Parity-Expanded Variational Analysis for Nonzero Momentum

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Introduction

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- Existing parity projection techniques are vulnerable to opposite parity contaminations at nonzero momentum
- We propose the Parity Expanded Variational Analysis (PEVA) technique to resolve this issue

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- Call states that transform negatively under parity in their rest frame "negative parity states" (B^-)

Conventional analysis

- Conventional baryon operators $\{\chi^i\}$ couple to states of both parities

$$\langle\Omega|\chi^i|B^+\rangle = \lambda_i^{B^+}\sqrt{\frac{m_{B^+}}{E_{B^+}}}\, u_{B^+}(p,s)$$

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&= \sum_{B^+,s} \mathrm{e}^{-E_{B^+}(\mathbf{p})\,t}\ \langle\Omega|\,\chi^i\,|B^+\rangle\,\langle B^+|\,\overline{\chi}^j\,|\Omega\rangle \\
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&- \sum_{B^-, s} \mathrm{e}^{-E_{B^-}(\mathbf{p})\, t}\, \lambda_i^{B^-}\, \overline{\lambda}_j^{B^-}\, \frac{m_{B^-}}{E_{B^-}(\mathbf{p})}\, \gamma_5\, u_{B^-}(p, s)\, \overline{u}_{B^-}(p, s) \gamma_5
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- Introduce $\Gamma_\pm = (\gamma_4 \pm \mathbb{I})/2$ and define $G_{ij}(\Gamma_\pm; \mathbf{p};\, t) := \mathrm{tr}\left(\Gamma_\pm\, \mathcal{G}_{ij}(\mathbf{p};\, t)\right)$

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Zero momentum

- At zero momentum, $E_B(\mathbf{0}) = m_B$, so

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- Can analyse states of each parity independently

Lattice results

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$$\chi_1 = \epsilon^{abc}\left[u^{a\top}(C\gamma_5)\,d^b\right]u^c$$
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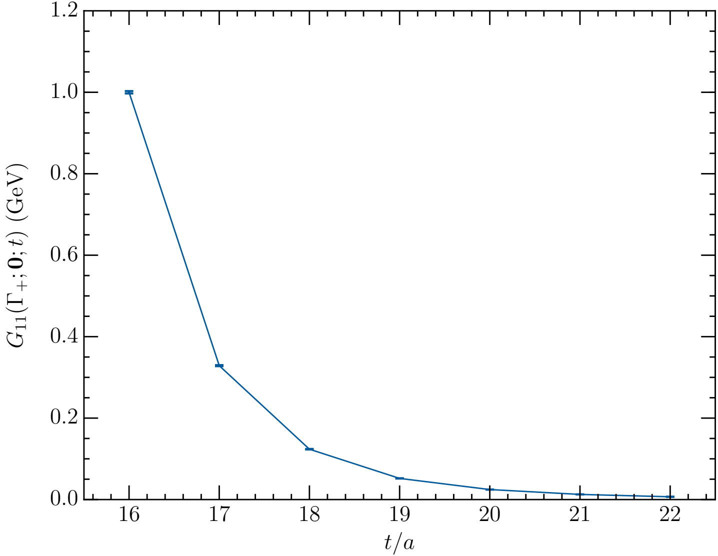
- Apply 16, 35, 100 and 200 sweeps of stout-link smearing in creating the propagators

Correlation function

$G_{11}(\Gamma_+; \mathbf{0};\ t)$



Nonzero momentum

$$G_{ij}(\Gamma_+;\mathbf{p};\ t) = \sum_{B^+} \mathrm{e}^{-E_{B^+}(\mathbf{p})\,t}\,\lambda_i^{B^+}\,\overline{\lambda}_j^{B^+}\,\frac{E_{B^+}(\mathbf{p}) + m_{B^+}}{2E_{B^+}(\mathbf{p})} + \sum_{B^-} \mathrm{e}^{-E_{B^-}(\mathbf{p})\,t}\,\lambda_i^{B^-}\,\overline{\lambda}_j^{B^-}\,\frac{E_{B^-}(\mathbf{p}) - m_{B^-}}{2E_{B^-}(\mathbf{p})}$$

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- $O(|\mathbf{p}|)$ opposite parity contaminations at nonzero momentum
- Can remove single opposite parity state with

$$\Gamma_\pm(\mathbf{p}) = \frac{1}{2}\left(\frac{m_{B^\mp}}{E_{B^\mp}(\mathbf{p})}\gamma_4 \pm \mathbb{I}\right)$$

Parity-Expanded Variational Analysis (PEVA)

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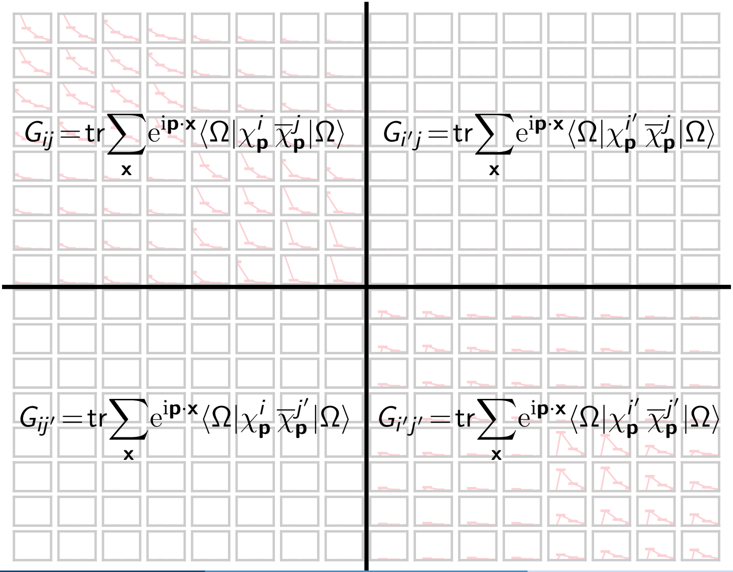
- Use $\{\chi^i_{\mathbf{p}}, \chi^{i'}_{\mathbf{p}}\}$ as expanded basis for variational analysis

PEVA correlation matrix

$\mathbf{p}^2 = 0\,\mathrm{GeV}^2$



$$G_{ij} = \mathrm{tr} \sum_{\mathbf{x}} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \Omega | \chi^i_{\mathbf{p}}\, \overline{\chi}^j_{\mathbf{p}} | \Omega \rangle$$

$$G_{i'j} = \mathrm{tr} \sum_{\mathbf{x}} \mathrm{e}^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \Omega | \chi^{i'}_{\mathbf{p}}\, \overline{\chi}^j_{\mathbf{p}} | \Omega \rangle$$

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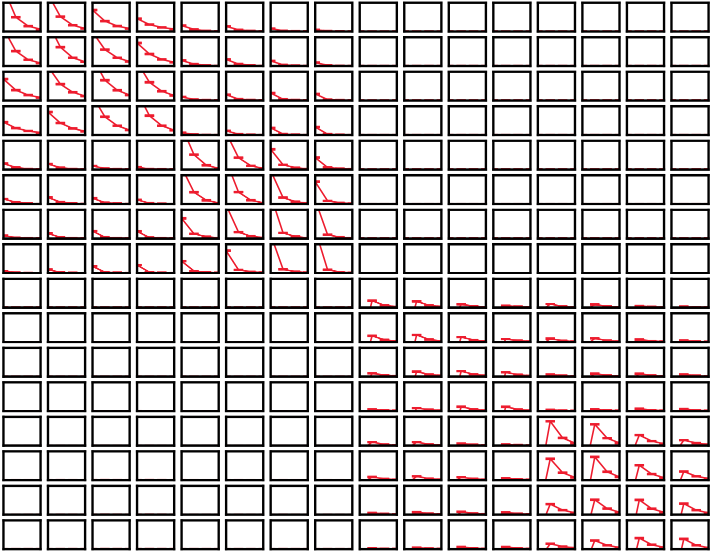
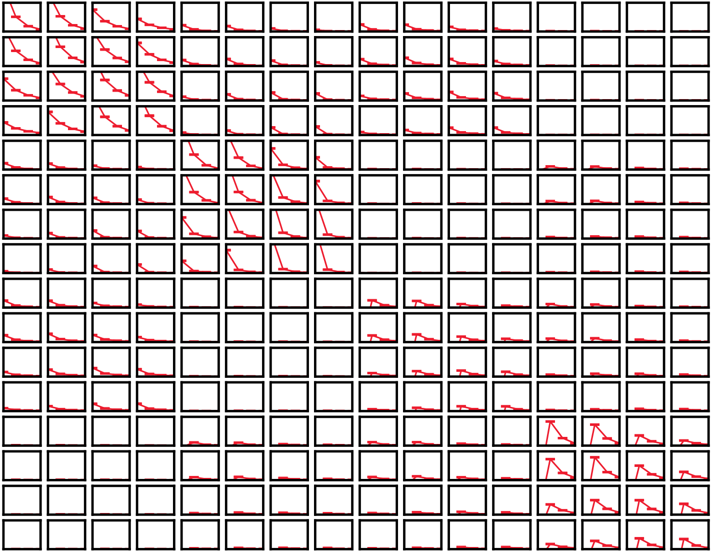


PEVA correlation matrix

$\mathbf{p}^2 = 0.166\,\mathrm{GeV}^2$



PEVA correlation matrix

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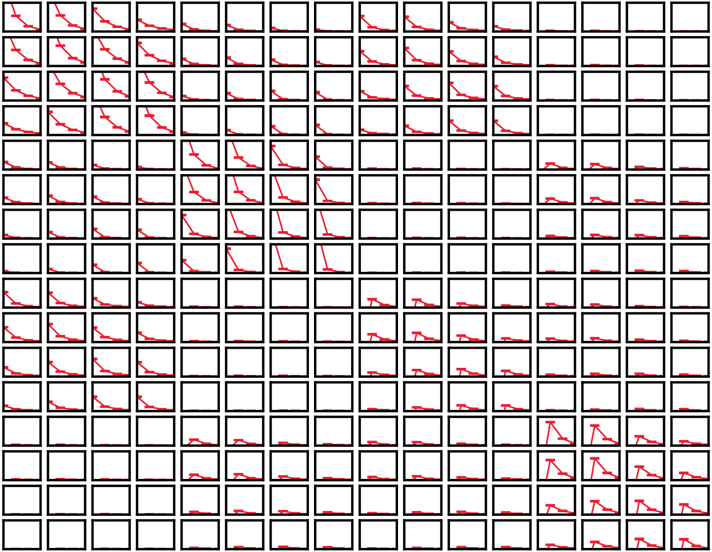

PEVA correlation matrix

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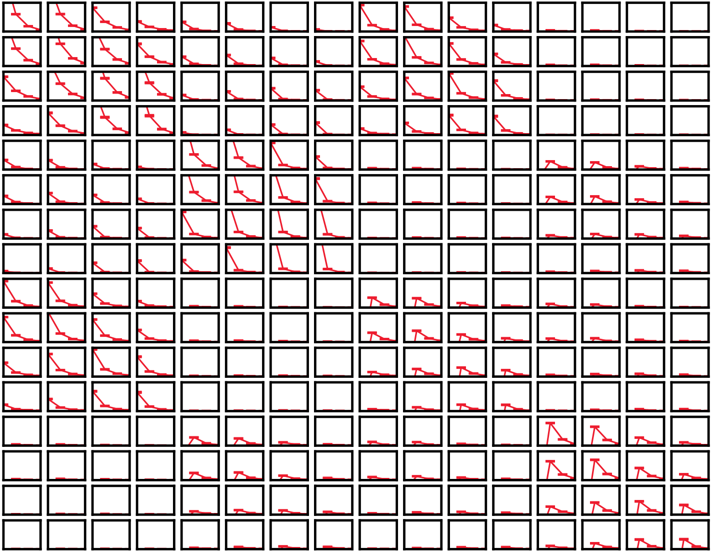
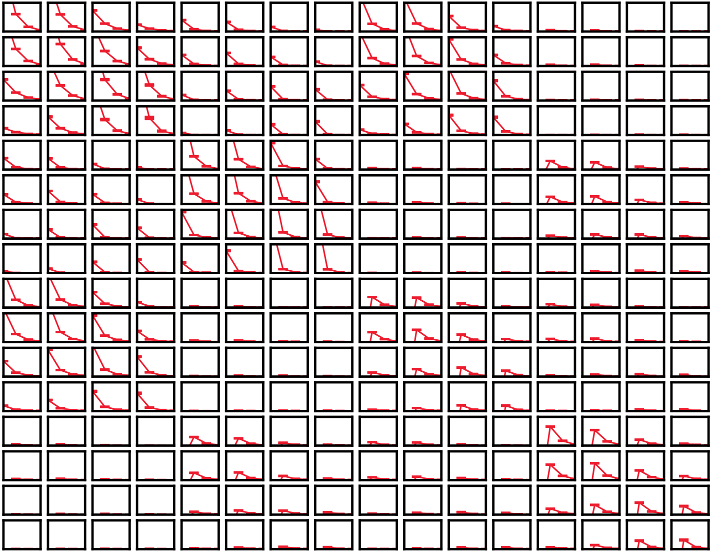
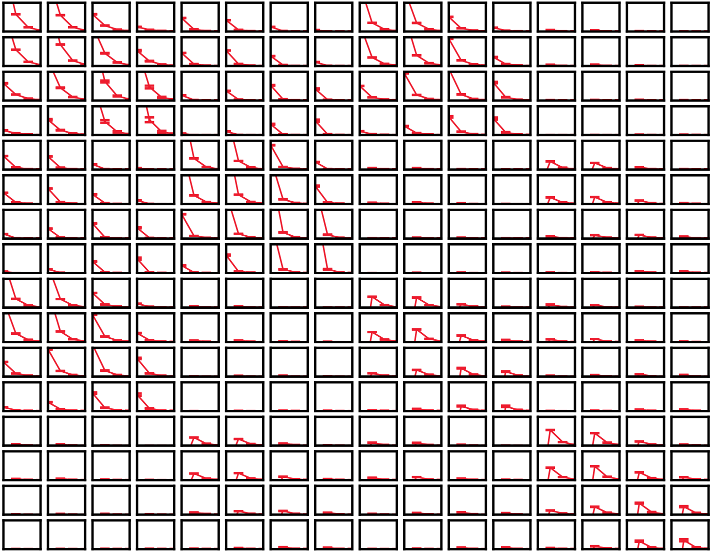


PEVA correlation matrix

$\mathbf{p}^2 = 0.664\,\mathrm{GeV}^2$



PEVA correlation matrix
$\mathbf{p}^2 = 0.830\,\mathrm{GeV}^2$



Effective energy

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$$
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G^\alpha(\mathbf{p};\, t) &:= \mathbf{v}^\alpha(\mathbf{p})\, G(\mathbf{p};\, t)\, \mathbf{u}^\alpha(\mathbf{p}) \\
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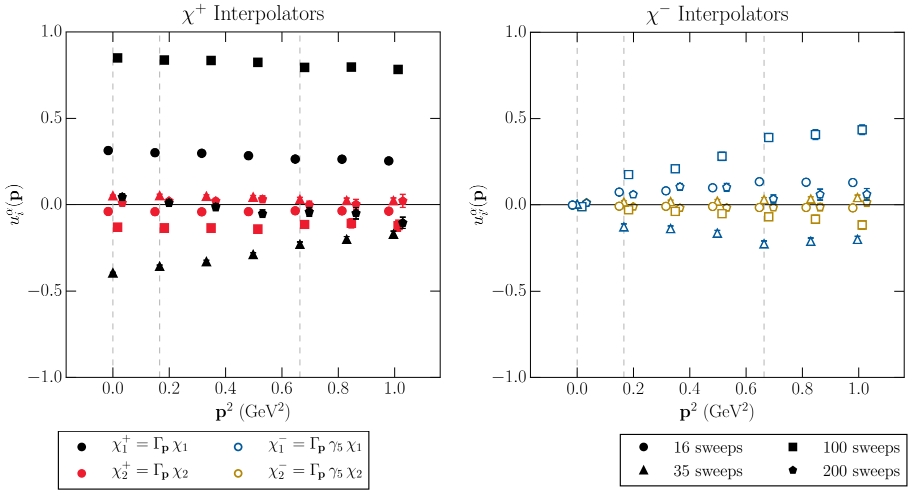
- Expect effective energy to approximately obey dispersion relation

$$E_\alpha^{\mathrm{eff}}(\mathbf{p}, t) \approx \sqrt{{m_\alpha}^2 + \mathbf{p}^2}$$

Eigenvector components

Ground state



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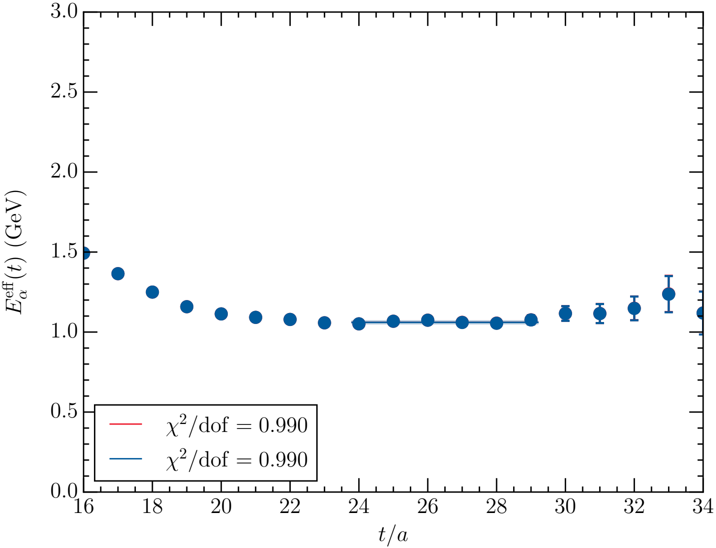
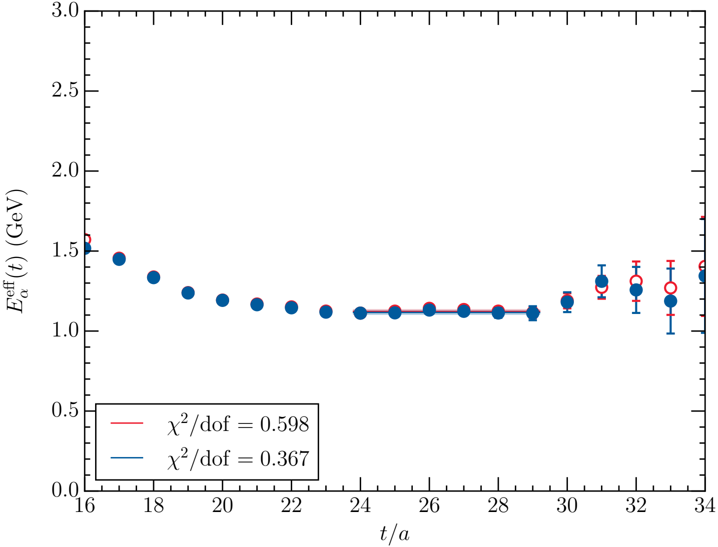


Effective energy

Ground state - $\mathbf{p}^2 = 0.166\,\mathrm{GeV}^2$



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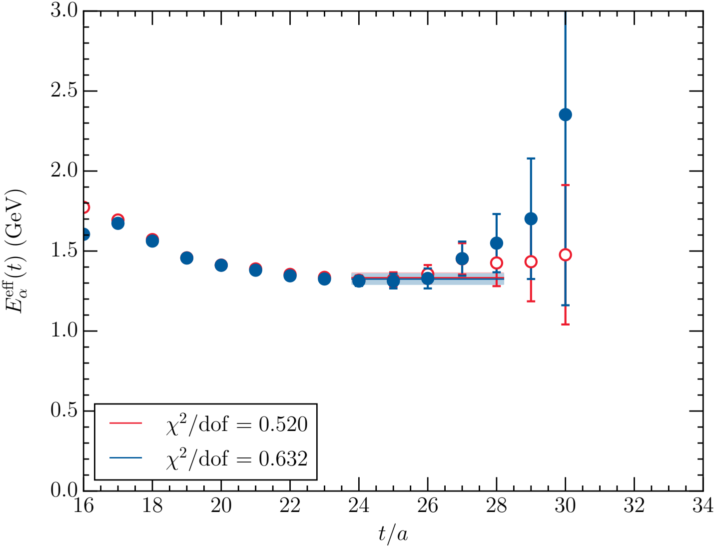
Ground state - $\mathbf{p}^2 = 0.664\,\mathrm{GeV}^2$



Effective energy
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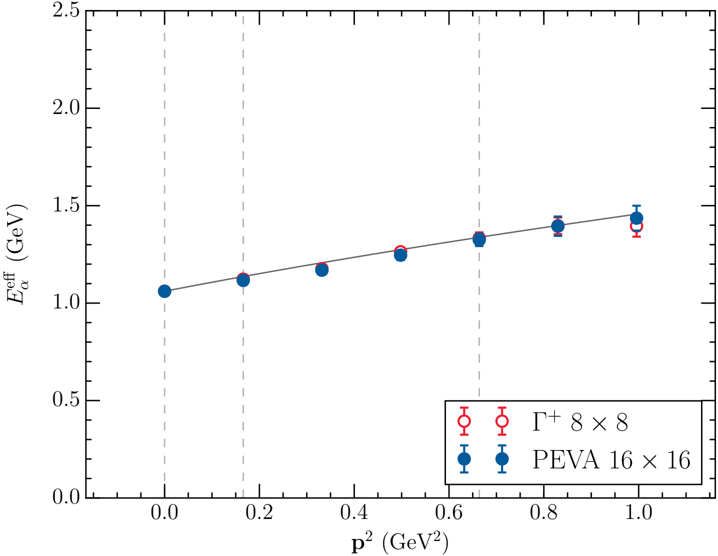
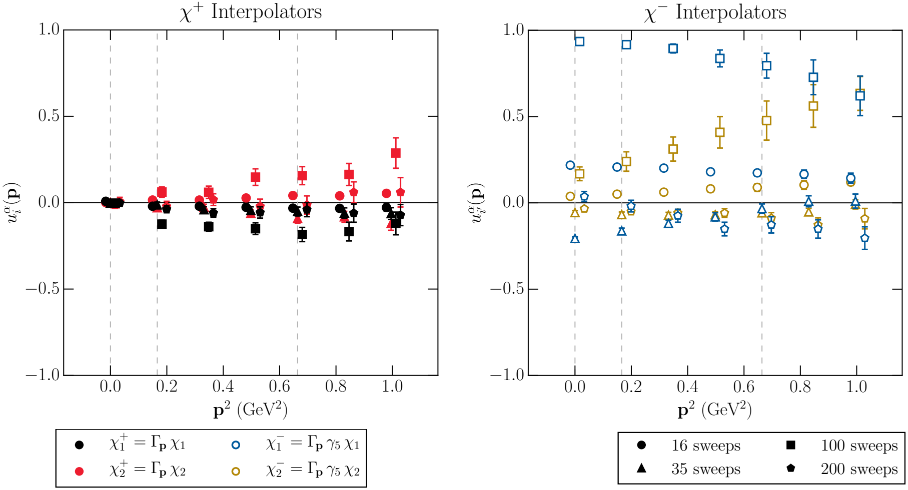


Eigenvector components

First negative parity excitation



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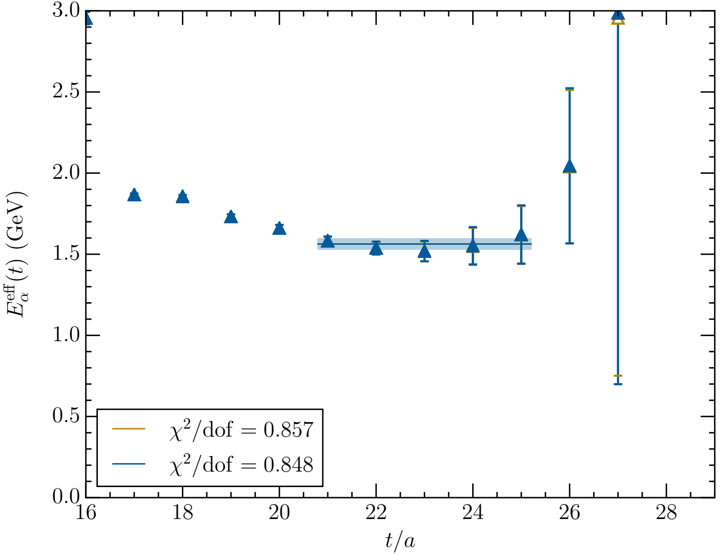

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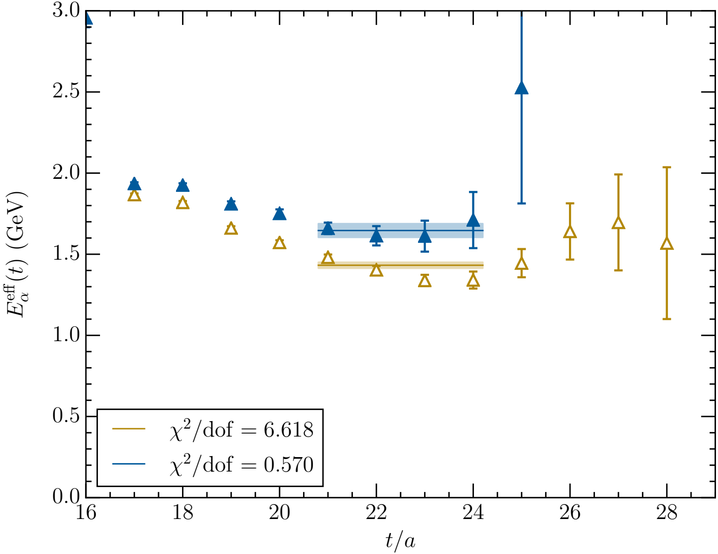

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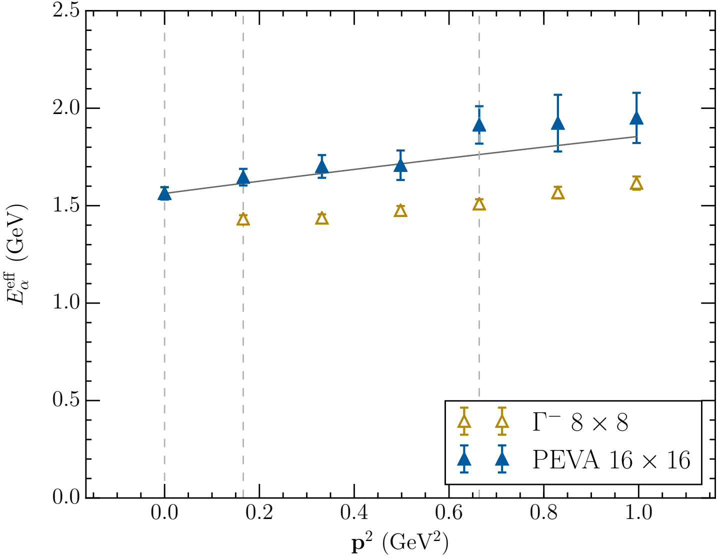

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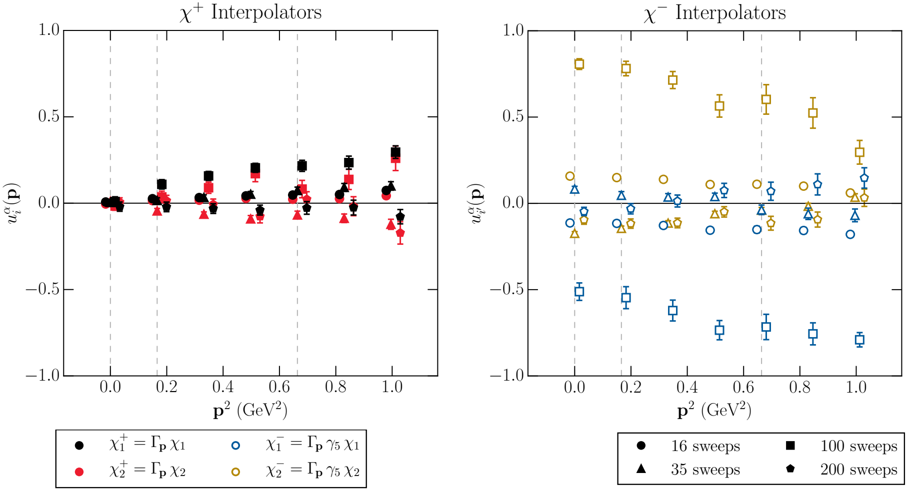

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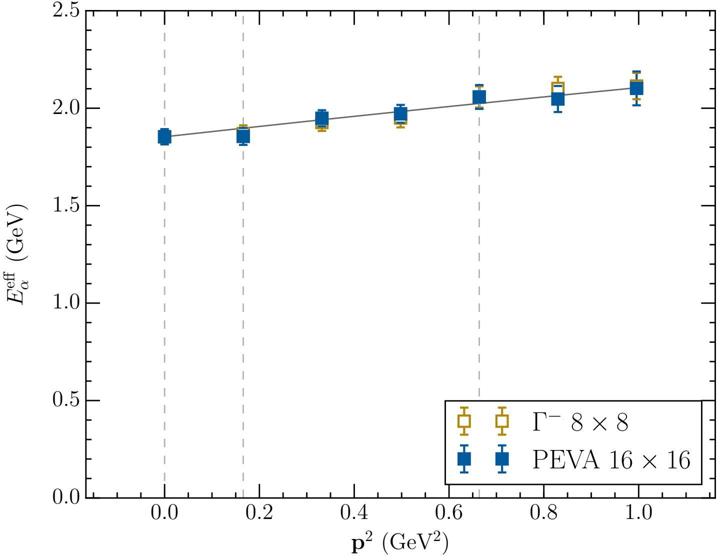

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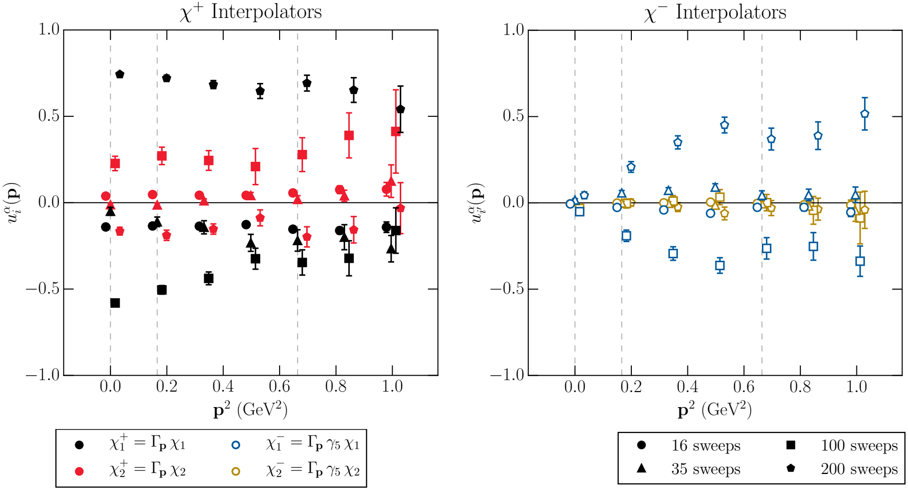

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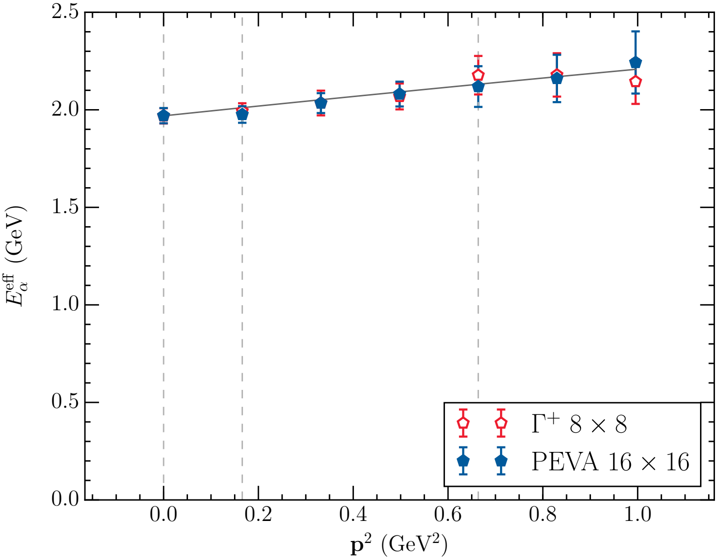
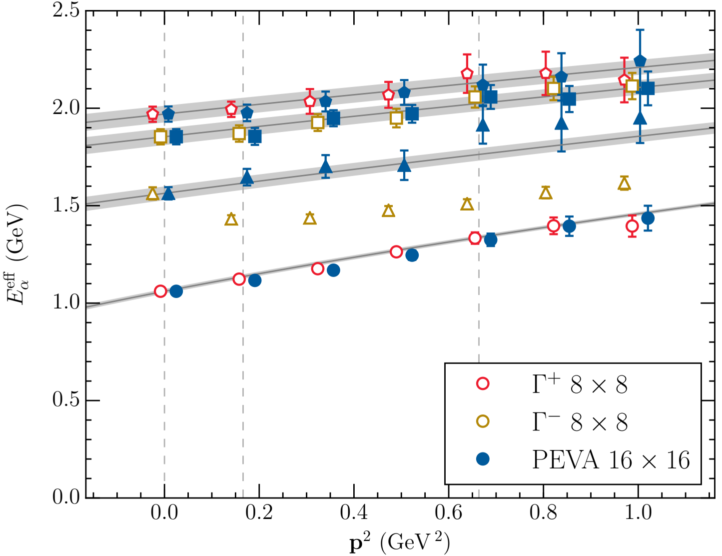


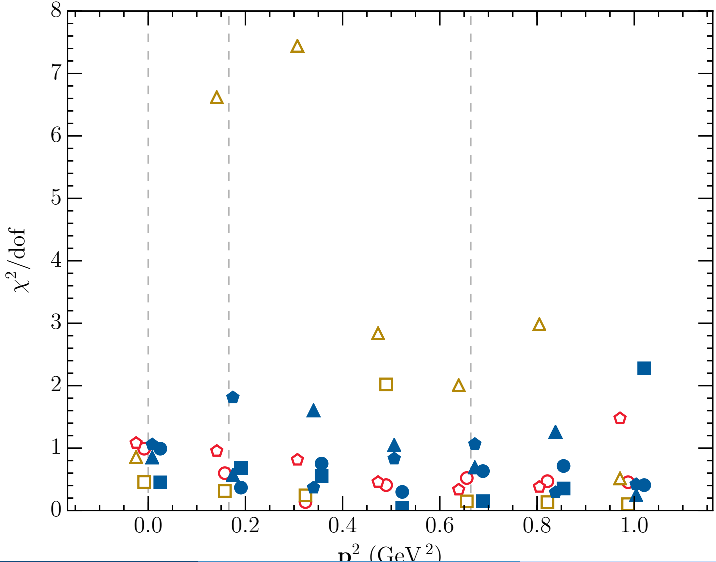
Effective energy

Nucleon spectrum



χ^2/dof

Nucleon spectrum



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- Could have even more significant effects in three point functions, where mixing could prevent current from accessing energy eigenstates

More information

"Parity-expanded variational analysis for nonzero momentum"
F. M. Stokes, W. Kamleh, D. B. Leinweber, M. S. Mahbub,
B. J. Menadue, B. J. Owen
Phys. Rev. D **92** (2015) 11, 114506
doi:10.1103/PhysRevD.92.114506
arXiv:1302.4152 (hep-lat).

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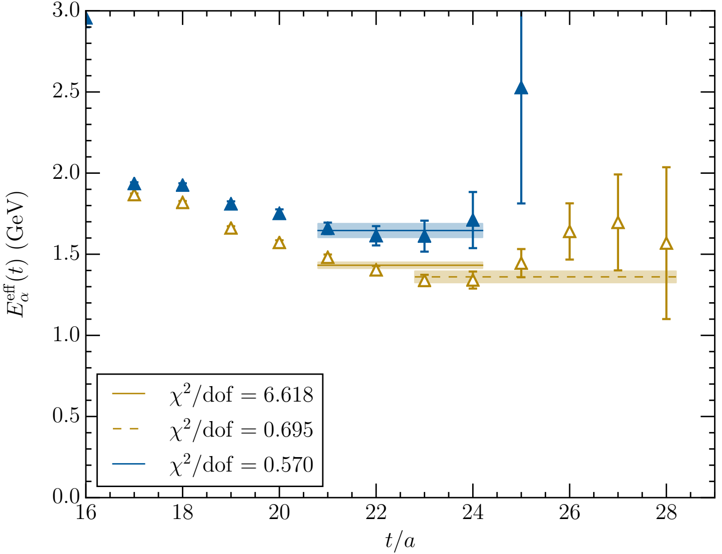

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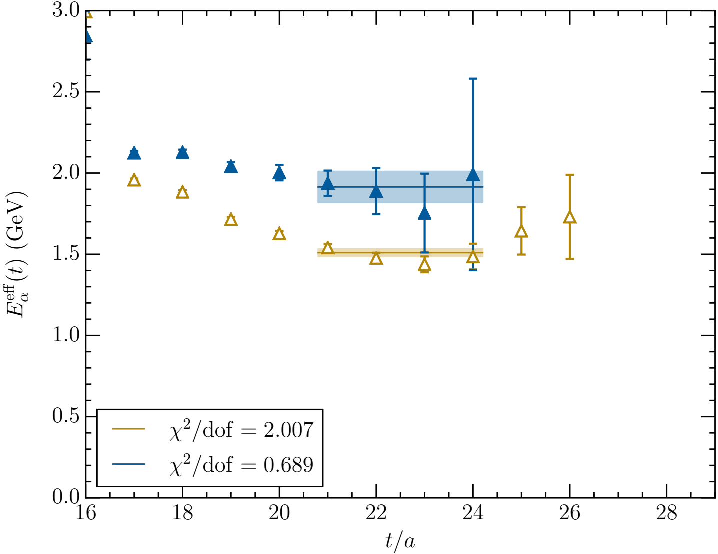

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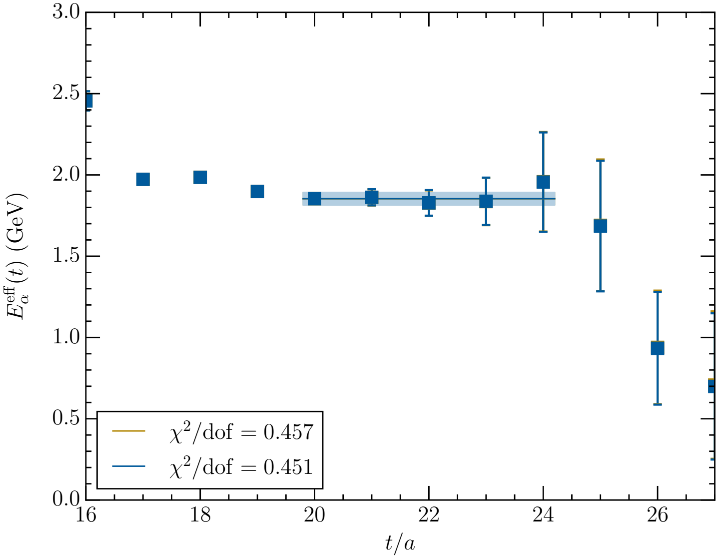

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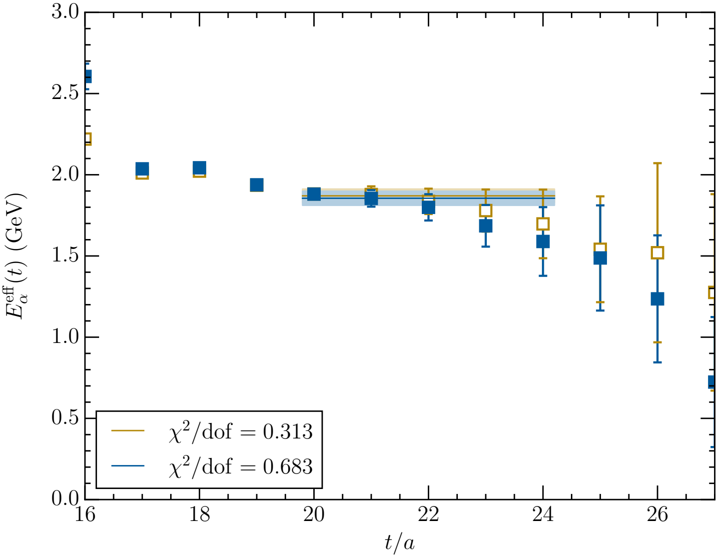

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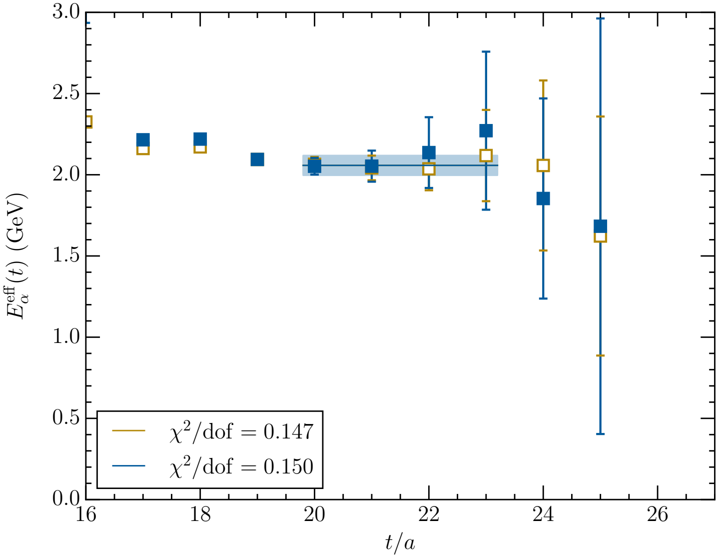

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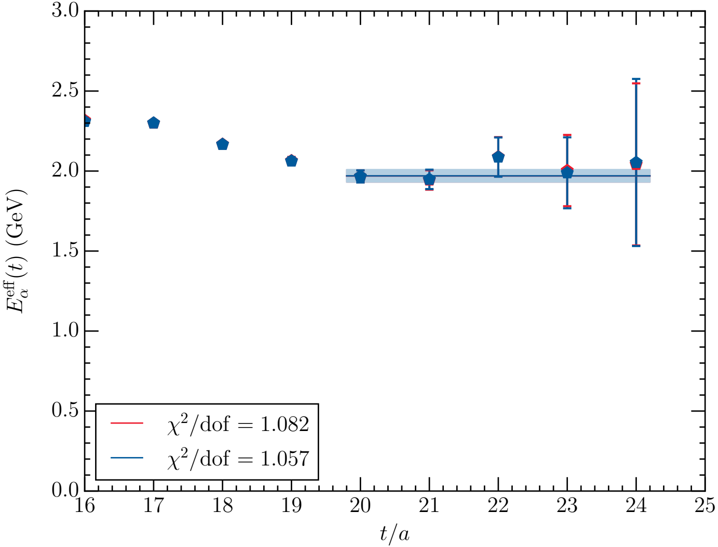

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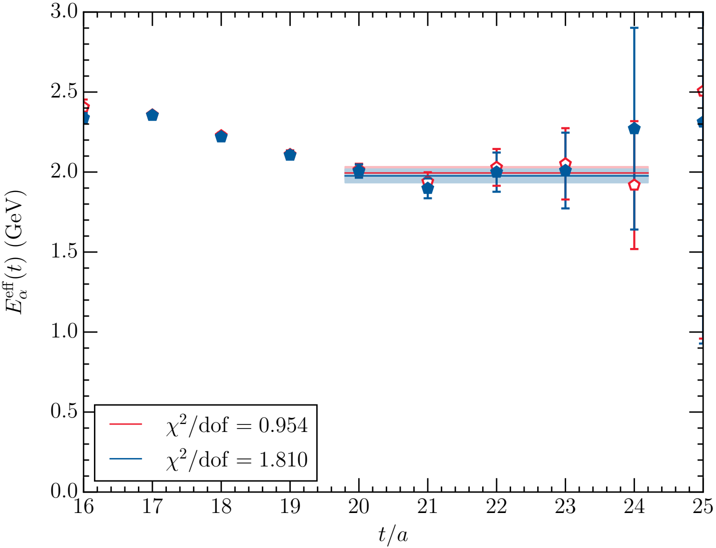
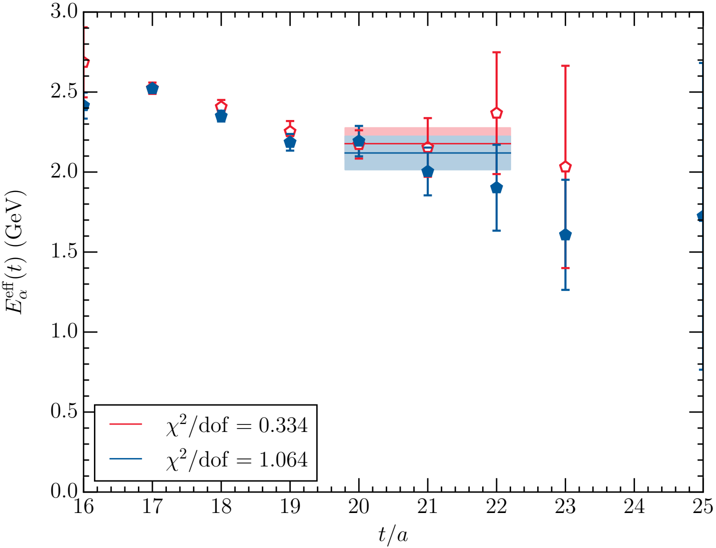


Effective energy

First positive parity excitation - $\mathbf{p}^2 = 0.664\,\mathrm{GeV}^2$



Variational analysis

- Seek operators $\{\phi^{\alpha}_{\mathbf{p}}\}$ that perfectly isolate energy eigenstates

$$\langle\Omega|\phi^{\alpha}_{\mathbf{p}}|B^{\beta}\rangle = \delta^{\alpha\beta}\sqrt{\frac{m_\alpha}{E_\alpha}}\,Z^{\alpha}\,\Gamma_{\mathbf{p}}\,u_\alpha(p,s)\,,$$

$$\langle B^{\beta}|\overline{\phi}^{\alpha}_{\mathbf{p}}|\Omega\rangle = \delta^{\alpha\beta}\sqrt{\frac{m_\alpha}{E_\alpha}}\,\overline{Z}^{\alpha}\,\overline{u}_\alpha(p,s)\,\Gamma_{\mathbf{p}}\,.$$

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- Can be written as linear combination of operators

$$\phi^\alpha_{\mathbf{p}} = \sum_i v^\alpha_i(\mathbf{p}) \; \chi^i_{\mathbf{p}} + \sum_{i'} v^\alpha_{i'}(\mathbf{p}) \; \chi^{i'}_{\mathbf{p}}$$

$$\overline{\phi}^\alpha_{\mathbf{p}} = \sum_i u^\alpha_i(\mathbf{p}) \; \overline{\chi}^i_{\mathbf{p}} + \sum_{i'} u^\alpha_{i'}(\mathbf{p}) \; \overline{\chi}^{i'}_{\mathbf{p}}$$

Variational analysis

- Then,

$$\begin{aligned} G(\mathbf{p};\ t)\,\mathbf{u}^{\alpha}(\mathbf{p}) &= \boldsymbol{\lambda}^{\alpha}\overline{Z}^{\alpha}\,\mathrm{e}^{-E_{\alpha}(\mathbf{p})\,t} \\ \mathbf{v}^{\alpha}(\mathbf{p})\,G(\mathbf{p};\ t) &= Z^{\alpha}\overline{\boldsymbol{\lambda}}^{\alpha}\,\mathrm{e}^{-E_{\alpha}(\mathbf{p})\,t} \end{aligned}$$

Variational analysis

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$$G(\mathbf{p};\, t)\, \mathbf{u}^{\alpha}(\mathbf{p}) = \boldsymbol{\lambda}^{\alpha} \overline{Z}^{\alpha}\, \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, t}$$
$$\mathbf{v}^{\alpha}(\mathbf{p})\, G(\mathbf{p};\, t) = Z^{\alpha} \overline{\boldsymbol{\lambda}}^{\alpha}\, \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, t}$$

- So $\mathbf{v}^{\alpha}(\mathbf{p})$ and $\mathbf{u}^{\alpha}(\mathbf{p})$ form the left and right generalised eigenvectors of $G(\mathbf{p};\, t + \Delta t)$ and $G(\mathbf{p};\, t)$

$$G(\mathbf{p};\, t + \Delta t)\, \mathbf{u}^{\alpha}(\mathbf{p}) = \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, \Delta t}\, G(\mathbf{p};\, t)\, \mathbf{u}^{\alpha}(\mathbf{p})$$
$$\mathbf{v}^{\alpha}(\mathbf{p})\, G(\mathbf{p};\, t + \Delta t) = \mathrm{e}^{-E_{\alpha}(\mathbf{p})\, \Delta t}\, \mathbf{v}^{\alpha}(\mathbf{p})\, G(\mathbf{p};\, t)$$

Variational analysis

- Then,

$$G(\mathbf{p};\, t)\, \mathbf{u}^\alpha(\mathbf{p}) = \boldsymbol{\lambda}^\alpha \overline{Z}^\alpha\, \mathrm{e}^{-E_\alpha(\mathbf{p})\, t}$$
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$$\mathbf{v}^\alpha(\mathbf{p})\, G(\mathbf{p};\, t + \Delta t) = \mathrm{e}^{-E_\alpha(\mathbf{p})\, \Delta t}\, \mathbf{v}^\alpha(\mathbf{p})\, G(\mathbf{p};\, t)$$

- Can construct correlator for a single energy eigenstate

$$\begin{aligned} G^\alpha(\mathbf{p};\, t) &:= \mathbf{v}^\alpha(\mathbf{p})\, G(\mathbf{p};\, t)\, \mathbf{u}^\alpha(\mathbf{p}) \\ &= Z^\alpha \overline{Z}^\alpha\, \mathrm{e}^{-E_\alpha(\mathbf{p})\, t} \end{aligned}$$