - z(M_l^2 - M_h^2) + M_l^2}{M^2}\right]$$

branch cut $P^2 = (M_h + M_l)^2$

Z_0 came from:

$$Z_0(M_l^2, M_h^2, P^2; M^2) = \int_0^1 dz \, \left[I_{\log}(M^2) - I_{\log}(H(z)) \right]$$

UV divergences cancel

$$I_{\log}(M^2) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 + M^2)^2}$$

$$H(z) = z(1-z)P^2 - z(M_l^2 - M_h^2)z + M_l^2$$

Infrared cutoff:

$$\frac{1}{(q^2+M^2)^2} = -\int_0^\infty d\tau\,\tau\,e^{-\tau(q^2+M^2)} \rightarrow -\int_0^{\tau_{\rm ir}^2} d\tau\,\tau\,e^{-\tau(q^2+M^2)}$$

$$\tau_{\rm ir} = \frac{1}{\Lambda_{\rm ir}}$$

Masses and leptonic decay constantes of heavy-light mesons*

Parameters:

$$m_G = 0.8 \text{ GeV} \qquad \alpha_{\text{IR}} = 0.93\pi \qquad \tau^2_{\text{ir,uv}} = 1/\Lambda^2_{\text{ir,uv}} \qquad \begin{aligned} \Lambda_{\text{ir}} &= 0.240 \text{ GeV} \\ \Lambda_{\text{uv}} &= 0.905 \text{ GeV} \end{aligned}$$

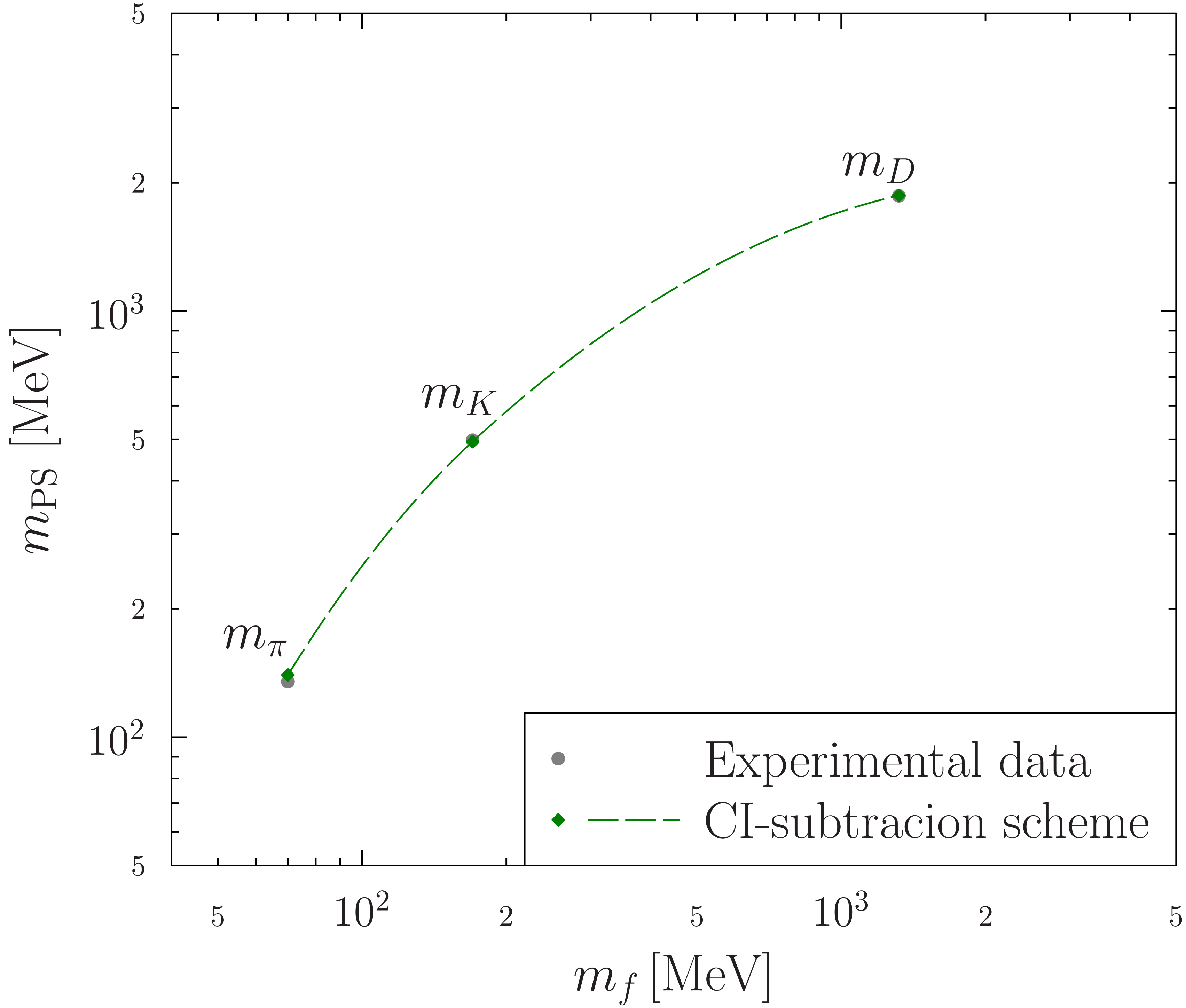
Results:

m_u	M_u	m_s	M_s	m_c	M_c
0.007	0.367	0.165	0.529	1.319	1.374

Meson	E	F	m_{PS}	f_{PS}	E^*	F^*	m^*_{PS}	f^*_{PS}	m^{exp}_{PS}	$f^{\text{exp}}_{PS}/\sqrt{2}$
π	3.609	0.476	0.140	0.102	3.593	0.474	0.140	0.101	0.135	0.092
K	3.959	0.615	0.494	0.111	3.805	0.586	0.492	0.110	0.498	0.110
D	5.213	1.285	1.869	0.174	5.105	0.718	1.864	0.093	1.865	0.146
D_s	5.748	1.572	1.900	0.162	5.331	0.977	1.974	0.096	1.968	0.184

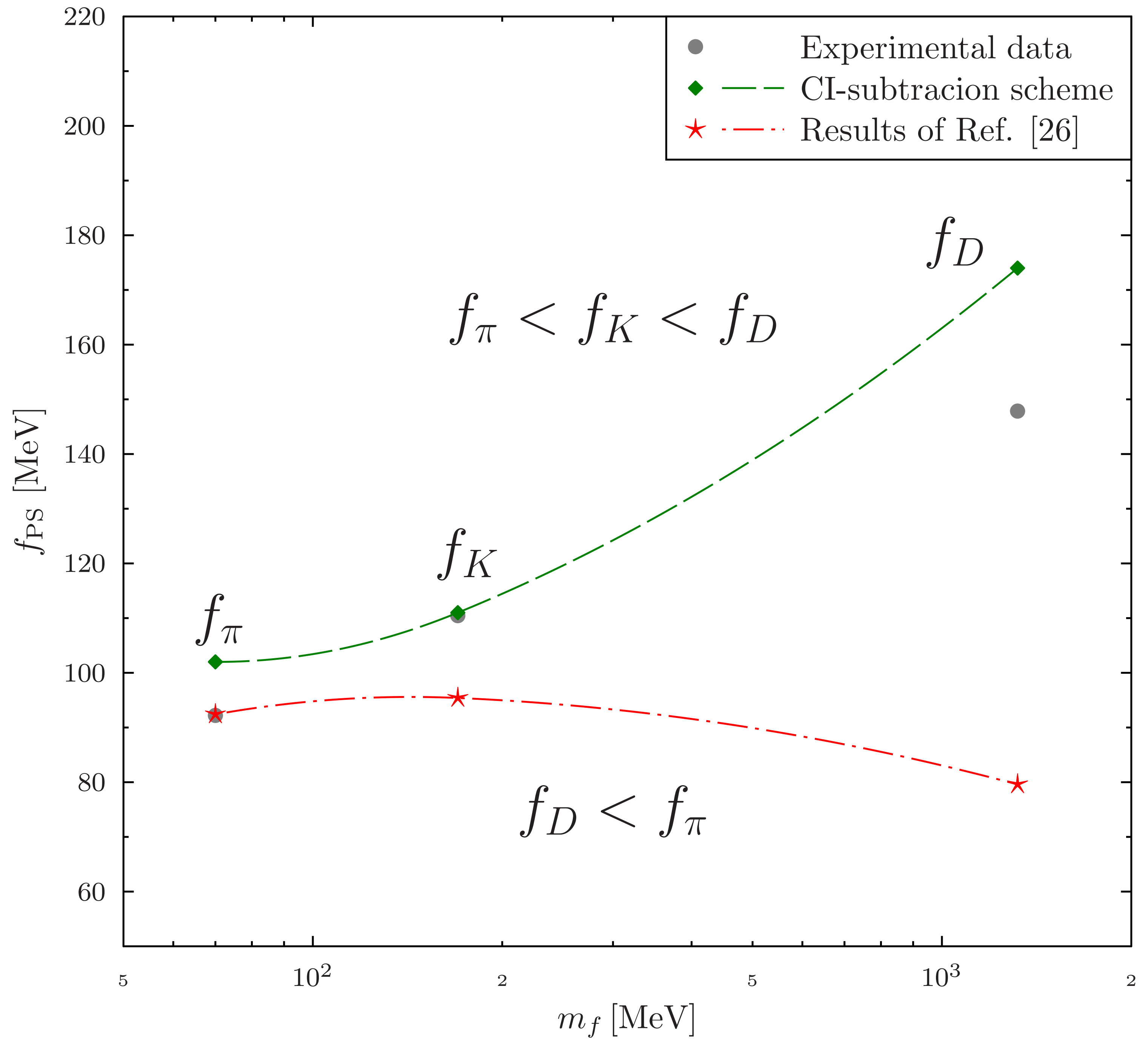
*F.E. Serna, M.A. Brito, GK

Masses



m_D
m_K
m_π
Experimental data
CI-subtracion scheme
m_{PS} [MeV]
m_f [MeV]
10^3
10^2
5
2

Leptonic decay constantes



Heavy-heavy

Heavy-heavy	E	F	m_H	E^*	F^*	m_H^*	m_H^{exp}
$\eta_c(1S)$	7.472	2.121	2.983	6.023	1.711	2.983	2.983
$\chi_{c_0}(1P)$	2.908	0	3.293	1.905	0	3.293	3.414

$$f_{\eta_c(1S)} = 0.236 \text{ GeV}$$

Lattice*: 0.255 GeV

*C.T.H. Davies et al. PRD 82, 114504 (2010)

Chiral chemical potential

Chiral unbalance produced by strong magnetic fields:

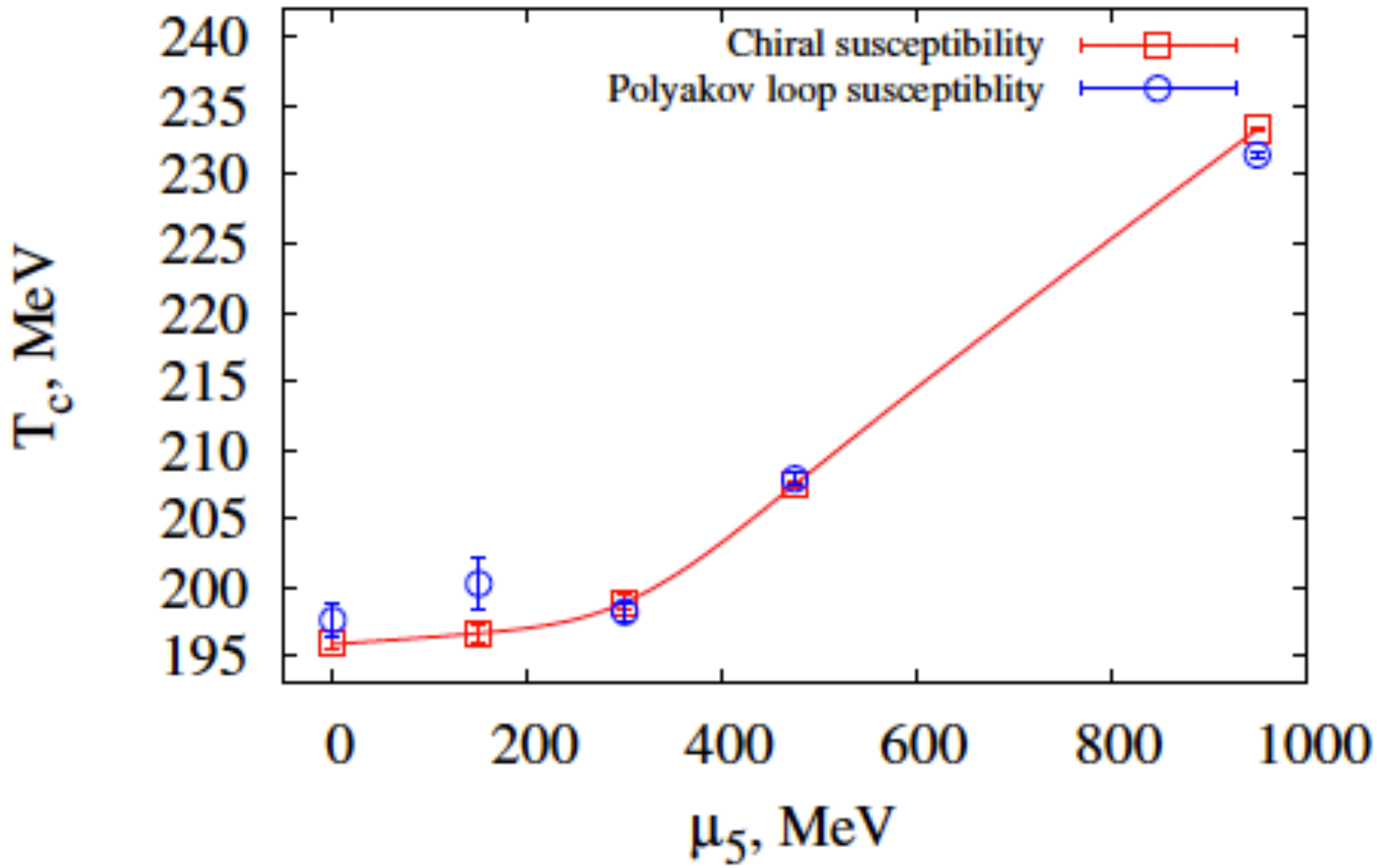
— heavy-ion collisions

— supernova collapse, neutron stars

— no sign problem in lattice simulations

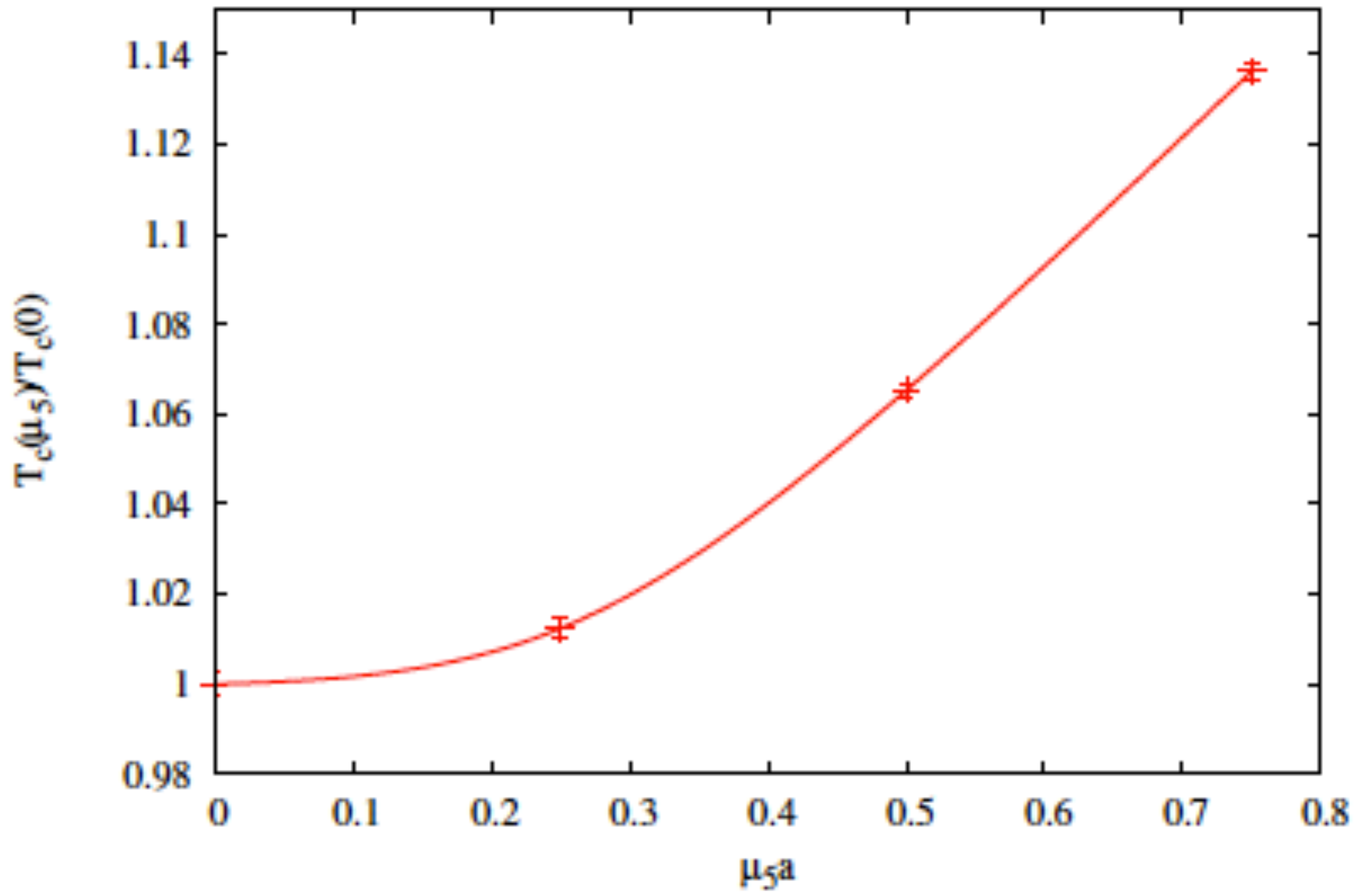
$$\mathcal{L} \sim \mu_5 \left(\bar{\psi}\gamma^0\gamma^5\psi\right) = \mu_5 \left(\psi_R^\dagger\psi_R - \psi_L^\dagger\psi_L\right)$$

SU(2) color



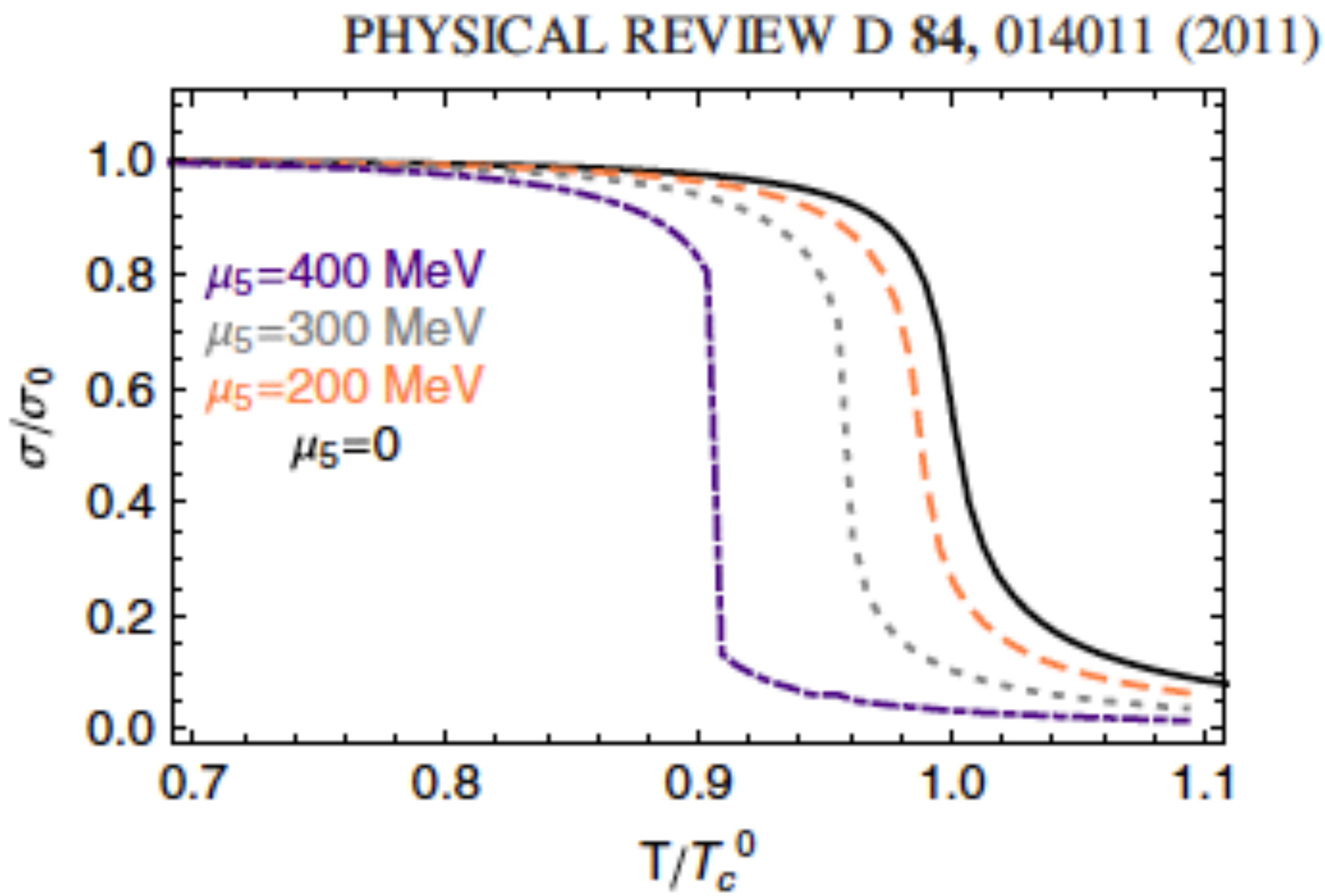
V.V. Braguta et al. JHEP 06 (2015) 094

SU(3) color



V.V. Braguta et al. Phys. Rev. D 93, 034509 (2016)

Chiral quark model



M. Ruggieri

NJL model

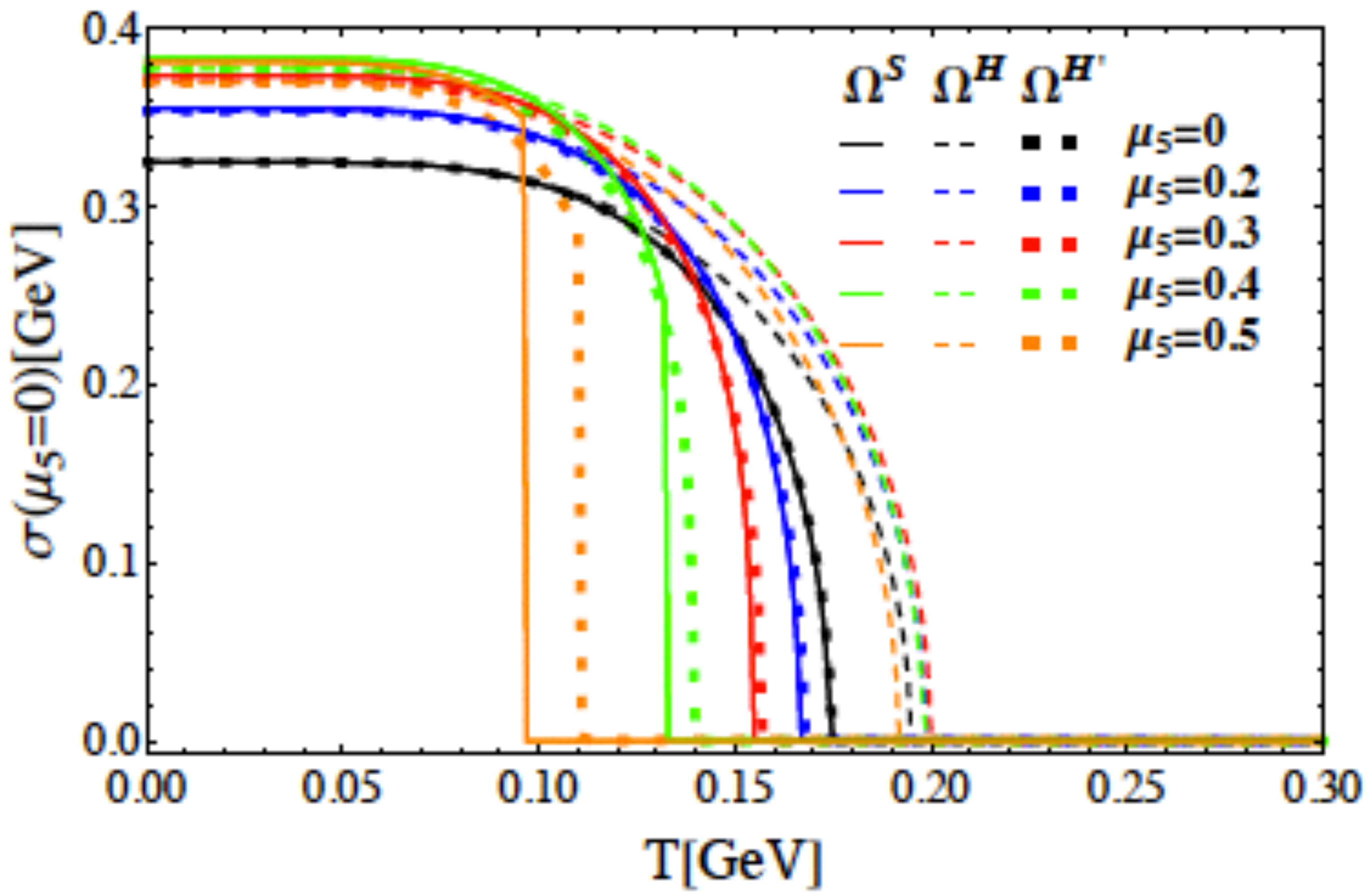


FIG. 2. The dynamical quark mass σ as a function T at $\mu_5 = 0,\ 0.2,\ 0.3,\ 0.4$ and 0.5 for Ω^S, Ω^H and $\Omega^{H'}$.

L. Yu, H. Liu and M. Huang, arXiv:1511.03073

Chiral quark model

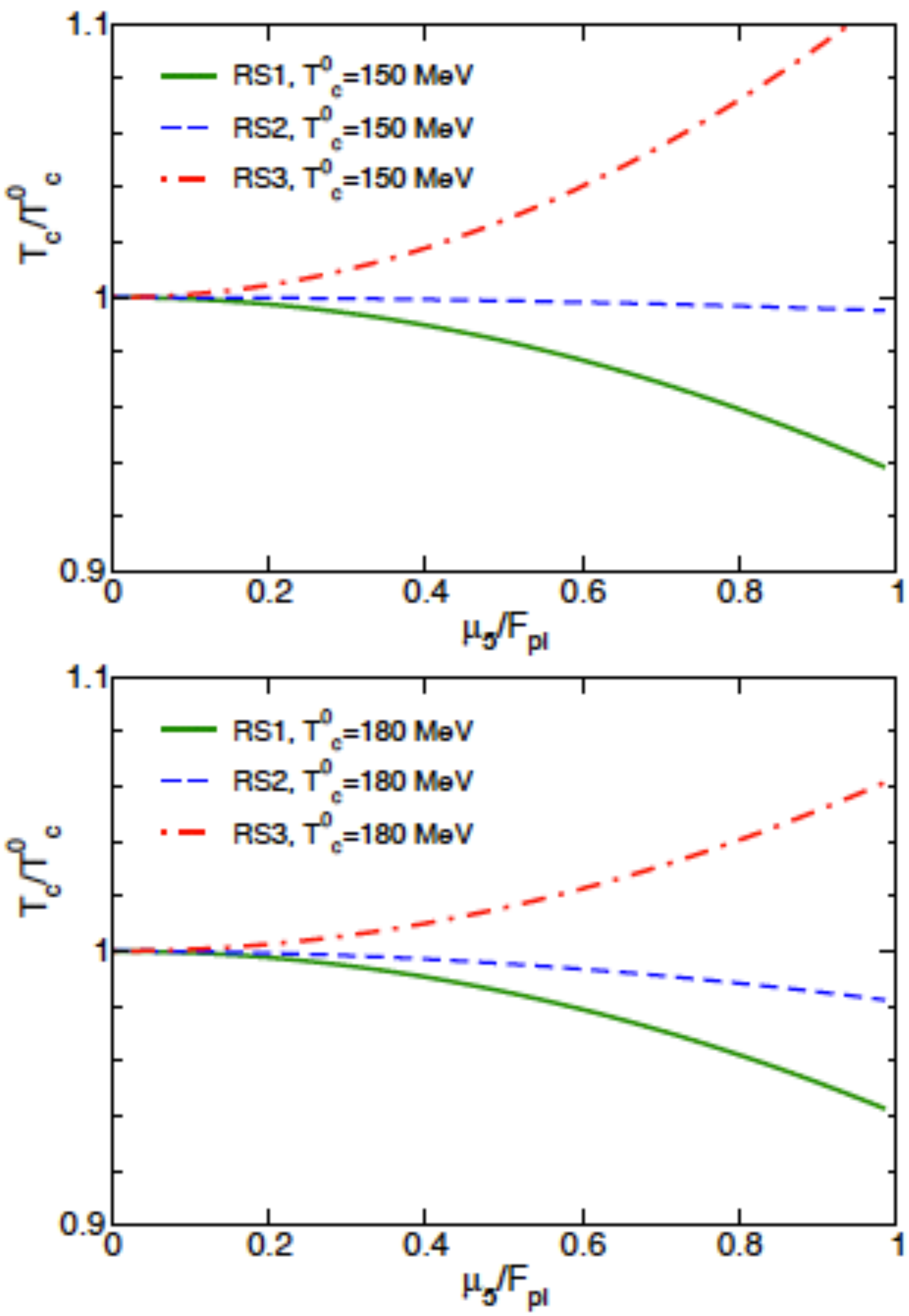


Figure 1: Critical temperature as a function of μ_5 for the three renormalization schemes discussed in this article: upper panel corresponds to $T_c^0 = 150$ MeV and lower panel to $T_c^0 = 180$ MeV.

$$T_c(\mu_5) = T_c^0 + \mu_5^2 \frac{3}{\pi^2 T_c^0}\left(\xi + \gamma_E + \frac{1}{2} - \log\frac{\pi T_c^0}{F_\pi}\right)$$

$$\begin{aligned} \xi &= 0, \quad \mathrm{RS1}, \\ \xi &= \frac{1}{2}, \quad \mathrm{RS2}, \\ \xi &= \frac{3}{2}, \quad \mathrm{RS3}. \end{aligned}$$

M. Ruggieri & G. X. Peng, arXiv:1602.03651

DSE — modified Maris-Tandy interaction

PHYSICAL REVIEW D 91, 056003 (2015)

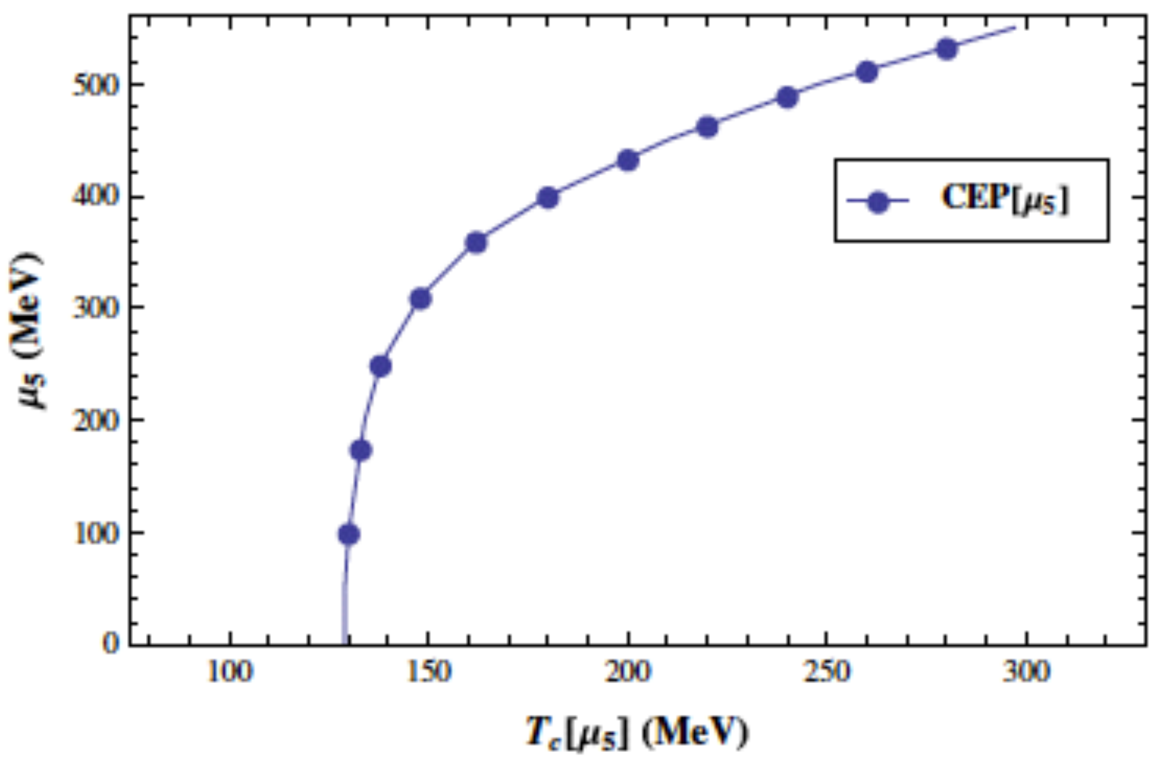


FIG. 3 (color online). The relation between μ_5 and the corresponding $T_c[\mu_5]$ in the $T - \mu$ plane.

Shu-Sheng Xu et al. PRD 91, 056003 (2015)

Chiral chemical potential

— How to make NJL agree with lattice*

NJL Lagrangian:

$$\mathcal{L} = \bar{\psi}\left(i\,\partial\!\!\!/ + \mu_5\gamma^0\gamma^5\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau\psi\right)^2\right]$$

Gap equation:

$$\frac{1}{4N_cN_fG} = \sum_{s=\pm1}\left\{\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{i}{k_0^2-(k+s\mu_5)^2-M^2} - \int_0^\infty \frac{d^3k}{(2\pi)^3}\frac{1}{1+e^{-\beta\omega_s}}\frac{1}{\omega_s}\right\}$$

$$\omega_s = \sqrt{M^2+(k+s\mu_5)^2}, \qquad s=\pm1$$

* R.L.S. Farias, D.C. Duarte, GK, R.O. Ramos

Use the identity (3 times)

$$
\frac{1}{k_0^2-(k\pm\mu_5)^2-M^2} = \frac{1}{k_0^2-k^2-M_0^2}
$$

$$
+ \frac{M^2-M_0^2\pm 2k\mu_5\mu_5^2}{(k_0^2-k^2-M_0^2)\left[k_0^2-(k\pm\mu_5)^2-M^2\right]}
$$

and obtain for the gap equation

$$
\begin{aligned}
\frac{1}{4N_cN_fG} &= 2\,I_{\rm quad}\left(M_0^2\right)+2(M^2-M_0^2-2\mu_5^2)\,I_{\rm log}\left(M_0^2\right) \\
&+ \frac{1}{16\pi^2}\left(M^2-M_0^2+\mu_5^2\right)^2-\frac{\mu_5^2}{4\pi^2} \\
&+ I_{fin}(M^2,M_0^2,\mu_5)+I_{fin}(M^2,M_0^2,-\mu_5)-I_{med}(M,T,\mu_5)
\end{aligned}
$$

where

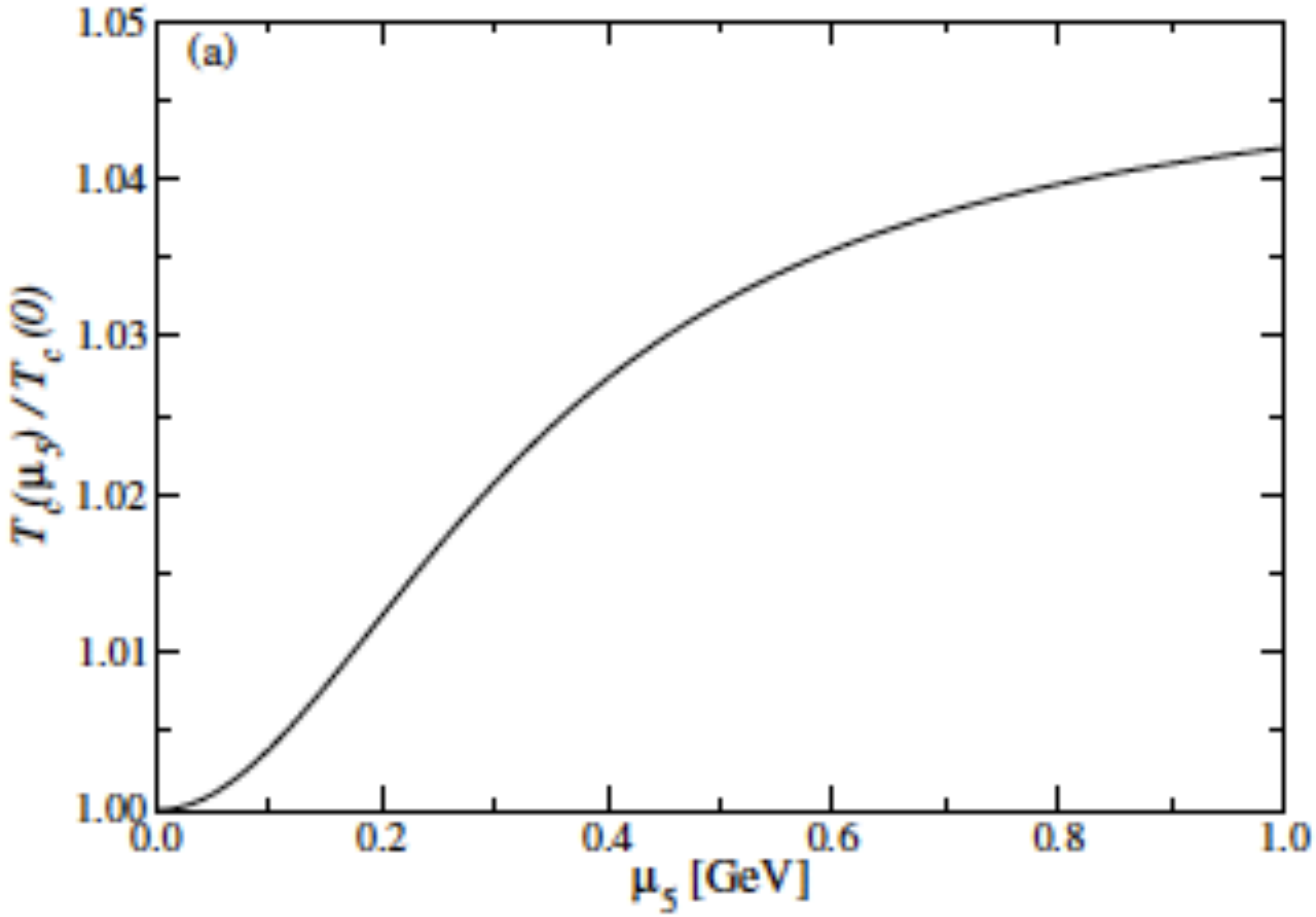
$$I_{\text{quad}}(M_0^2) = i\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k_0^2 - k^2 - M_0^2} \qquad I_{\log}(M_0^2) = i\int_\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{(k_0^2 - k^2 - M_0^2)^2}$$

$$I_{fin}(M^2, M_0^2, \mu_5) = i\int \frac{d^4k}{(2\pi)^4}\frac{(M^2 + \mu_5^2 + 2k\mu_5 - M_0^2)^3}{(k_0^2 - k^2 - M_0^2)^3[k_0^2 - (k+\mu_5)^2 - M^2]}$$

$$I_{med}(M, T, \mu_5) = \sum_{s=\pm 1}\int_0^\infty \frac{d^3k}{(2\pi)^3}\frac{1}{1 + e^{-\beta\omega_s}}\frac{1}{\omega_s}$$

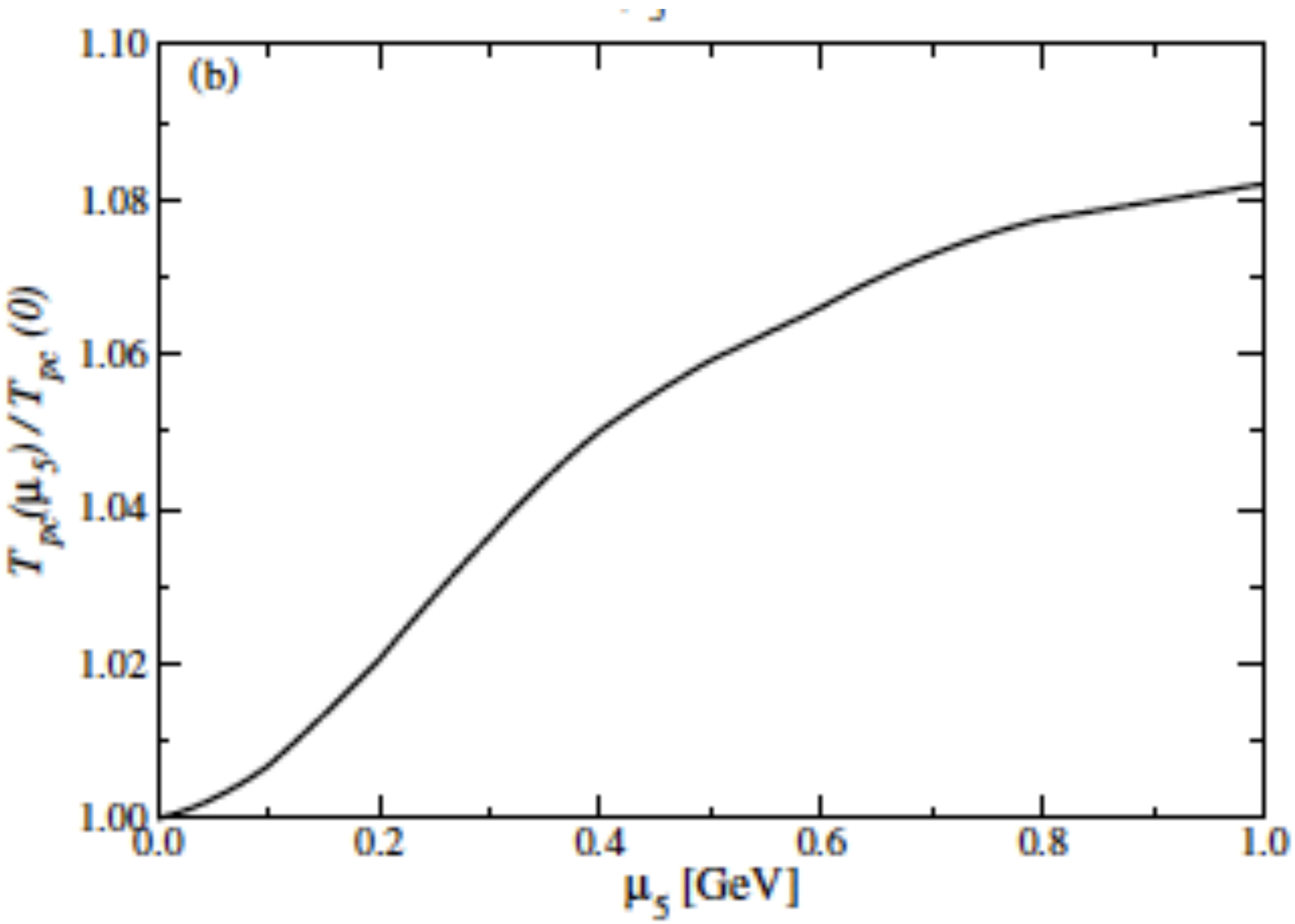
Critical temperature

— chiral limit $m_0 = 0$

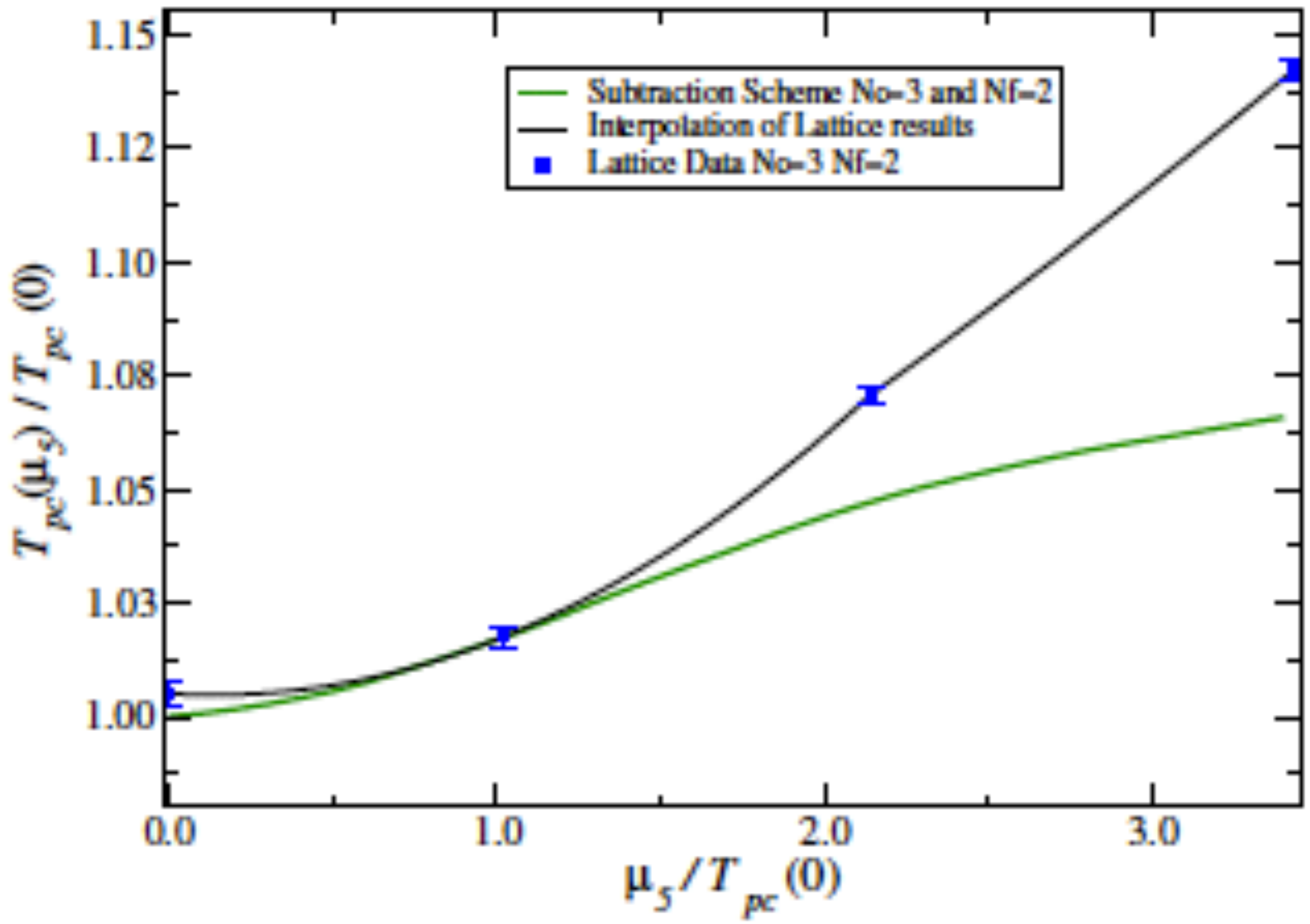


Critical temperature

— $m_0 = 5$ MeV



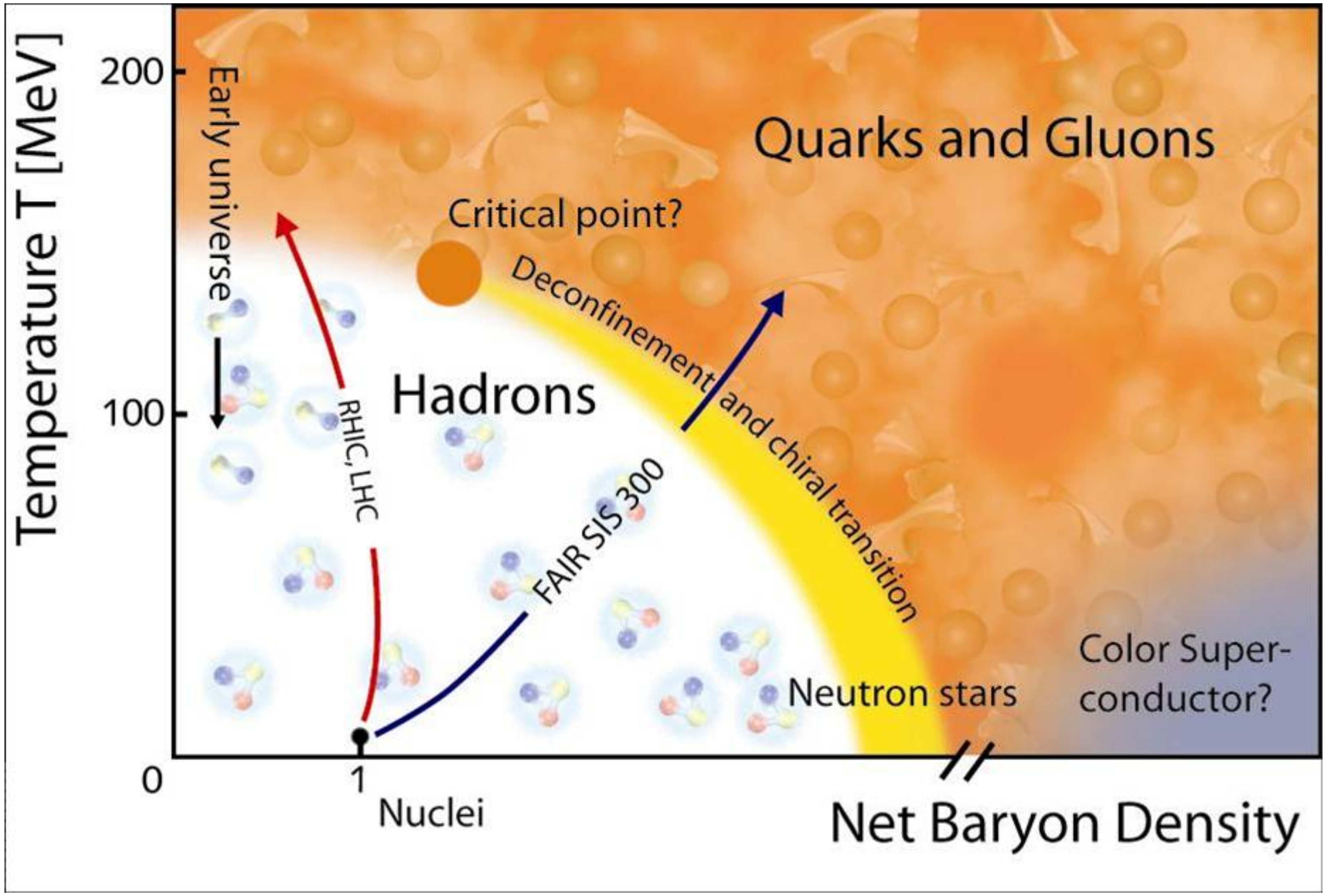
Comparison with lattice



Color superconductivity

High-density quark matter

— instability around the Fermi surface, gap in the energy spectrum



Perturbative QCD

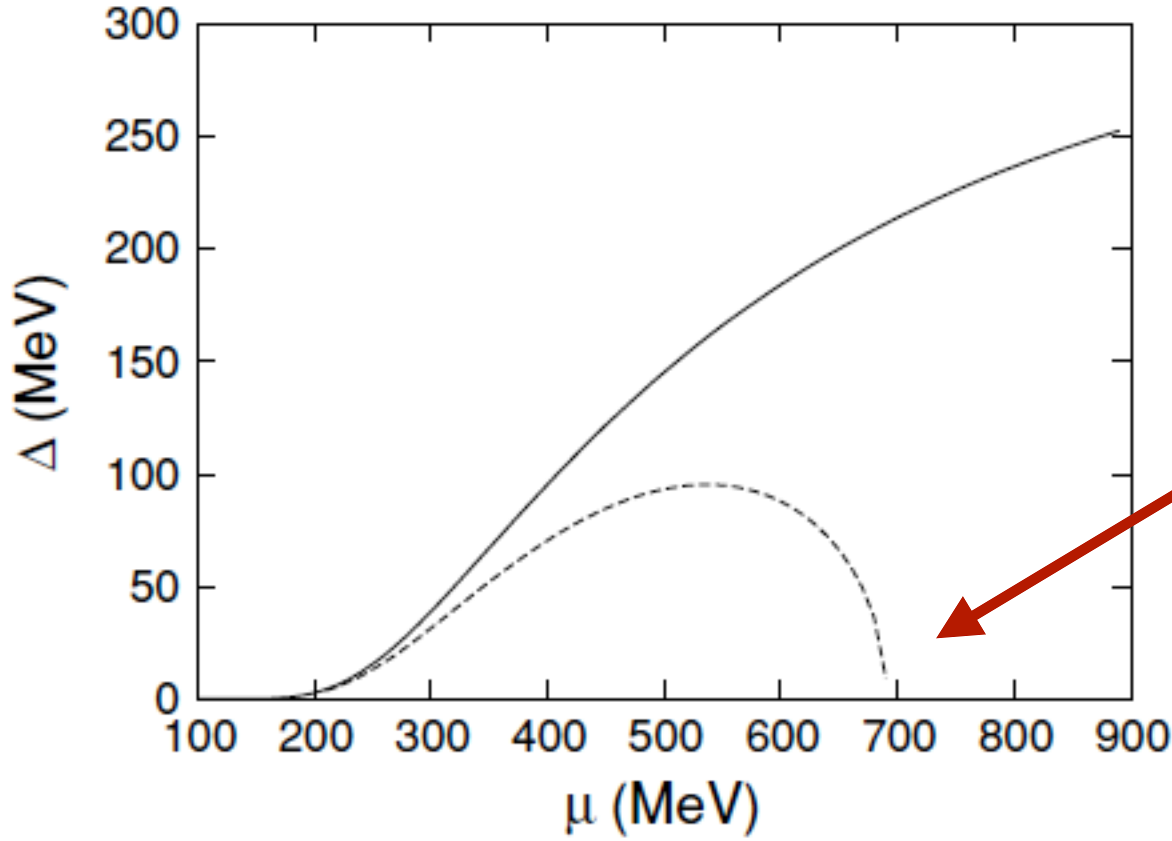
— gap increases with chemical potential

$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

$$g = g(\mu)$$

Superconducting gap

$$1 = \lambda G i \int_\Lambda \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k_0^2 - (k+\mu)^2 - \Delta^2} + (\mu \to -\mu)\right]$$



NJL model

For $\Delta \sim \Lambda$
- gap vanishes

Chiral symmetry is restored

Separate finite and divergent parts

$$1 = \lambda G i \int_\Lambda \frac{d^4k}{(2\pi)^4}\left[\frac{1}{k_0^2 - (k+\mu)^2 - \Delta^2} + (\mu \to -\mu)\right]$$

$$= \lambda\, G[2I_1(M^2, \Delta^2) - 4\mu^2 I_2(M^2, \Delta^2) \quad + I_{\text{fin}}(\Delta^2, \mu) + I_{\text{fin}}(\Delta^2, -\mu)]$$

where

$$I_1(M^2,\Delta^2) = -\frac{\langle\bar{\psi}\psi\rangle}{12M} - (\Delta^2 - M^2)\frac{f_\pi^2}{12M^2} - \frac{1}{(4\pi)^2}\left[\Delta^2 - M^2 - \Delta^2 \ln\left(\frac{\Delta^2}{M^2}\right)\right]$$

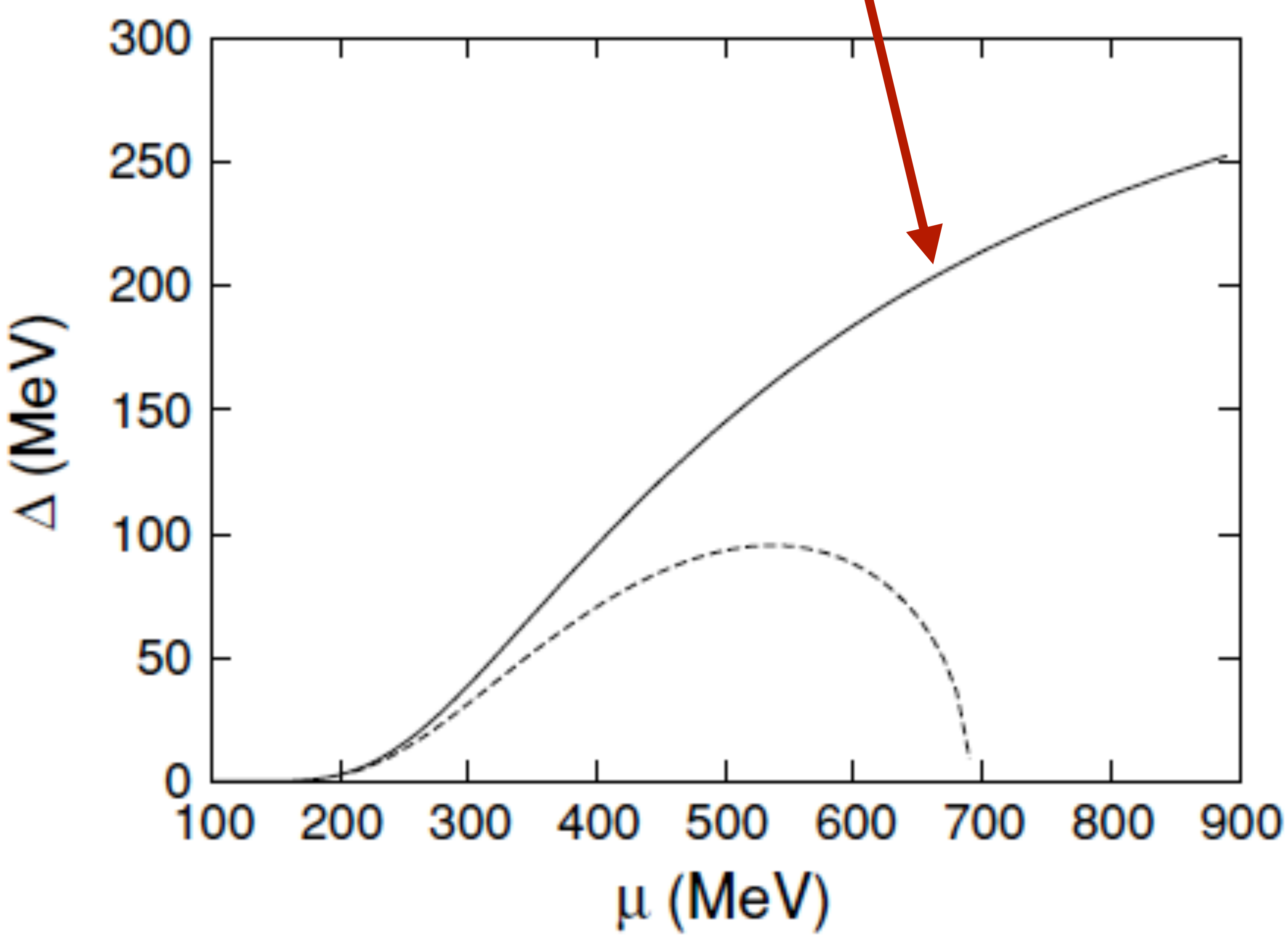
$$I_2(M^2,\Delta^2) = \frac{f_\pi^2}{12M^2} - \frac{1}{(4\pi)^2}\ln\left(\frac{\Delta^2}{M^2}\right)$$

$$I_{\text{fin}}(\Delta^2,\mu) = i\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left(k_0^2 - k^2 - \Delta^2\right)^3}\left[2\mu^2(\mu^2 - 4\Delta^2) + \frac{(\mu^2 + 2k\mu)^3}{\left(k_0^2 - (k+\mu)^2 - \Delta^2\right)}\right]$$

Result

pert QCD $g = g(\mu)$

$$\boxed{\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)}$$



Δ grows with μ

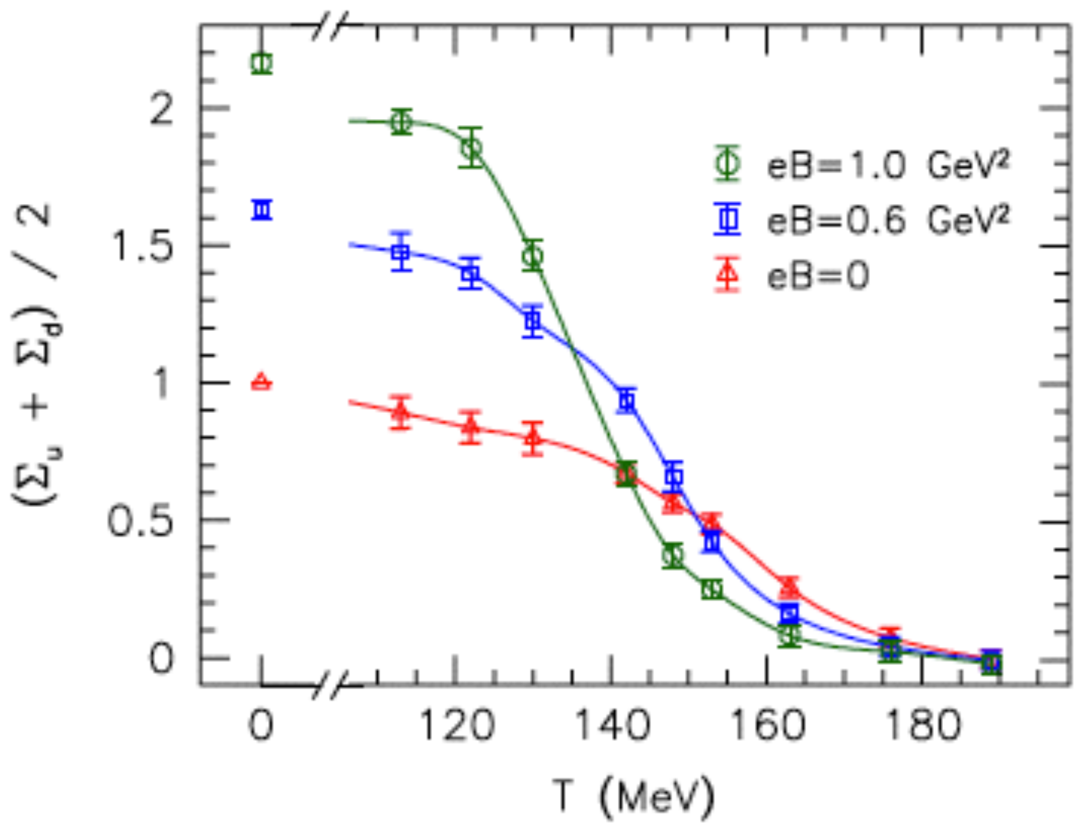
R.L.S. Farias, G. Dallabona, GK, O.A. Battitel, PRC 73, 018201 (2006)

How about asymptotic freedom?

— change the coupling

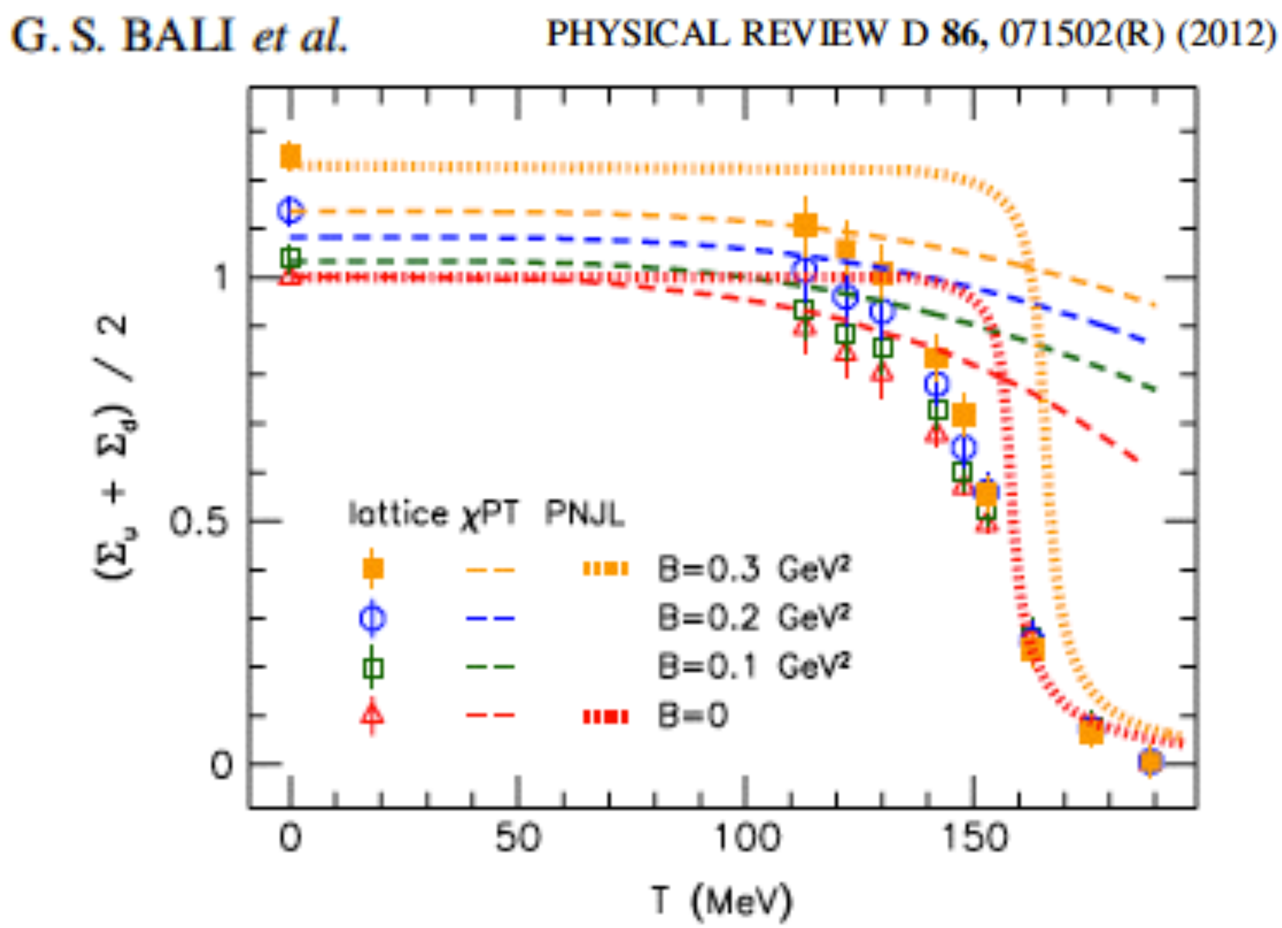
Example

— inverse magnetic catalysis



(pseudo)critical temperature for chiral restoration decreases as B increases

Models (NJL, PNJL, LSM, …) predict opposite effect



Asymptotic freedom

$$\boxed{\frac{1}{\alpha_s} \sim b \ln \frac{|eB|}{\Lambda^2_{QCD}}}$$

V. A. Miransky and I. A. Shovkovy, Phys. Rev. D **66**, 045006 (2002)

Change the NJL coupling*

— rough fit to lattice data for condensates

$$G(B,T) = G(B)\left(1 - \gamma \frac{|eB|}{\Lambda^2_{\rm QCD}} \frac{T}{\Lambda_{\rm QCD}}\right)$$

$$G(B) = \frac{G_0}{1 + \alpha \ln\left(1 + \beta \frac{eB}{\Lambda^2_{\rm QCD}}\right)}$$

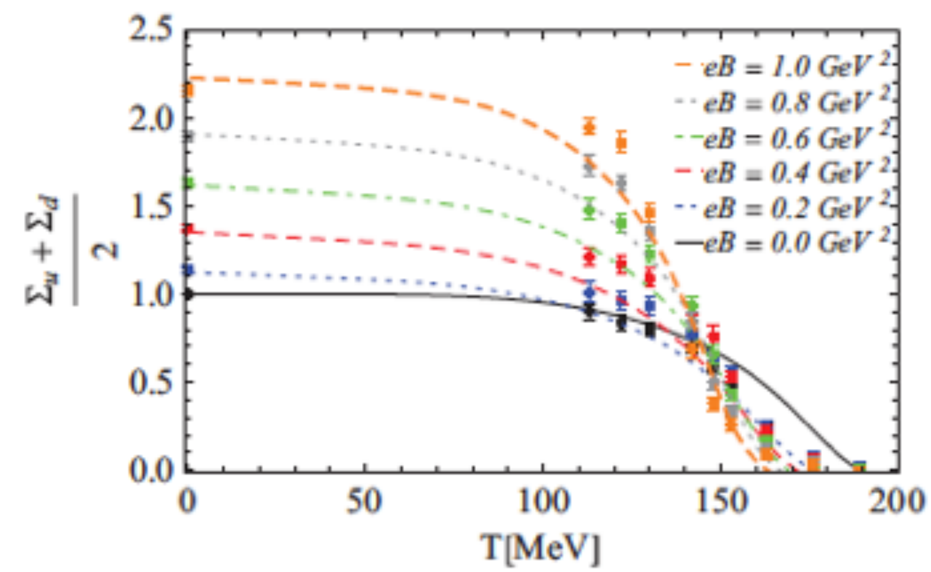


FIG. 1. (Color online) The condensate average as a function of the temperature. Data points are from the lattice simulations of Ref. [13].

* R.L.S. Farias, K.P. Gomes, G. Krein and M.B. Pinto, Phys. Rev. C **90**, 025203 (2014)

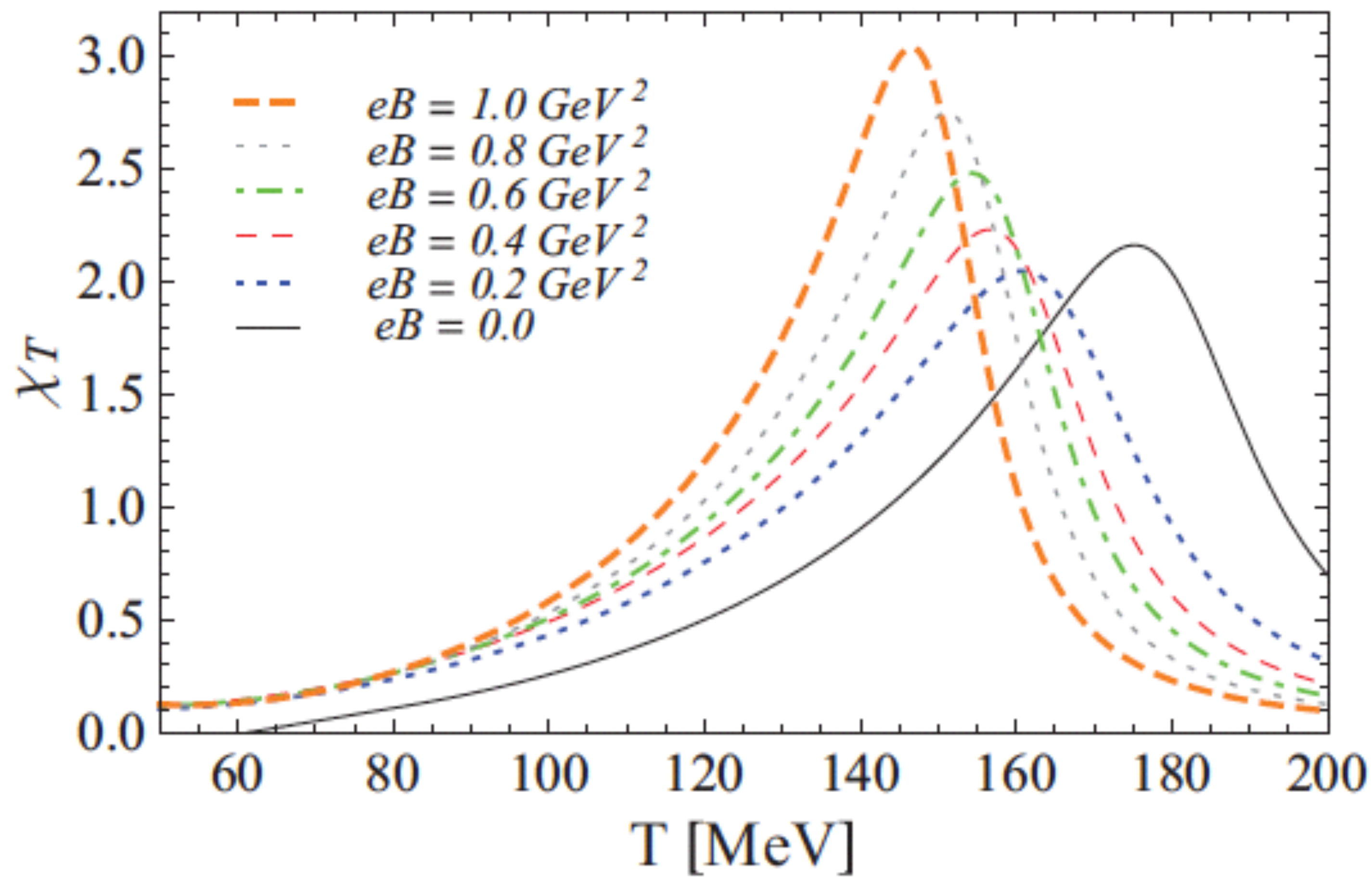


FIG. 3. (Color online) The normalized thermal susceptibility as a function of the temperature for different values of the magnetic field B.

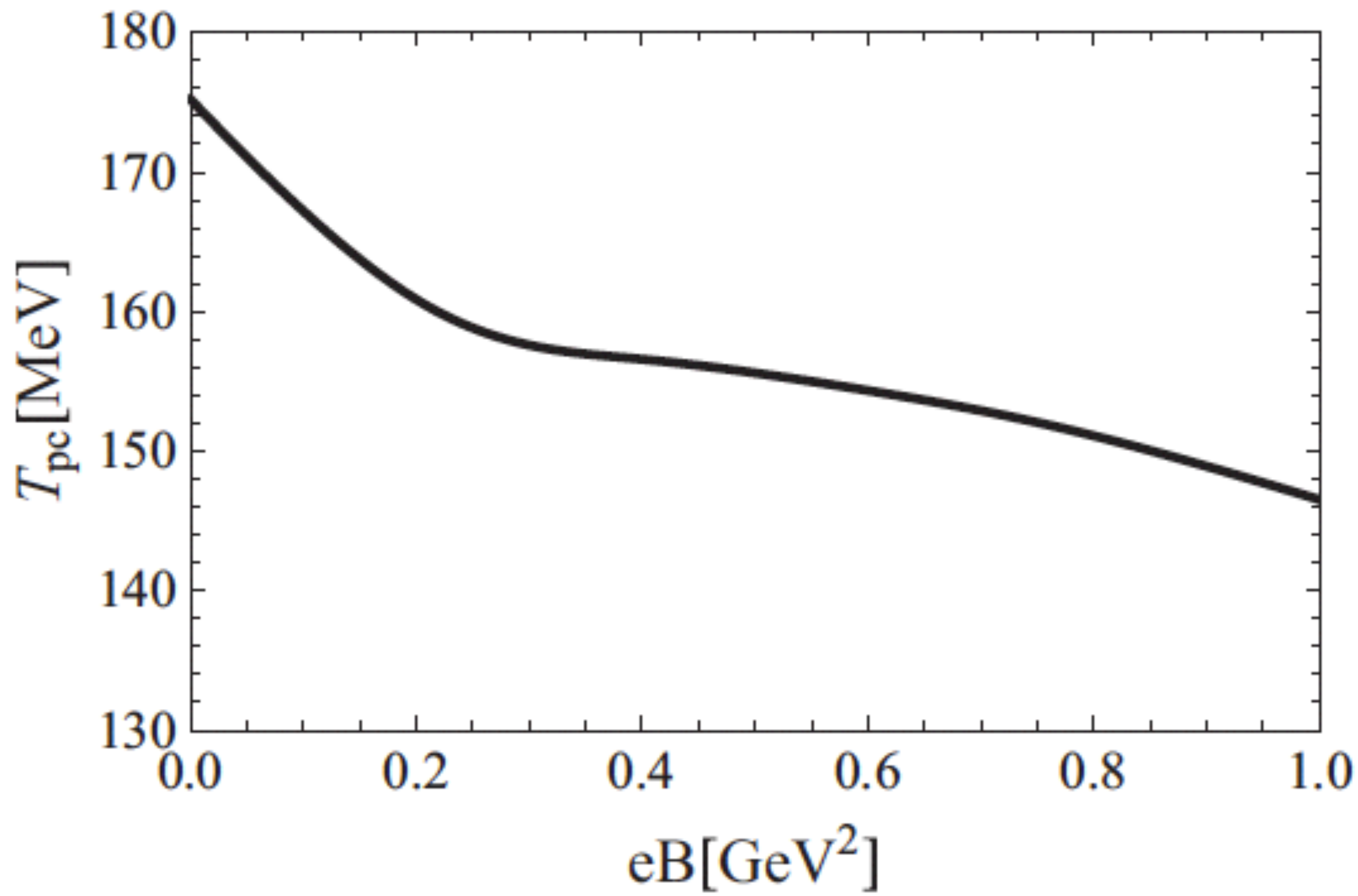
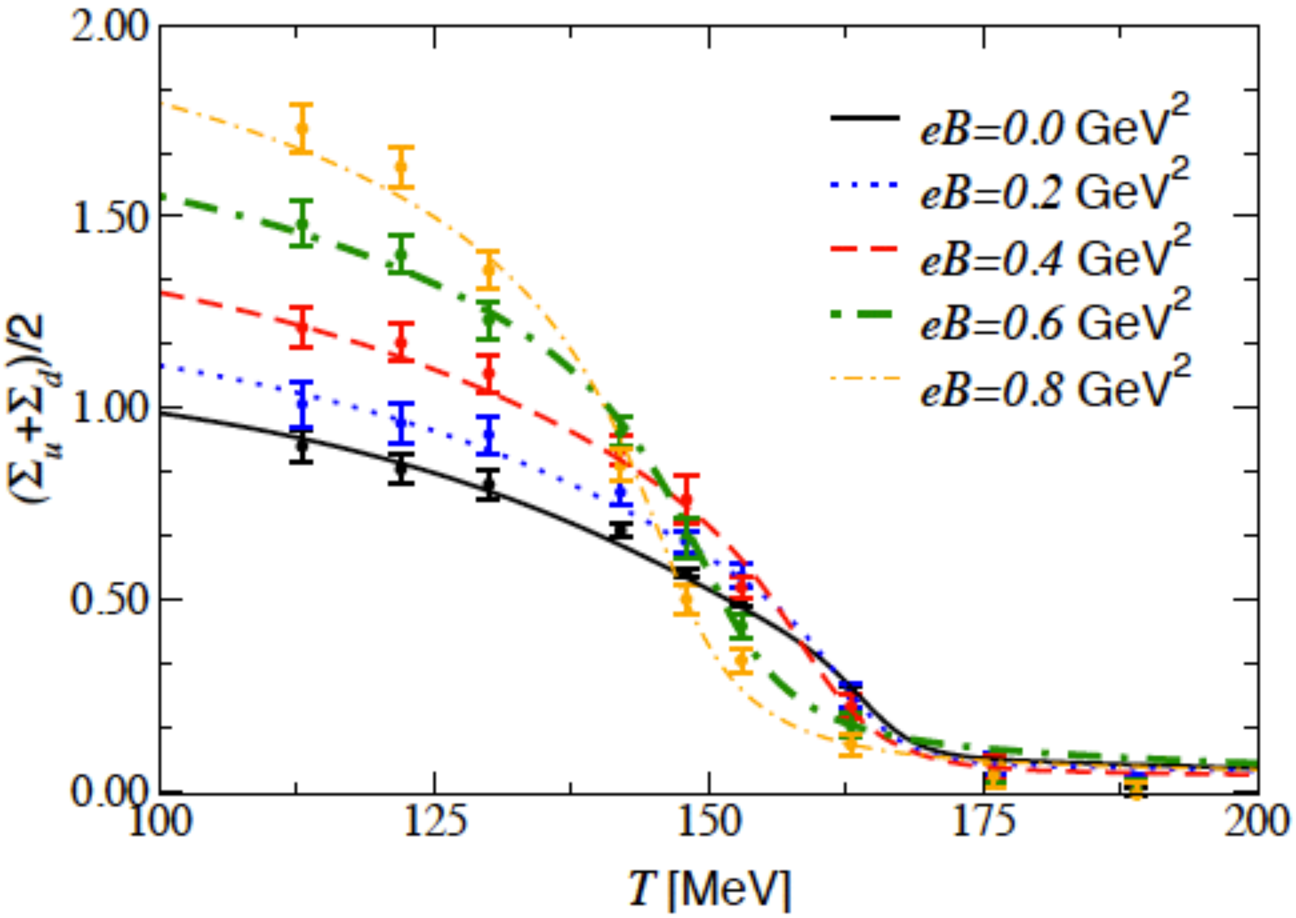


FIG. 4. The pseudocritical temperature for the chiral transition of magnetized quark matter as a function of magnetic field.

Change the NJL coupling*

— more precise fit to lattice data for condensates



* R.L.S. Farias, V.S. Tomoteo, S.S. Avancini, M.B. Pinto, GK

Predictions for thermodynamics

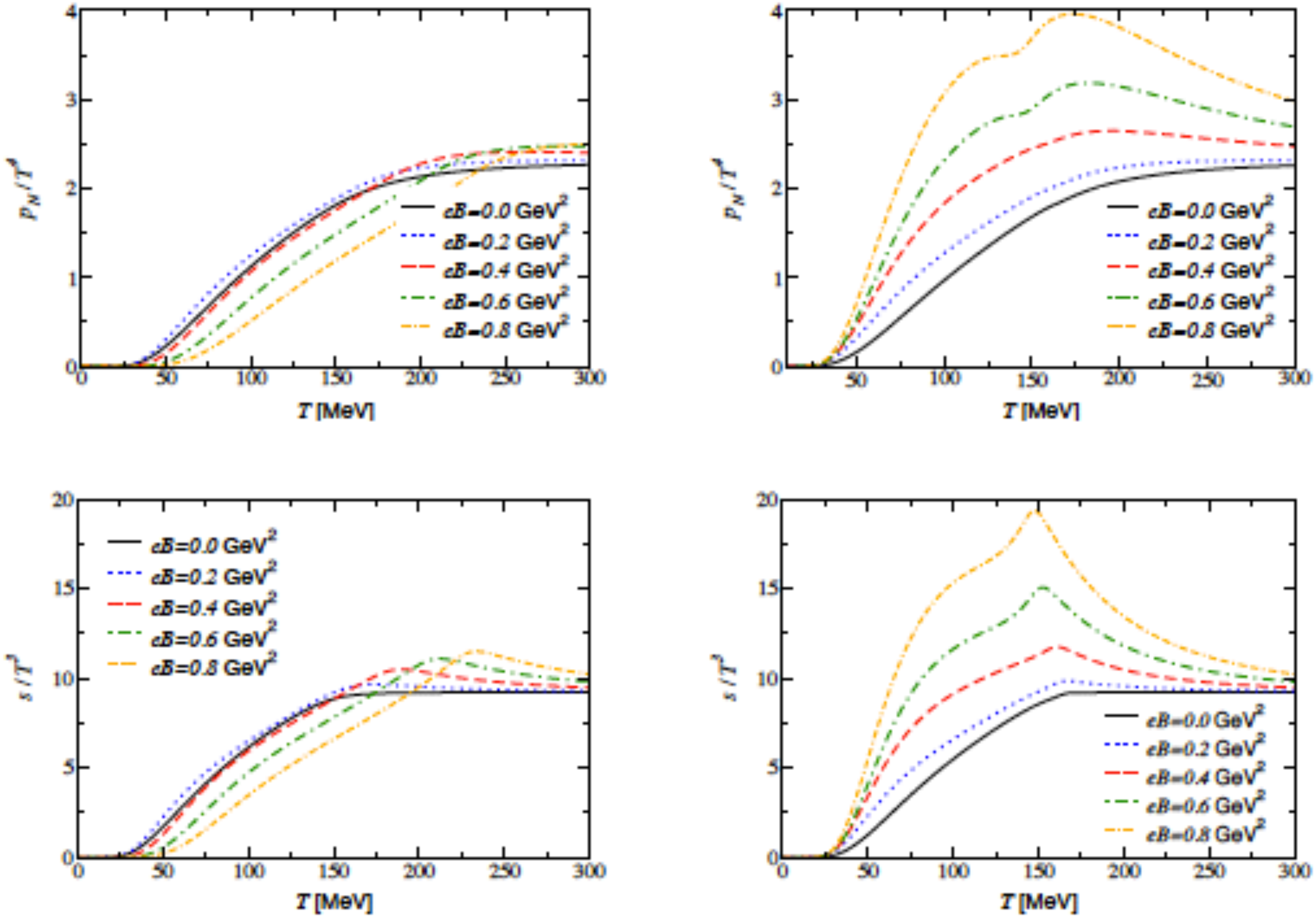


Figure 3: Normalized pressure and entropy density as functions of temperature for different values of the magnetic field calculated with G (left) and $G(B,T)$ (right).

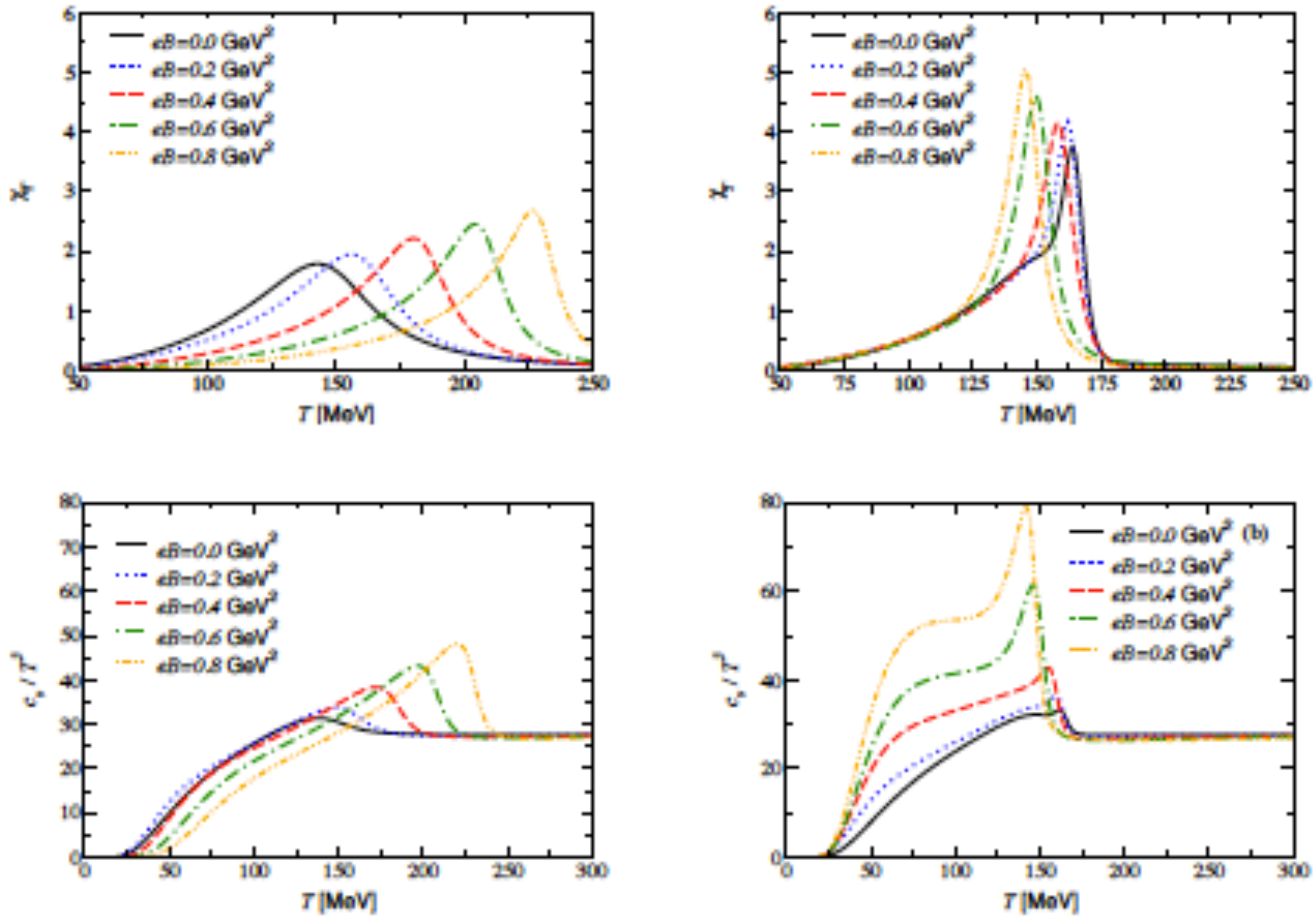


Figure 5: The thermal susceptibility and specific heat as functions of temperature for different values of the magnetic field obtained with G (left) and $G(B,T)$ (right).

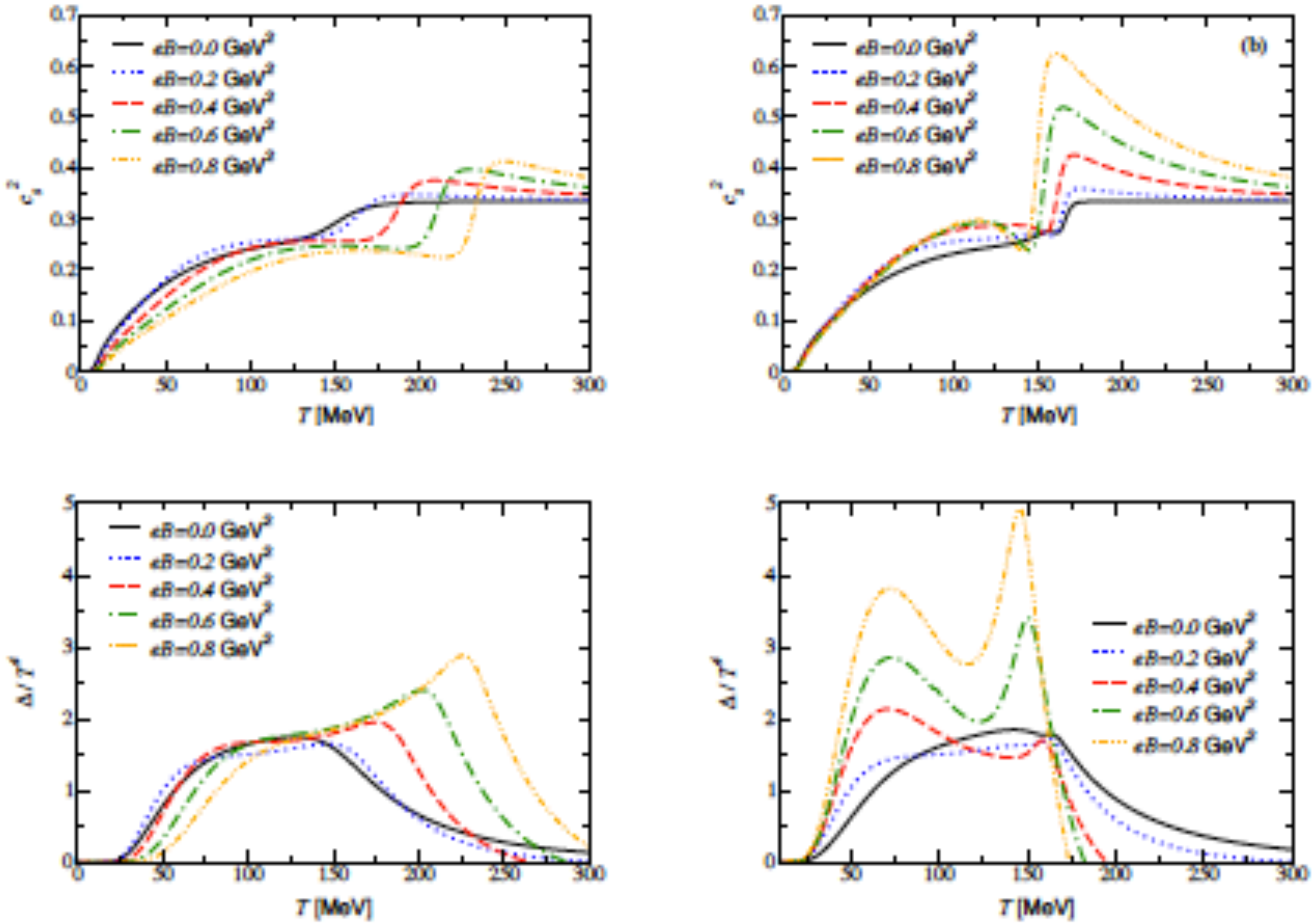


Figure 6: The sound velocity squared and interaction measure as functions of temperature for different values of the magnetic field obtained with G (left) and $G(B,T)$ (right).

Perspectives

— NJL is not a replacement of full QCD, of course

— Symmetries can be preserved

— Confinement, asymptotic freedom can be mimicked