Argonne NATIONAL LABORATORY

DSEs and their Application to Hadronic Physics

Craig Roberts, Physics Division

U.S. DEPARTMENT OF ENERGY

An auspicious meeting

- Tony and I met whilst we were both postgraduates at Flinders University … in 1984

An (in)auspicious meeting

- Tony and I met whilst we were both postgraduates at Flinders University … in 1984
- We shared a PhD advisor, who took us sailing on his yacht with the intent of testing us … sink or swim was his supervisory method

First Review Article on Dyson-Schwinger Equations

Information | References (157) | Citations (734) | Files | Plots

Dyson-Schwinger equations and their application to hadronic physics

Craig D. Roberts (Argonne, PHY) , Anthony G. Williams (Adelaide U. & Florida State U. & Florida State U., SCRI)

Mar 8, 1994 - 110 pages

Prog.Part.Nucl.Phys. 33 (1994) 477-575

In *Faessler, A. (ed.): Progress in particle and nuclear physics, vol. 33* 477-575. and Adelaide Univ. - ADP-93-225-T142 (93,rec.Mar.94) 110 p. Argonne Nat. Lab. - PHY-7668-TH-93 (93,rec.Mar.94) 110 p

DOI: 10.1016/0146-6410(94)90049-3

ADP-93-225-T-142, ANL-PHY-7668-TH-93

e-Print: hep-ph/9403224 | PDF

Abstract

We review the current status of nonperturbative studies of gauge field theory using the Dyson-Schwinger equation formalism and its application to hadronic physics. We begin with an introduction to the formalism and a discussion of renormalisation in this approach. We then review the current status of studies of Abelian gauge theories [e.g., strong coupling quantum electrodynamics] before turning our attention to the non-Abelian gauge theory of the strong interaction, quantum chromodynamics. We discuss confinement, dynamical chiral symmetry breaking and the application and contribution of these techniques to our understanding of the strong interactions.

Note: 110 pages, LaTeX. Replaced only to facilitate retrieval. Also available at /u/ftp/pub/Review.uu via anonymnous-ftp Journal-ref: Prog. Part. Nucl. Phys. 33 (1994) 477

Keyword(s): INSPIRE: review | Dyson-Schwinger equation | gauge field theory: nonperturbative | renormalization | quantum electrodynamics | dimension: 3 | dimension: 4 | symmetry breaking: chiral | quantum chromodynamics | propagator | quark: confinement | critical phenomena | Bethe-Salpeter equation | effective action | bibliography

Record added 1994-03-08, last modified 2015-12-18

Excerpt from *Table of Contents*

These themes, *inter alia*, have long entertained Tony.

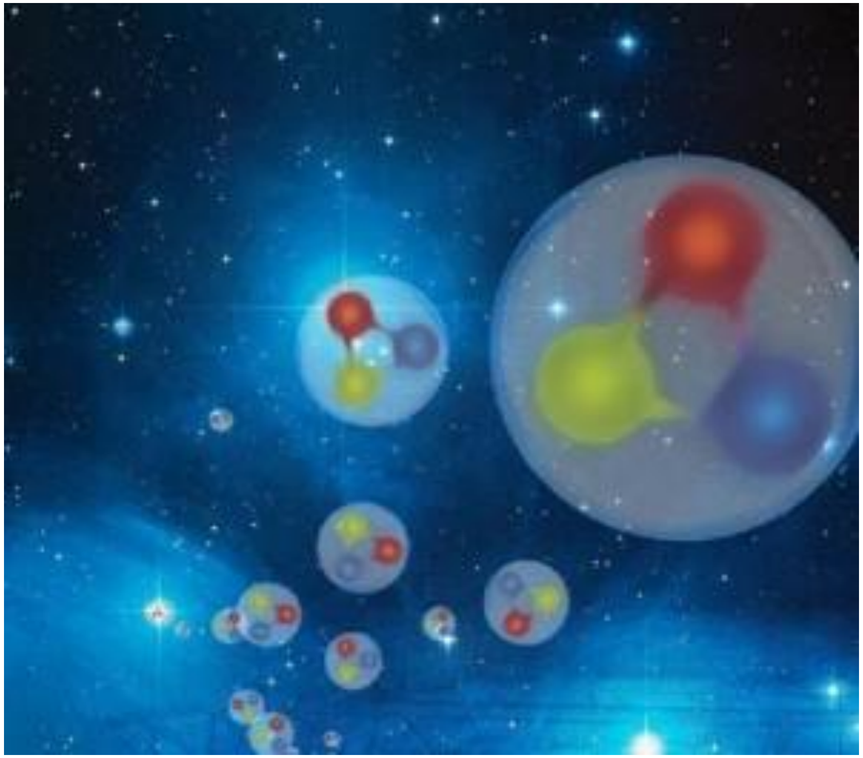
He has made game-changing contributions in each of these areas.

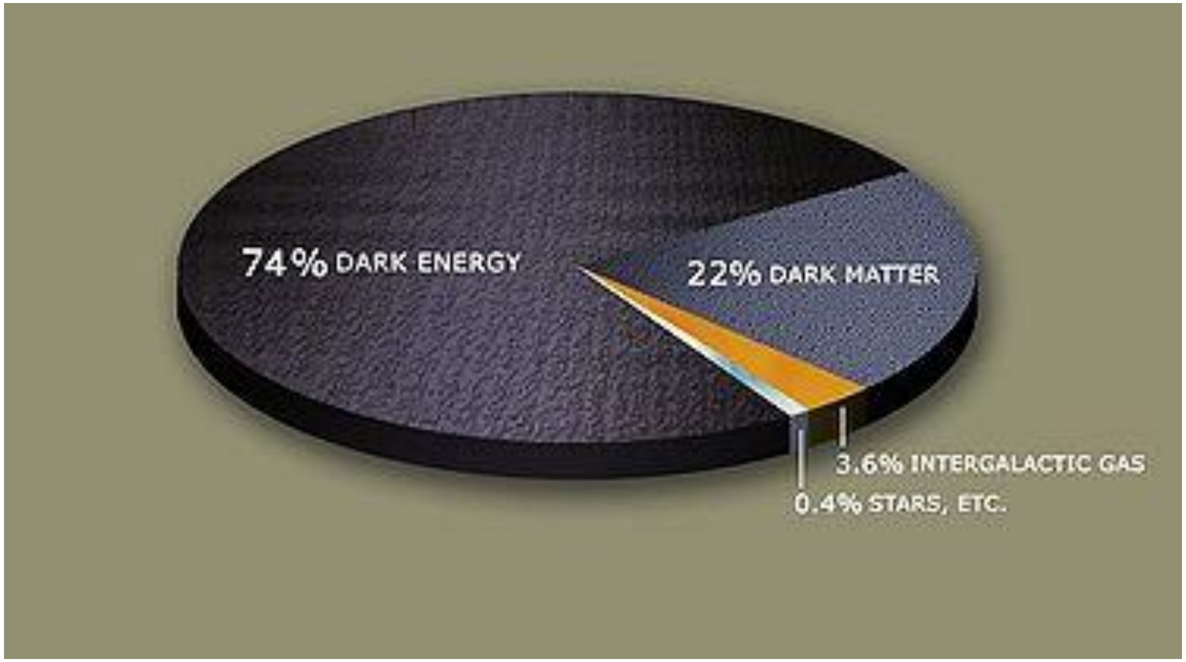
In particular, perhaps, to the understanding of DCSB

Craig Roberts. DSEs and Hadronic Physics (85pp)

Quantum Chromodynamics

- *QCD*: The piece of the Standard Model that describes strong interactions.
- The physics of neutrons, protons, pions, etc. – i.e., Hadron Physics – is a nonperturbative problem in *QCD*
- Notwithstanding the 2013 Nobel Prize in Physics, the origin of 98% of the visible mass in the Universe is – *somehow* – found within *QCD*





Top Open Questions in Physics

Excerpt from the top-10

- ***Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?***

  Quantum chromodynamics is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.

Overarching Science Challenge for the coming decade

Discover meaning of confinement, its relationship to DCSB – the Origin of Visible Mass – and the connection between them

What is QCD?

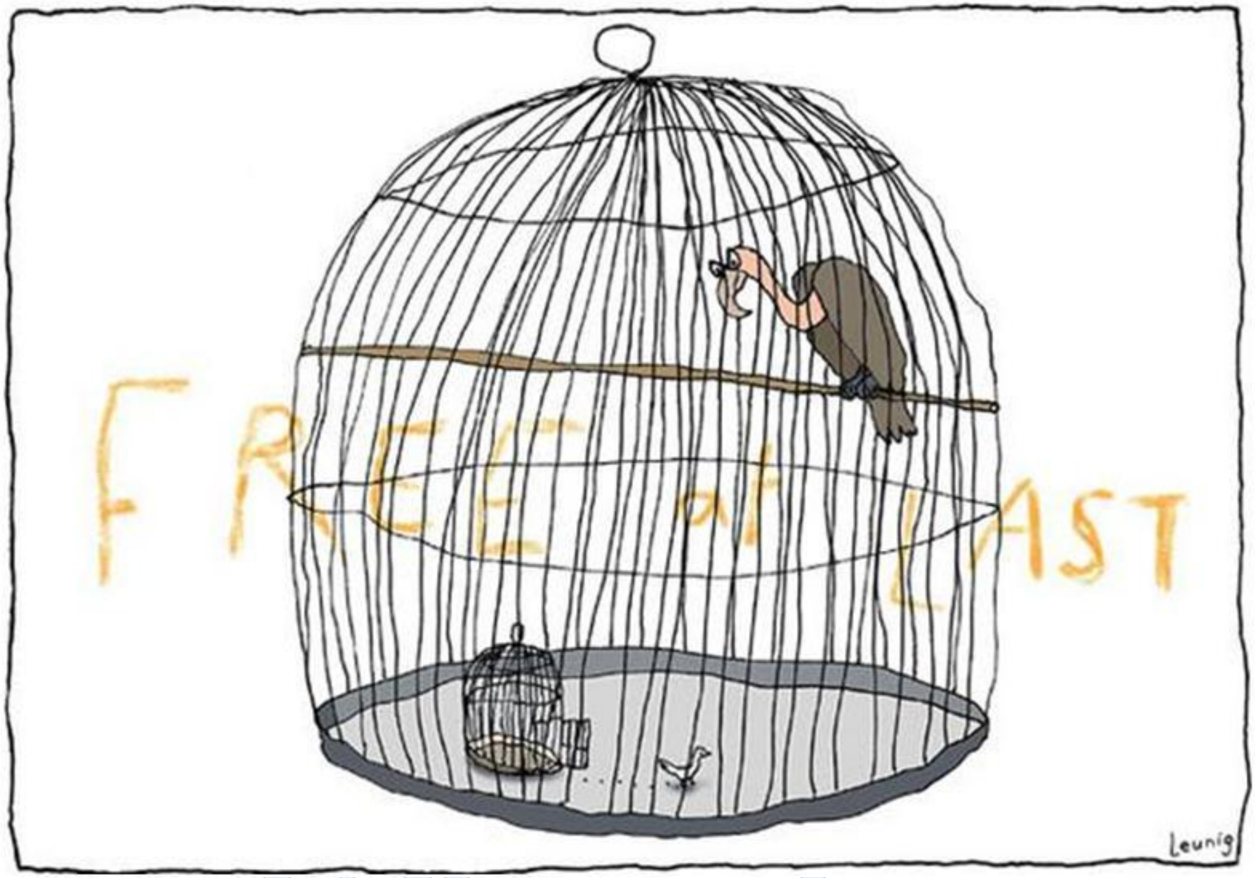
QCD is a *Theory*

(*not* an effective theory)

- Very likely a self-contained, nonperturbatively renormalisable and hence well defined Quantum Field Theory

  This is not true of QED – cannot be defined nonperturbatively
- No confirmed breakdown over an enormous energy domain:

  0 GeV < E < 8 TeV
- Increasingly probable that any extension of the Standard Model will be based on the paradigm established by QCD
  - Extended Technicolour: electroweak symmetry breaks via a fermion bilinear operator in a strongly-interacting non-Abelian theory. (Andersen *et al*. "Discovering Technicolor" Eur.Phys.J.Plus 126 (2011) 81)
  - Higgs sector of the SM becomes an effective description of a more fundamental fermionic theory, similar to the Ginzburg-Landau theory of superconductivity wikipedia.org/wiki/Technicolor_(physics)



What is Confinement?

CLAY MATHEMATICS INSTITUTE

Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove the confinement conjecture

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x} - \vec{y}|), \tag{2}$$

as long as $|\vec{x} - \vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

CLAY MATHEMATICS INSTITUTE

Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on* $\mathbb{R}^4$ *and has a mass gap* $\Delta > 0$. *Existence includes establishing axiomatic properties at least as strong as those cited in* **[45, 35]**.

Confinement?

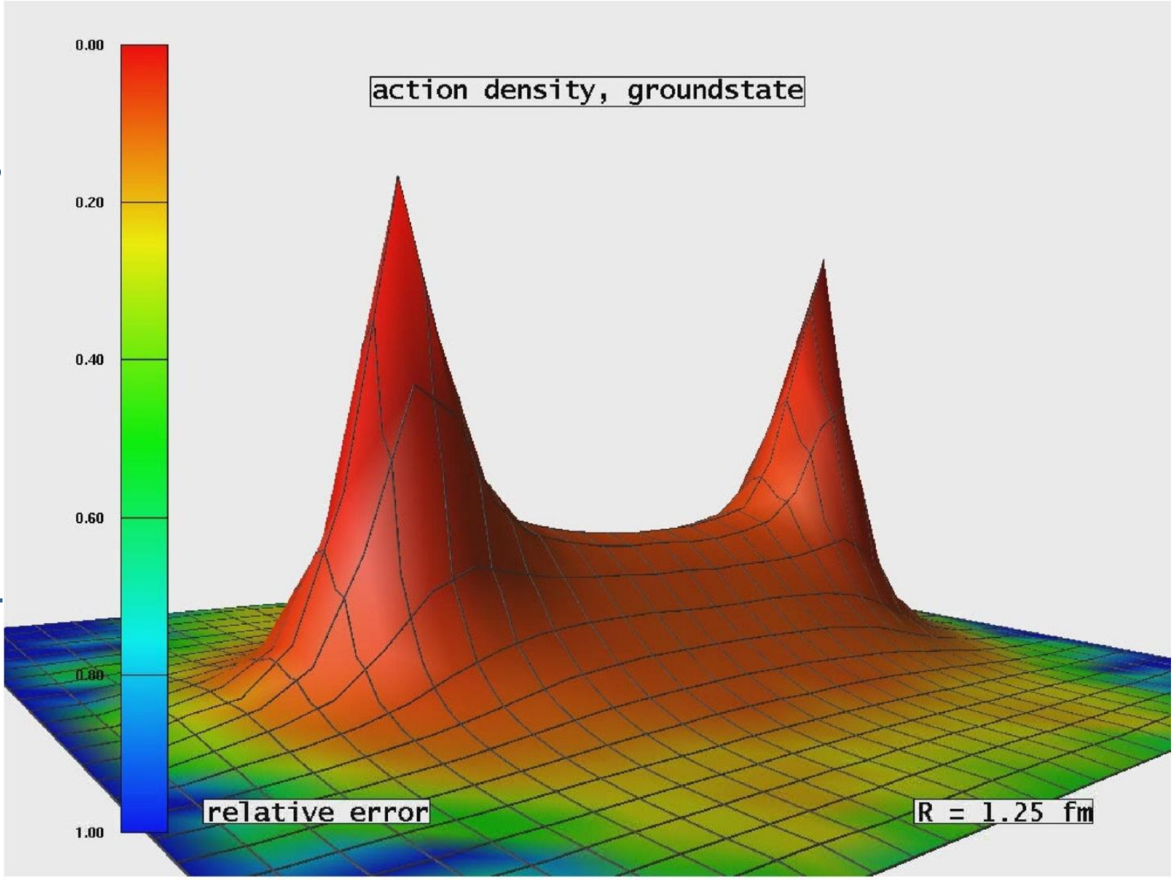
Light quarks & Confinement

- Folklore … *Hall-D Conceptual Design Report(5)*

  "The color field lines between a quark and an anti-quark form flux tubes.

  A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

  This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."



Light quarks & Confinement

- Problem:
  16 tonnes of force
  makes a lot of pions.

Light quarks & Confinement

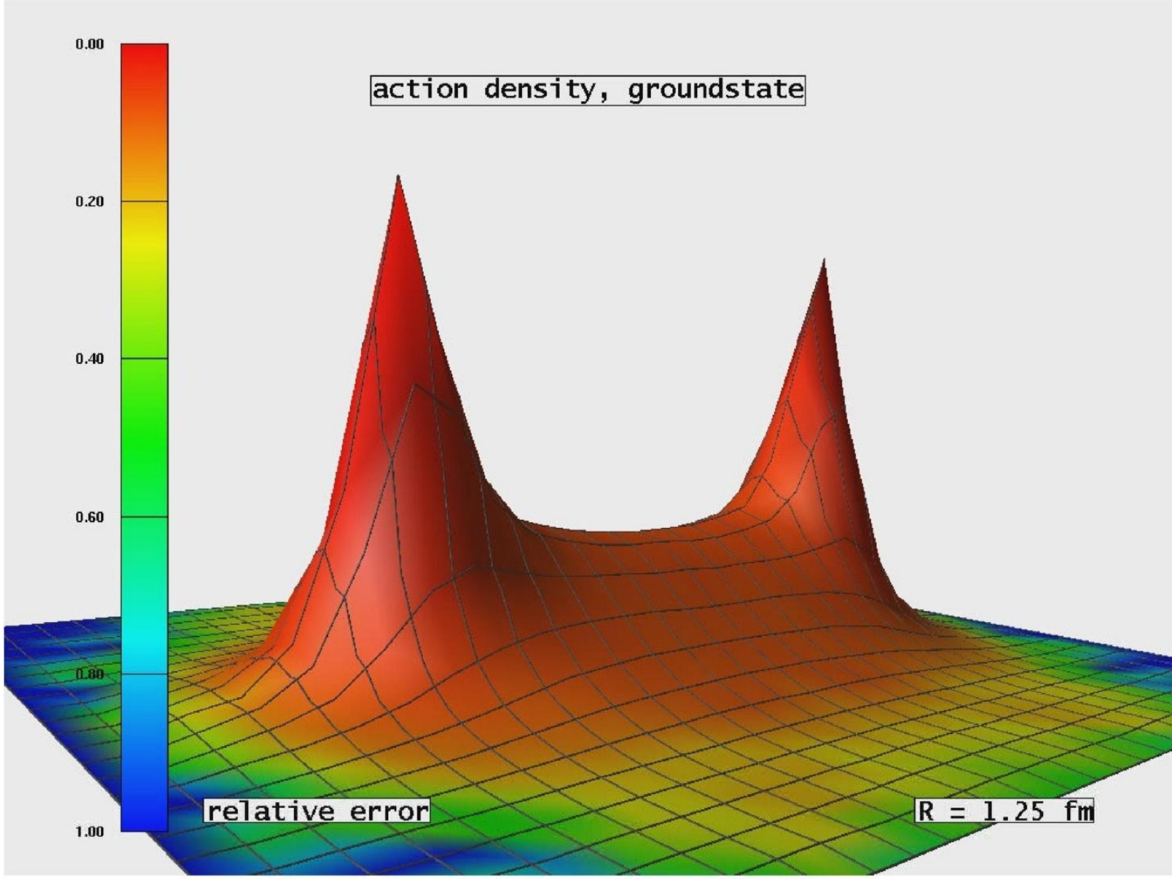
- Problem:

16 tonnes of force

makes a lot of pions.

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



G. *Bali* et al., *PoS LAT2005 (2006) 308*

Light quarks & Confinement

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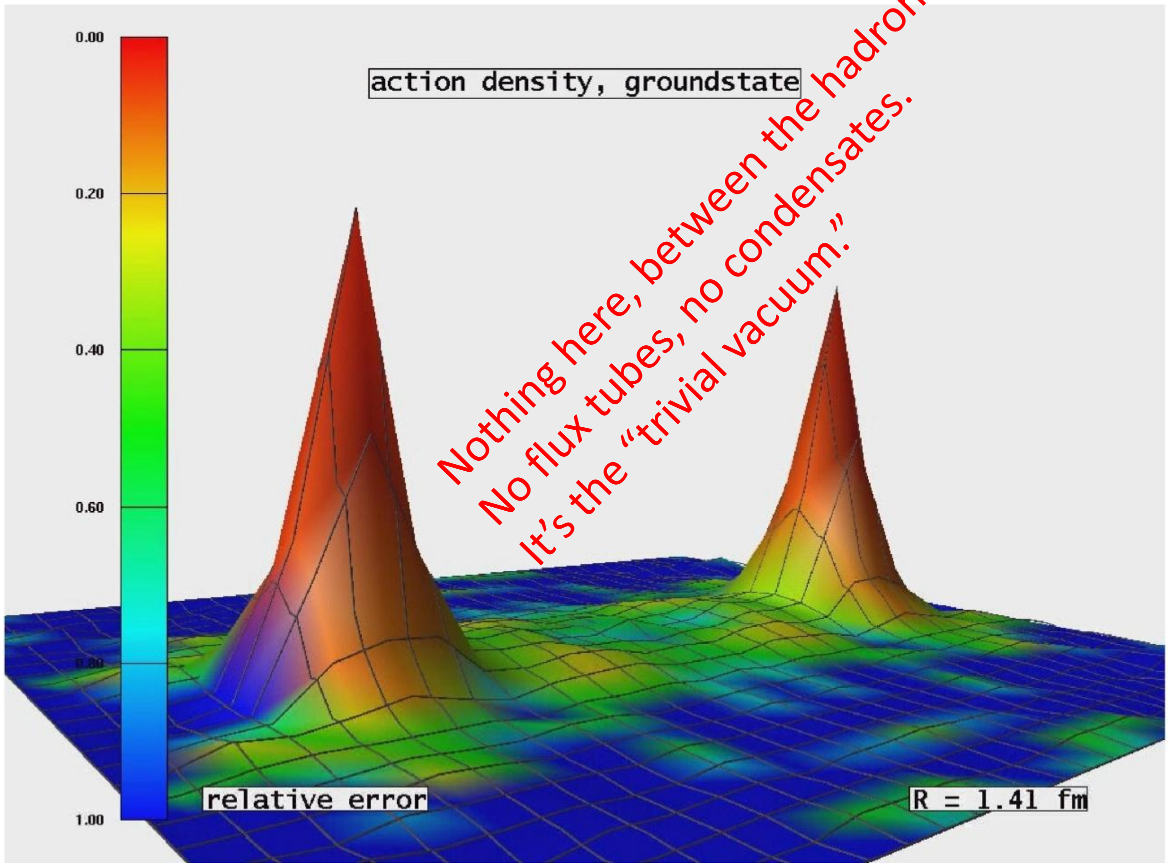

- *Existence of a mass-gap in the pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV
- However, with

  $\Delta^2/m_\pi^2 > 100$,

  can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- If *answer is* not *simply **no***, then it must be that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- Pion must play critical role in any explanation of confinement; and any discussion that omits reference to the pion's role is *practically irrelevant*.

142 MILLENNIUM PRIZE PROBLEMS

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

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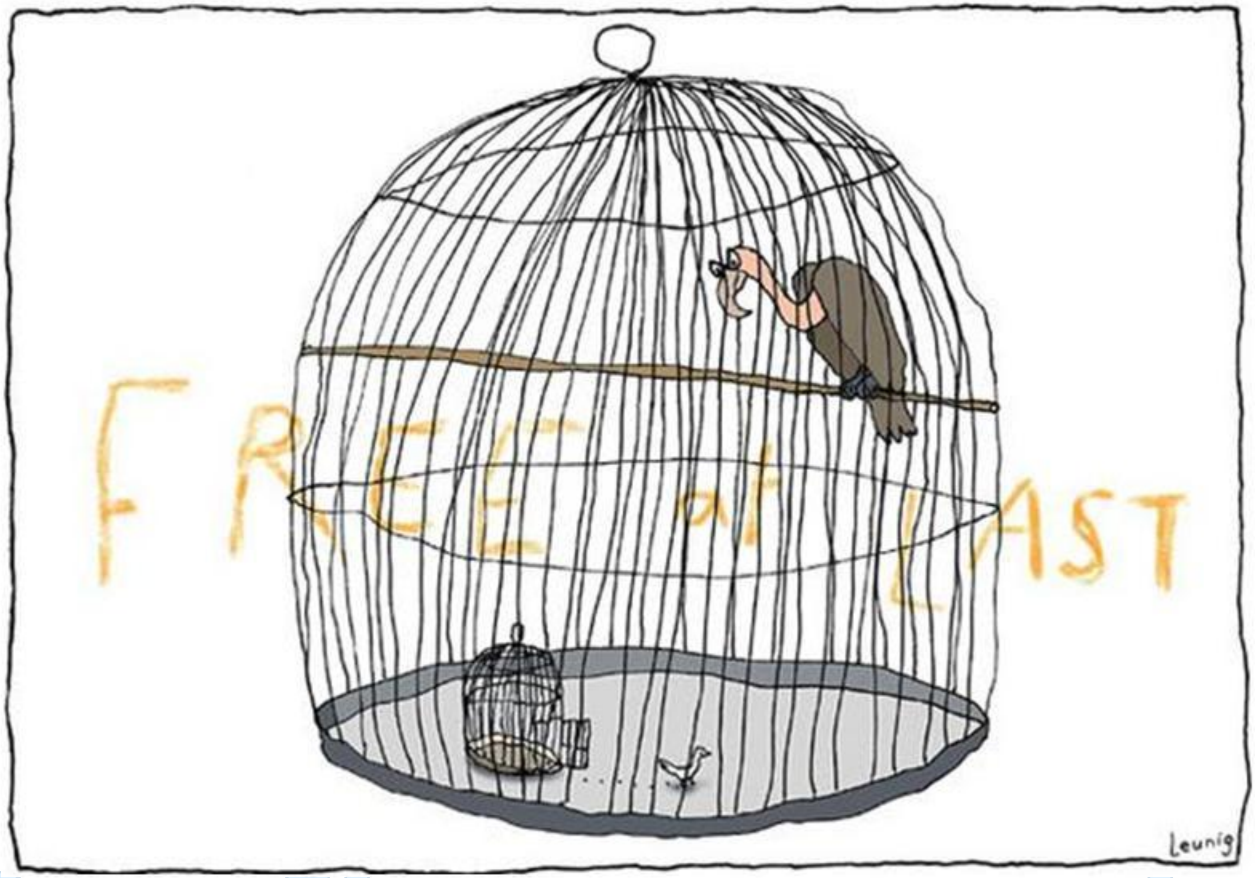
$$(2) \qquad |\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x}-\vec{y}|),$$

as long as $|\vec{x}-\vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Craig Roberts. DSEs and Hadronic Physics (85pp)



Confinement is dynamical!

On the implications of confinement

Information | References (20) | Citations (127) | Files | Plots

On the implications of confinement

G. Krein (Florida State U., SCRI) , Craig D. Roberts (Argonne, PHY) , Anthony G. Williams (Florida State U., SCRI)

Oct 1990 - 14 pages

Int.J.Mod.Phys. A7 (1992) 5607-5624
DOI: 10.1142/S0217751X92002544
FSU-SCRI-90-168, PHY-6746-TH-90

Keyword(s): INSPIRE: quark: confinement | quantum chromodynamics | Dyson-Schwinger equation | quark: propagator | gluon: propagator | renormalization | axiomatic field theory | color: confinement | quantum chromodynamics: model | gluon: condensation | propagator: pole | n-point function | perturbation theory | field theory: Euclidean | analytic properties | quark quark: interaction | interaction: quark quark

Record added 1990-12-10, last modified 2015-04-24

Received: 30 September 1991

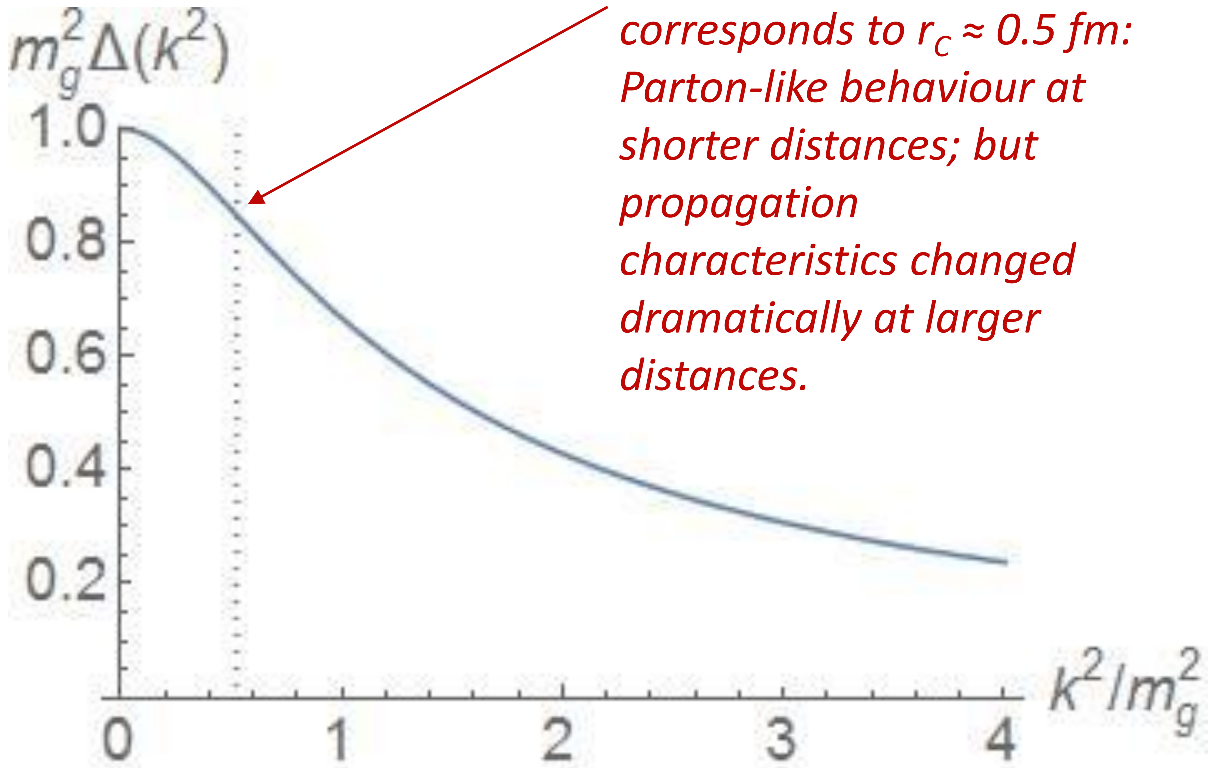
We consider some implications of confinement starting from the basic observation that cross-sections for the production of colored asymptotic states, such as free quarks and gluons, from color singlet initial states must be zero if QCD is to be confining. We discuss two pictures of confinement: the failure of the cluster decomposition property and the absence of a pole at timelike momenta in the propagator of a confined particle. We use QCD-based models as a framework to relate the failure of the cluster decomposition property to other ideas, such as the role of a nonzero gluon condensate. Our primary interest is to address the question of the absence of a mass pole through a study of model Schwinger-Dyson equations. These equations contain some of the dynamical information that is present in the study of the cluster decomposition property. We discuss the problems with this idea and its study using the Schwinger-Dyson equations.

PACS: 12.38.Aw, 11.30.Qc, 11.30.Rd

Confinement

- All DSE and lattice solutions for Landau-gauge gluon quark propagators exhibit an inflection point in k^2

$\Rightarrow$ Violate reflection positivity = sufficient for confinement because no Hamiltonian describing observables produces states with negative norm

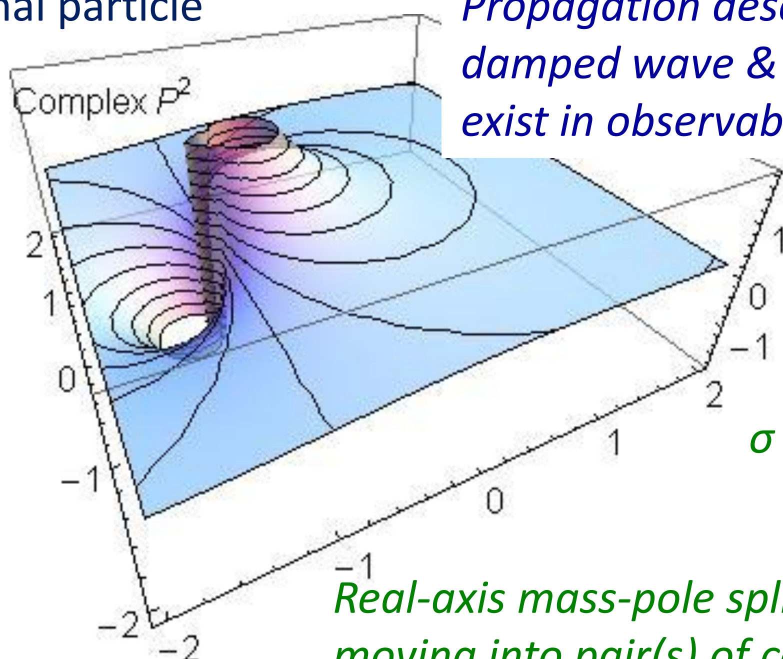


Inflexion point corresponds to $r_C \approx 0.5$ fm: Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.

Confinement

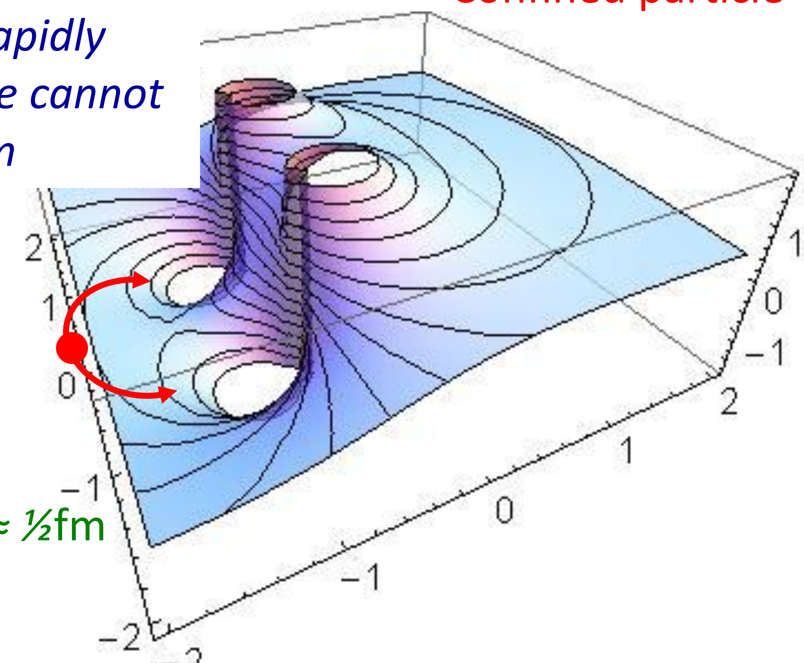
- QFT Paradigm:
  - Confinement is expressed through a *dramatic* change in the analytic structure of propagators for coloured states
  - It can be read from a plot of the dressed-propagator for a coloured state

Normal particle



Propagation described by rapidly damped wave & hence state cannot exist in observable spectrum

Confined particle

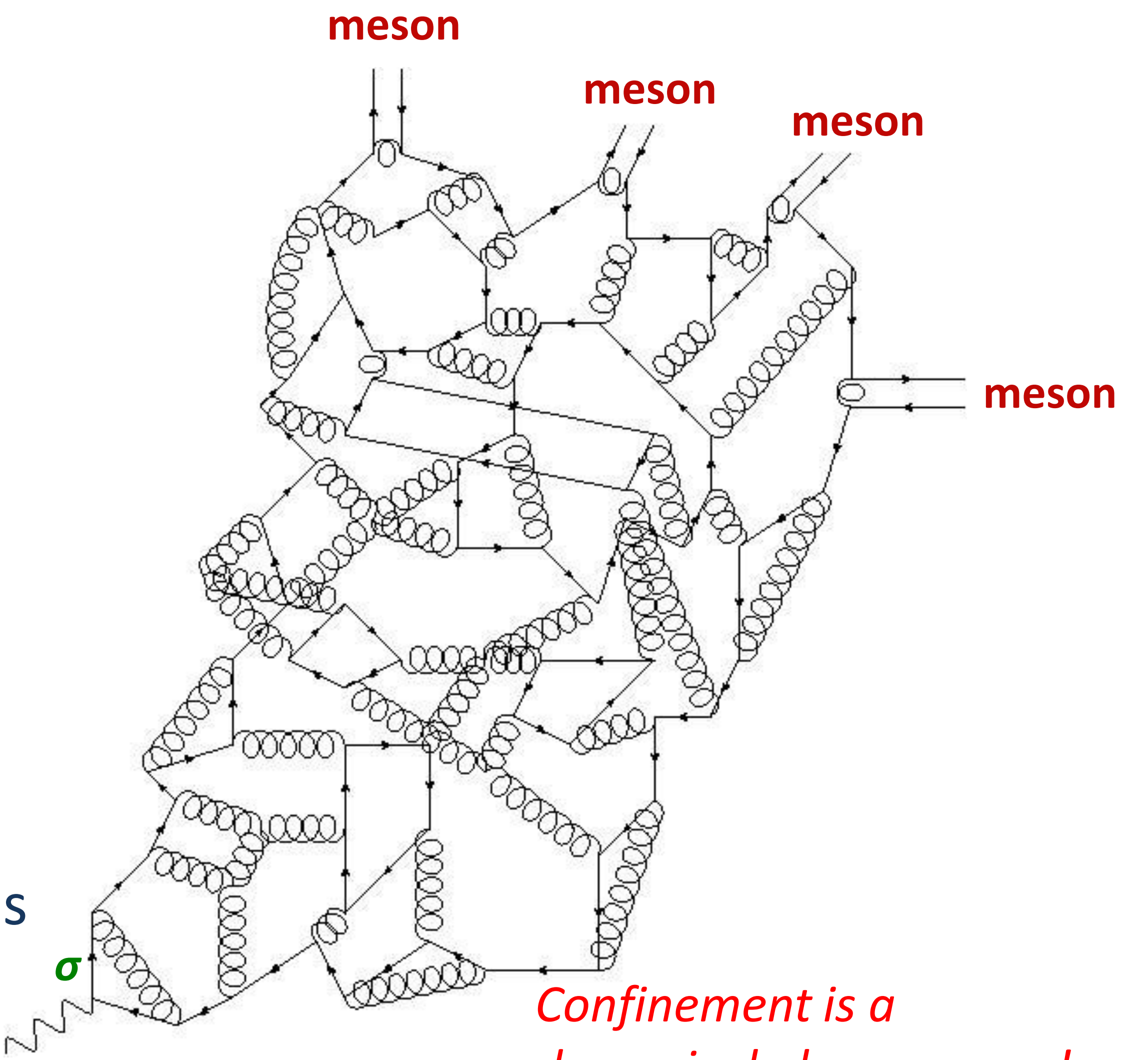


$\sigma \approx 1/\mathrm{Im}(m)$
$\approx 1/2\Lambda_{QCD} \approx$ ½fm

Real-axis mass-pole splits,
moving into pair(s) of complex conjugate singularities, (or qualitatively analogous structures chracterised by a dynamically generated mass-scale)

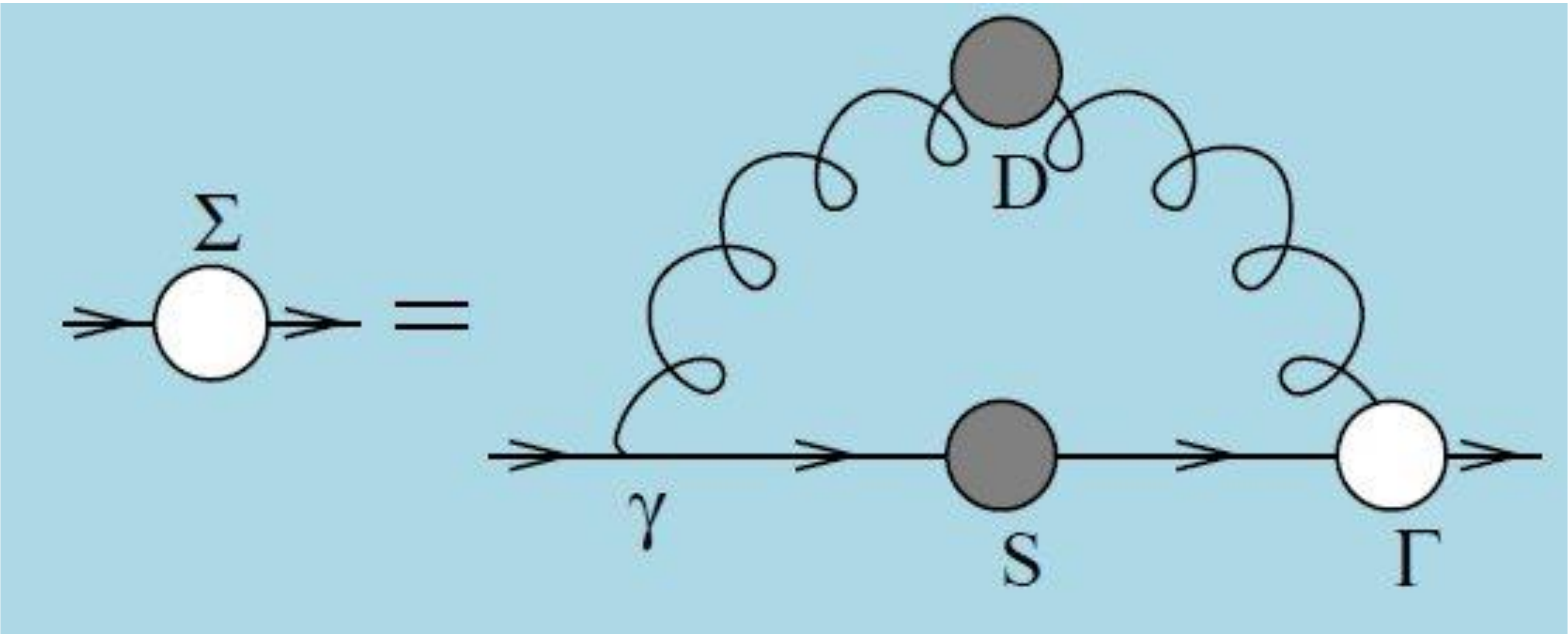
Quark Fragmentation

- A quark begins to propagate
- But after each “step” of length ***σ***, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Quark Gap Equation

Gap Equation: Gluon two-point function

➢ Twenty years of study and dispute over the infrared behaviour of $D_{\mu\nu}(k)$

Information | References (21) | Citations (125) | Files | Plots

Gluon propagator in the infrared region

UKQCD Collaboration (Derek B. Leinweber , Jon Ivar Skullerud, Anthony G. Williams (Adelaide U.) , Claudio Parrinello (Liverpool U.)) Hide

Mar 1998 - 13 pages

Phys.Rev. D58 (1998) 031501
DOI: 10.1103/PhysRevD.58.031501
ADP-98-4-T284, LTH-421
e-Print: hep-lat/9803015 | PDF
Experiment: LATTICE-UKQCD

Abstract
The gluon propagator is calculated in quenched QCD for two different lattice sizes (16^3x48 and 32^3x64) at beta=6.0. The volume dependence of the propagator in Landau gauge is studied. The smaller lattice is instrumental in revealing finite volume and anisotropic lattice artefacts. Methods for minimising these artefacts are developed and applied to the larger lattice data. New structure seen in the infrared region survives these conservative cuts to the lattice data. This structure serves to rule out a number of models that have appeared in the literature. A fit to a simple analytical form capturing the momentum dependence of the nonperturbative gluon propagator is also reported.

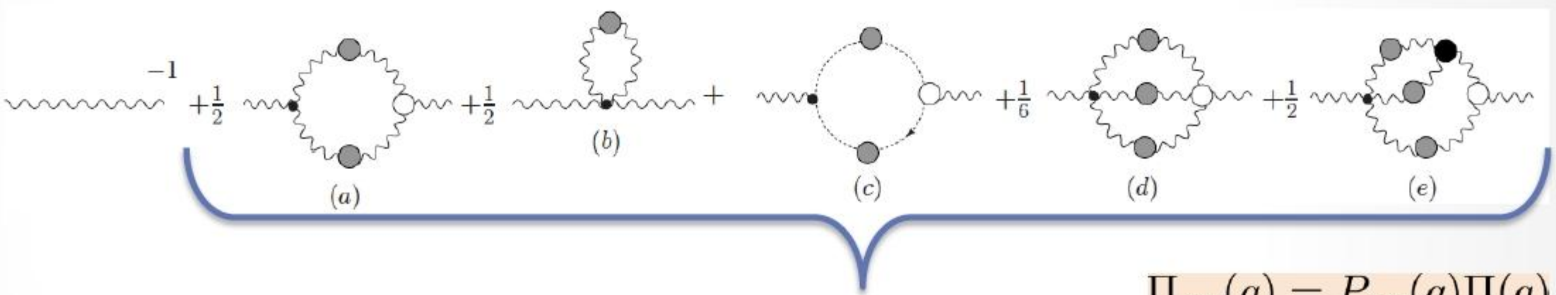
Note: 13 pages, 9 figures, using RevTeX. Submitted to Phys. Rev. D. This and related papers can also be obtained from http://www.physics.adelaide.edu.au/~jskuller/papers/ Report-no: ADP-98-4/T284; LTH 421
Keyword(s): INSPIRE: lattice field theory | gauge field theory: SU(3) | approximation: quenching | gluon: propagator | effect: finite size | lattice: anisotropy | Landau gauge | infrared problem | momentum dependence | numerical calculations: Monte Carlo
Record added 1998-03-23, last modified 2015-12-20

➢ *Crucial first step toward the answer, followed by development of coherent picture*

Non-perturbative comparison of QCD effective charges, A.C. Aguilar, D. Binosi, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D80 (2009) 085018

Gluons, too, have a gap equation

$\Delta^{-1}_{\mu\nu}(q) =$



$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

- Pinch-technique + background field method … reordering of diagrammatic summations in the self-energy – $\Pi_{\mu\nu}$ – ensures that subclusters are individually transverse and gluon-loop and ghost-loop contributions are separately transverse
- STIs → WGTIs
- Enables systematic analysis and evaluation of truncations and straightforward comparison of results with those of lQCD

Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi, L. Chang, J.Papavassiliou, C.D. Roberts, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons also become massive!

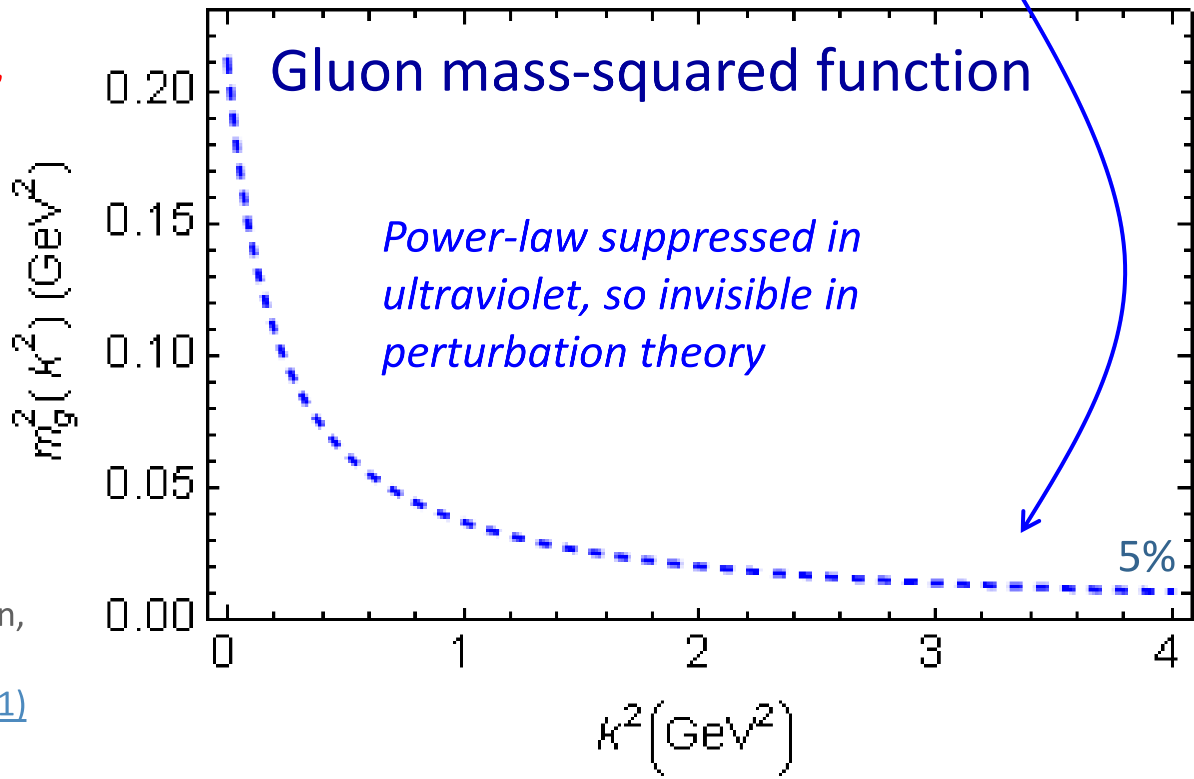
- Running gluon mass

$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\; m_g^2(0) = (0.46\,\text{GeV})^2$$

$$m_g^2(k^2) = \frac{\mu_g^4}{\mu_g^2 + k^2}$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!



Interaction model for the gap equation, S.-x.Qin, L.Chang, Y-x.Liu, C.D.Roberts and D. J. Wilson, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Massive Gauge Bosons!

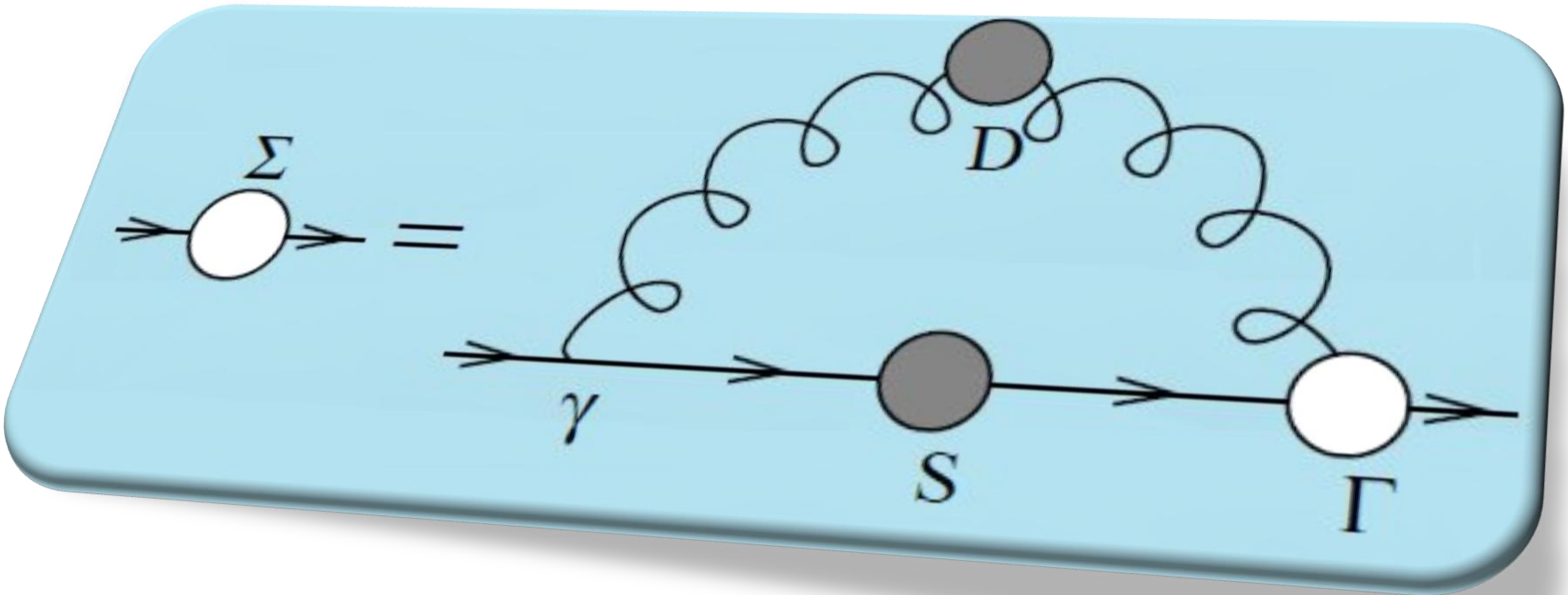
- Gauge boson cannibalism

  … a new physics frontier … within the Standard Model
- Asymptotic freedom means
  … ultraviolet behaviour of QCD is controllable
- Dynamically generated masses for gluons and quarks means that QCD dynamically generates its own infrared cutoffs
  - Gluons and quarks with

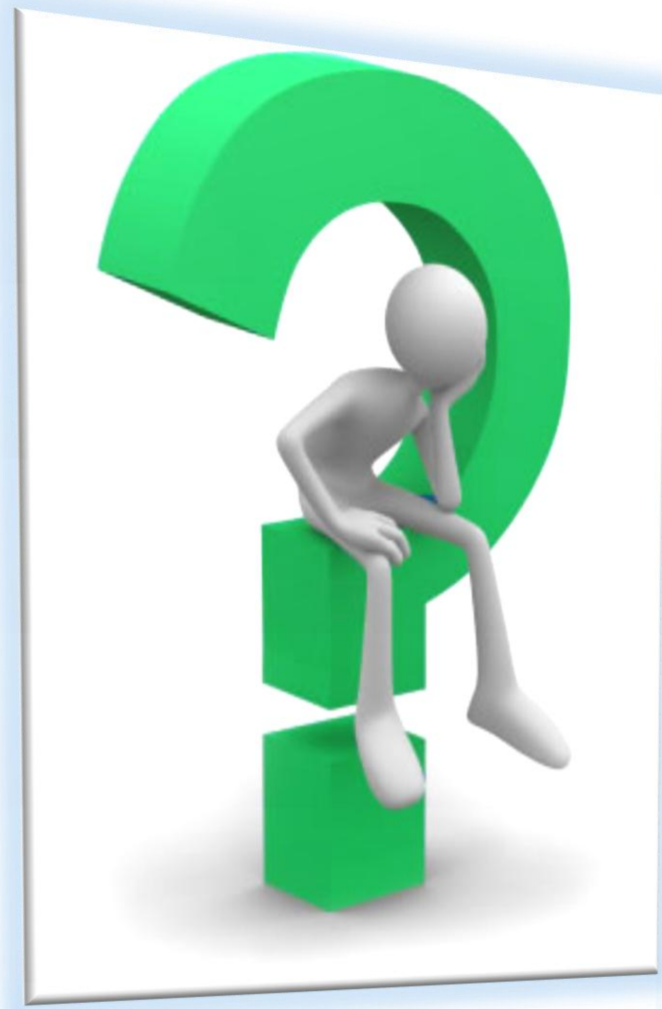
    wavelength $\lambda > 2/\text{mass} \approx 1$ fm

    decouple from the dynamics … ***Confinement?!***
- How does that affect observables?
  - It will have an impact in
    any continuum study
  - Must play a role in gluon saturation ...
    In fact, perhaps it's a harbinger of gluon saturation?

Electron Ion Collider:
The Next QCD Frontier



Gap Equation
- Critical open question



What is Γ_μ?

Gap Equation: Gluon-quark three-point function

- First use of dressed vertex in the gap equation
- Until this time it had been typical to use $\Gamma_\mu = \gamma_\mu$, without thinking too much about what errors, if any, that introduced.

Information | References (23) | Citations (23) | Files | Plots

Dynamical Chiral Symmetry Breaking in Quantum Chromodynamics With Confinement and Asymptotic Freedom

G. Krein, P. Tang, Anthony G. Williams (Washington U., Seattle)

Sep 26, 1988 - 17 pages

Phys.Lett. B215 (1988) 145
DOI: 10.1016/0370-2693(88)91087-8
DOE-ER-40427-30-N8

Information | References (47) | Citations (60) | Files | Plots

Quark Propagator in an Ansatz Approach to {QCD}

Anthony G. Williams, G. Krein (Washington U., Seattle) , Craig D. Roberts (Melbourne U.)

Dec 1989 - 23 pages

Annals Phys. 210 (1991) 464-485
DOI: 10.1016/0003-4916(91)90051-9
FSU-SCRI-89-151

Keyword(s): INSPIRE: quark: propagator | hadron: model | symmetry: chiral | dynamical symmetry breaking | Dyson-Schwinger equation | Landau gauge | pi: decay constant | decay constant: pi | quark: condensation | quantum chromodynamics: Lambda parameter | numerical calculations | gluon: propagator | quark: confinement | pi: decay | regularization | infrared problem | approximation

Record added 1989-12-21, last modified 2015-04-25

Ward-Green-Takahashi Identities

…udies and the latter in studying atmospheric ionization at ground level. These increases in ionization are considered to be due to radioactive matter brought down with the rain. Between 0935 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Doan and Wait and McNish the 35 percent increase in the soft component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhaven (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

[1] A. Dauvillier, Comptes Rendus **229**, 1096 (1949).
[2] Forbush, Stinchcomb, and Schein, Bull. Am. Phys. Soc. **25**, No. 1, 15 (1950).
[3] I. L. Chakraborty and S. D. Chatterjee, Ind. J. Phys. **23**, 525 (1949).
[4] Forbush, Gill, and Vallarta, Rev. Mod. Phys. **21**, 44 (1949).
[5] R. L. Doan, Phys. Rev. **49**, 107 (1936).
[6] G. R. Wait and A. G. McNish, Monthly Weather Rev. **62**, 1 (1934).

An Identity in Quantum Electrodynamics

J. C. Ward
The Clarendon Laboratory, Oxford, England
February 27, 1950

It has been recently proved by Dyson[1] that all divergencies in the S-matrix of electrodynamics may be removed by a renormalization of mass and charge. Dyson defines certain fundamental divergent operators Γ_μ, S_F', D_F' and gives a procedure for the calculation of their finite parts $\Gamma_{\mu 1}$, S_{F1}', D_{F1}' by a process of successive approximation. It is then shown that

$$\Gamma_\mu = Z_1^{-1}\Gamma_{\mu 1}(e_1), \quad S_F' = Z_2 S_{F1}(e_1), \quad D_F' = Z_3 D_{F1}(e_1),$$
$$e_1 = Z_1^{-1} Z_2 Z_3^{\frac{1}{2}} e,$$

where Z_1, Z_2, and Z_3 are certain infinite constants and e_1 is the renormalized electronic charge. Dyson conjectured that $Z_1 = Z_2$, and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part W, may be associated a set of proper vertex parts V^i obtained by inserting a photon line in one of the electron lines of W. Now consider the operators $\Lambda_\mu(V^i, p, p)$ in which the two external electron momentum variables p have been set equal, and the external photon variable made to vanish. Then $\Lambda_\mu(V^i, p, p)$ may be obtained from $\Sigma(W, p)$ by replacing S_F by $S_F\gamma_\mu S_F$ at one electron line of W. Because of the identity

$$-(1/2\pi)\partial S_F/\partial p_\mu = S_F\gamma_\mu S_F,$$

on summing $\Lambda_\mu(V^i, p, p)$ over all vertex parts V^i associated with W, one finds

$$\Sigma_{V^i}\Lambda_\mu(V^i, p, p) = -(1/2\pi)(\partial\Sigma(W, p)/\partial p_\mu).$$

(One can verify that any closed loop in W gives zero total effect.) Finally summing over all proper electron self-energy parts W, one finds

$$\Lambda_\mu(p, p) = -(1/2\pi)(\partial\Sigma^*(p)/\partial p_\mu).$$

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$\Lambda_\mu = Z_1^{-1}[(1-Z_1)\gamma_\mu + \Lambda_{\mu c}], \quad \Sigma^* = Z_2^{-1}[(Z_2-1)S_F^{-1} + S_F^{-1}S_c/2\pi].$$

We have

$$-(1/2\pi)Z_2^{-1}\{(Z_2-1)2\pi\gamma_\mu + \gamma_\mu S_c + (\gamma_\lambda p_\lambda - iK_0)(\partial S_c/\partial p_\mu)\}$$
$$= Z_1^{-1}\{(1-Z_1)\gamma_\mu + \Lambda_{\mu c}(p, p)\}.$$

Now put

$$\gamma_\lambda p_\lambda = iK_0, \quad (p_\lambda)^2 = -K_0^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi)Z_2^{-1}(Z_2-1)2\pi\gamma_\mu = Z_1^{-1}(1-Z_1)\gamma_\mu$$

which reduces immediately to $Z_1 = Z_2$.

[1] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

The Partial Molal Entropy of Superfluid in Pure He[4] below the λ-Point

O. K. Rice
Department of Chemistry, University of North Carolina, Chapel Hill, North Carolina
March 3, 1950

In a recent article[1] (the notation of which is retained here, except that subscripts $4n$ and $4s$ refer to normal fluid and superfluid, respectively, in place of 1 and 2), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relation[2] which states that the total molal entropy S at any temperature is the mole fraction x_{4n} of normal fluid times the molal entropy S_λ at the λ-point

$$S = x_{4n}S_\lambda = (1 - x_{4s})S_\lambda, \tag{1}$$

using the empirical relation for S as a function of temperature

$$S = S_\lambda(T/T_\lambda)^r \tag{2}$$

(with $r \sim 5.6$), and assuming that the partial molal enthalpy of superfluid, $\bar{H}_{4s}$, is independent of temperature (at essentially constant pressure), and independent of x_{4s} (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_{4s} = S_\lambda x_{4n}/(r+1). \tag{3}$$

However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below T_λ the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant S_λ, thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for S from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If $\bar{H}_{4s}$ is independent of x_{4s}, then $\bar{H}_{4n}$ must be also, and we have $\bar{H}_{4n} = H_{4n}$, where H_{4n} is the enthalpy of pure normal helium. We can write for the total molal enthalpy[3]

$$H = x_{4n}H_{4n}. \tag{4}$$

We will now proceed to derive an expression for $\bar{S}_{4s}$ in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since $F = H - TS$ and $\mu_{4s} = \bar{H}_{4s} - T\bar{S}_{4s} = -T\bar{S}$… the condition for internal equilibrium, $F = \mu_{4s}$, gives

$$\bar{S}_{4s} = S - H/T.$$

H.J. Munczek, Phys. Rev. D **52** (1995) 4736
A. Bender, C.D. Roberts and L. von Smekal, Phys.Lett. B **380** (1996) 7

Lei Chang and C.D. Roberts, Phys. Rev. Lett. 103 (2009) 081601

Longitudinal Axial-Vector Ward-Green-Takahashi Identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Axial-Vector vertex
Satisfies an inhomogeneous Bethe-Salpeter equation

Quark propagator satisfies a gap equation

Kernels of these equations are completely different
But they must be intimately related

- This class of identities have been known for more than 60 years
- Used for since mid-1980s as guide to constructing a symmetry-preserving kernel for the Bethe-Salpeter equation
- ***For the last 6 years we've known how to construct a symmetry preserving kernel given an arbitrary quark-gluon vertex***

Takahashi. (1985), Canonical quantization and generalized Ward relations: Foundation of nonperturbative approach, Print-85-0421 (Alberta).

Transverse Ward-Takahashi identity, anomaly and Schwinger-Dyson equation - Kondo, Kei-Ichi Int.J.Mod.Phys. A12 (1997) 5651-5686 hep-th/9608100 CHIBA-EP-94, OUTP-96-30-P

Transverse Ward-Takahashi identity for the fermion boson vertex in gauge theories - He, Han-Xin *et al.* Phys.Lett. B480 (2000) 222-228

Transverse vector vertex function and transverse Ward-Takahashi relations in QED - He, Han-Xin Commun.Theor.Phys. 46 (2006) 109-112

Transverse Ward-Takahashi relation for the fermion-boson vertex function in four-dimensional Abelian gauge theory - He, Han-Xin Int.J.Mod.Phys. A22 (2007) 2119-2132

Nonperturbative fermion boson vertex function in gauge theories - He, Han-xin hep-th/0202013

Checking the transverse Ward-Takahashi relation at one loop order in 4-dimensions - Pennington, M.R. *et al.* J.Phys. G32 (2006) 2219-2234 hep-ph/0511254 DCPT-05-130, IPPP-05-65

Transverse Ward-Takahashi relation for the fermion-boson vertex to one-loop order - He, Han-Xin *et al.* .J.Mod.Phys. A21 (2006) 2541-2551

Transverse Ward-Green-Takahashi Identities

Physics Letters B
Volume 722, Issues 4–5, 24 May 2013, Pages 384–388

Practical corollaries of transverse Ward–Green–Takahashi identities

Si-xue Qin[a, b], Lei Chang[c], Yu-xin Liu[a], Craig D. Roberts[d, e], Sebastian M. Schmidt[f]

- Longitudinal WGT identity expresses properties of the divergence of the vertex
- Transverse identities relate to its *curl* (as Faraday's law of induction involves an electric field)

$$q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) + 2im\,\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) + A^V_{\mu\nu}(k,p),$$

$$q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) + V^A_{\mu\nu}(k,p),$$

- The last two terms in each identity arise in computing the momentum space expression of a nonlocal axial-vector/vector vertex, whose definition involves a gauge-field-dependent line integral
- But … practical progress can be made without knowing their precise forms

Physics Letters B

Volume 722, Issues 4–5, 24 May 2013, Pages 384–388

Practical corollaries of transverse Ward–Green–Takahashi identities

Si-xue Qin[a, b], Lei Chang[c], Yu-xin Liu[a], Craig D. Roberts[d, e], Sebastian M. Schmidt[f]

- Using symmetries alone, it is readily established that DCSB forces dressed fermions to possess anomalous chromo- and electro-magnetic moments, which are large on the domain within which DCSB is effective
- This is the "final" word.
- Evidence had slowly been accumulating since 1985

Anomalous Magnetic Moment Of Light Quarks And Dynamical Symmetry Breaking - Singh, J.P. Phys.Rev. D31 (1985) 1097-1108

Anomalous quark chromomagnetic moment induced by instantons - Kochelev, N.I. Phys.Lett. B426 (1998) 149-153 hep-ph/9610551 KOBE-FHD-96-01, C96-09-02.3

The Anomalous magnetic moment of quarks - Bicudo, Pedro J.A. *et al.* Phys.Rev. C59 (1999) 1107-1112 hep-ph/9806243

Dressed-quark anomalous magnetic moments - Chang, Lei *et al.* Phys.Rev.Lett. 106 (2011) 072001 arXiv:1009.3458 [nucl-th]

Dynamical chiral symmetry breaking and the fermion--gauge-boson vertex - Bashir, A. *et al.* Phys.Rev. C85 (2012) 045205 arXiv:1112.4847 [nucl-th]

Dynamically generated AMM

- Simple vertex in perturbation theory $\Gamma_\mu = \gamma_\mu$
  → 12 distinct terms when strong interactions are turned on
- Amongst them, one with a unique structure; i.e., an anomalous magnetic moment term

$$\propto \sigma_{\mu\nu} k_\nu \frac{dB(k^2)}{dk^2}$$

- Follows, algebraically, that gauge theories coupled to fermions with a dynamically generated mass MUST possess an anomalous (chromo/electro)-magnetic moment, whose magnitude is driven by the strength of DCSB

Modern $\Gamma_\mu(q;k)$

Rapid Communication

Tracing masses of ground-state light-quark mesons

Phys. Rev. C **85**, 052201(R) – Published 7 May 2012

Lei Chang and Craig D. Roberts

- Describes the best-informed vertex available today
  - Contains all the Ball-Chiu terms

    They are the unique kinematic-singularity-free solution of the longitudinal vector WGT identity
  - And two of the terms critical for expressing the CAMMs

$$\Gamma_\mu^{\rm acm}(p_1,p_2) = \Gamma_\mu^{\rm acm_4}(p_1,p_2) + \Gamma_\mu^{\rm acm_5}(p_1,p_2)\,, \quad (8)$$

with $(k = p_1 - p_2,\, T_{\mu\nu} = \delta_{\mu\nu} - k_\mu k_\nu/k^2,\, a_\mu^{\rm T} := T_{\mu\nu} a_\nu)$

$$\Gamma_\mu^{\rm acm_4} = [\ell_\mu^{\rm T}\gamma\cdot k + i\gamma_\mu^{\rm T}\sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p_1,p_2)\,, \quad (9)$$

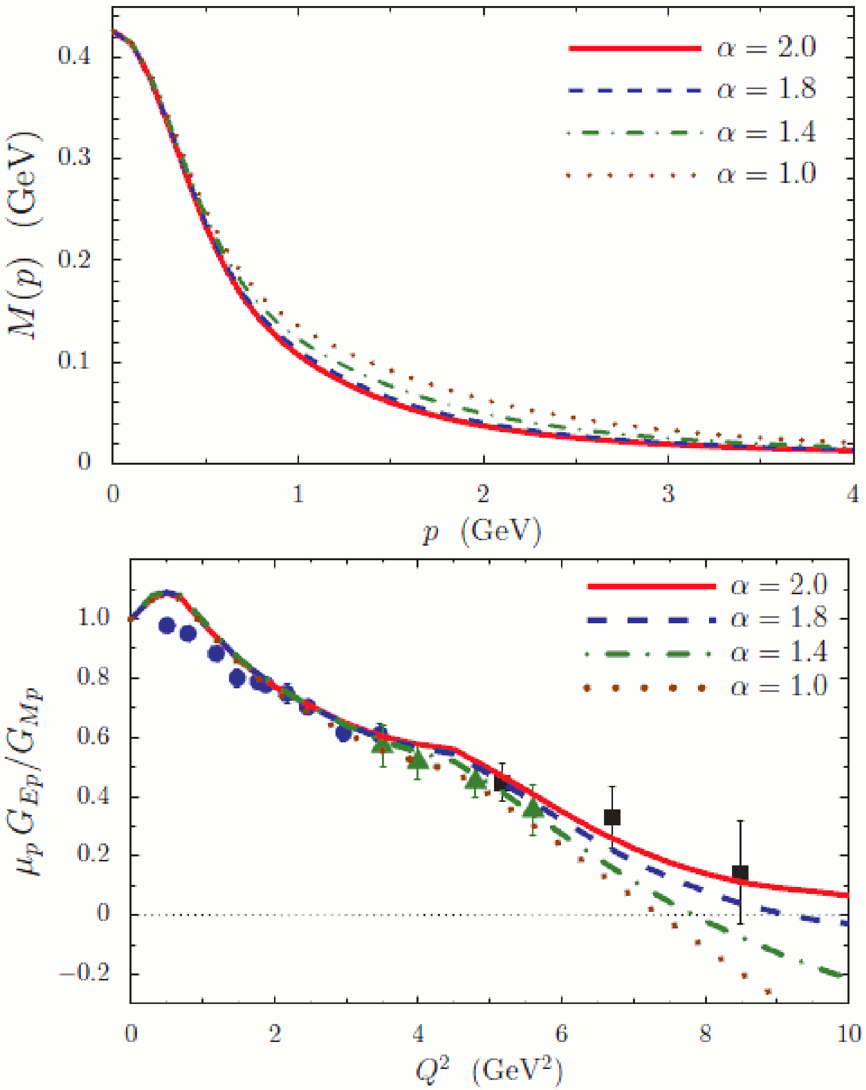
$$\Gamma_\mu^{\rm acm_5} = \sigma_{\mu\nu}k_\nu\tau_5(p_1,p_2)\,, \quad (10)$$

$$\tau_4 = \frac{2\tau_5(p_1,p_2)}{\mathcal{M}(p_1^2,p_2^2)}\,, \quad (11)$$

- The chromo AMM is crucial to explaining the splitting between parity partners, such as a_1-ρ mass splitting, and connecting it with DCSB

I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution,
arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Electromagnetic AMM



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- Apparently small changes in M(p) within the domain 1<p(GeV)<3 have striking effect on the proton's electric form factor
- The possible existence and location of the zero is determined by behaviour of $Q^2F_2^p(Q^2)$, proton's Pauli form factor
- Like the pion's PDA, $Q^2F_2^p(Q^2)$ measures the rate at which dressed-quarks become parton-like:
  - ✓ $F_2^p=0$ for bare quark-partons
  - ✓ Therefore, G_E^p can't be zero on the bare-parton domain

Mesons and Rainbow-Ladder Truncation: *Requiescat in Pace*

- For reasons that are now fully understood, Rainbow-Ladder truncation ($\Gamma_\mu = \gamma_\mu$) provides accurate results (15% over ≈ 100 observables) for properties of ground-states in the π, ρ, K, $Q_{bar}Q$, N, Δ channels
- Equally, however, we know it is critically flawed for
  - scalar, axial-vector and tensor mesons
  - excited states in all channels
  - exotic states
  - heavy-light systems
  - etc.
- Like quenched lattice-QCD

the time for RL analyses is passing

Craig Roberts. DSEs and Hadronic Physics (85pp)

Continuum-QCD & *ab initio* predictions

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

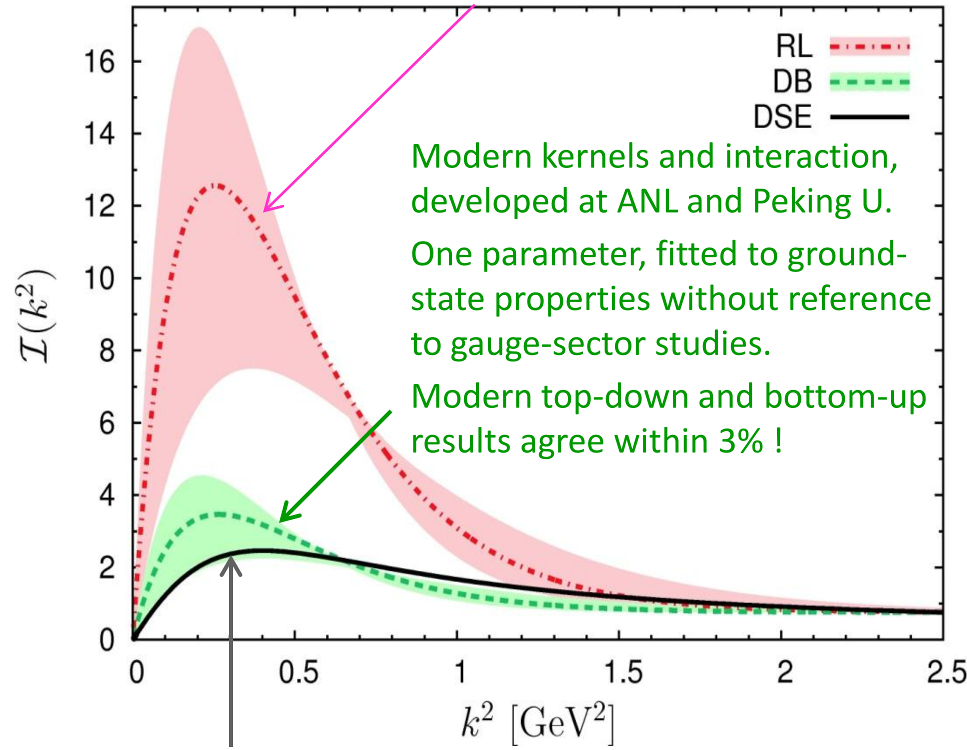
D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

Top down & Bottom up

- ***Bottom-up scheme** – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- ***Top-down approach*** – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *Serendipitous collaboration, conceived at one-week ECT* Workshop on* DSEs in Mathematics and Physics, *has united these two approaches*

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation

"Maris-Tandy" interaction. Developed at ANL & KSU in 1997-1998. More-than 700 citations – *but* quantitative disagreement with gauge-sector solution.



Top-down result = gauge-sector prediction

Bridging a gap between continuum-QCD
& ab initio *predictions of hadron observables*

Top down & Bottom up

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

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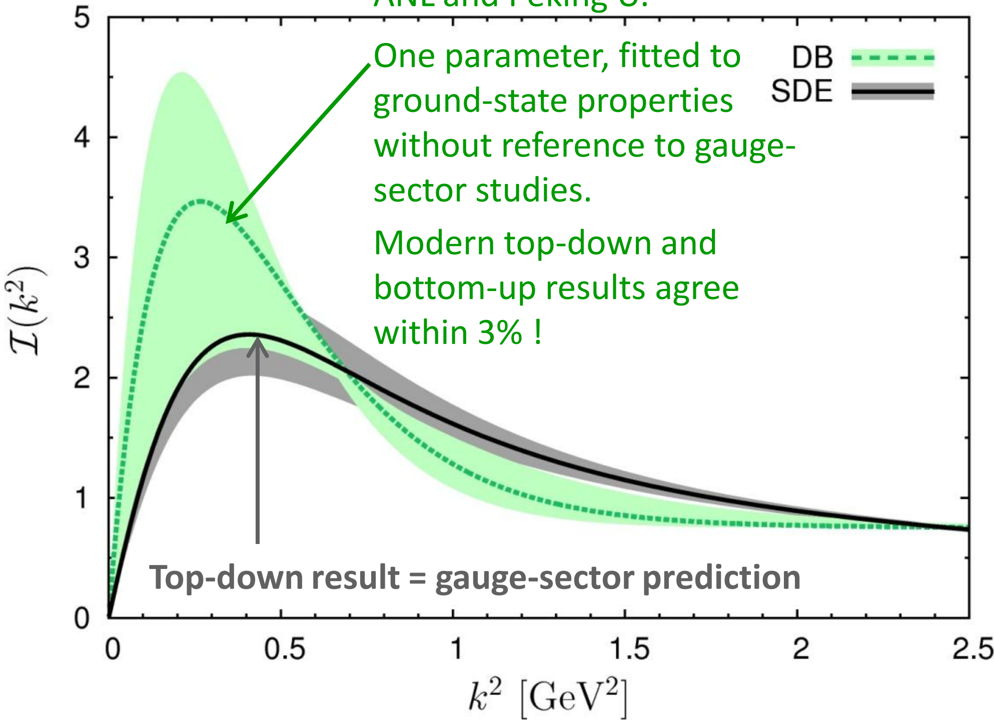
Modern kernels and interaction, developed at ANL and Peking U.



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Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

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C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B **742** (2015) 183*

Top down & Bottom up

- *<u>Bottom-up scheme</u> – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *<u>Top-down</u>* *computat...*
  direct and...
  equations...
- *Serendipit...*
  at one-we...
  in Mathematics and Physics, *has united these two approaches*

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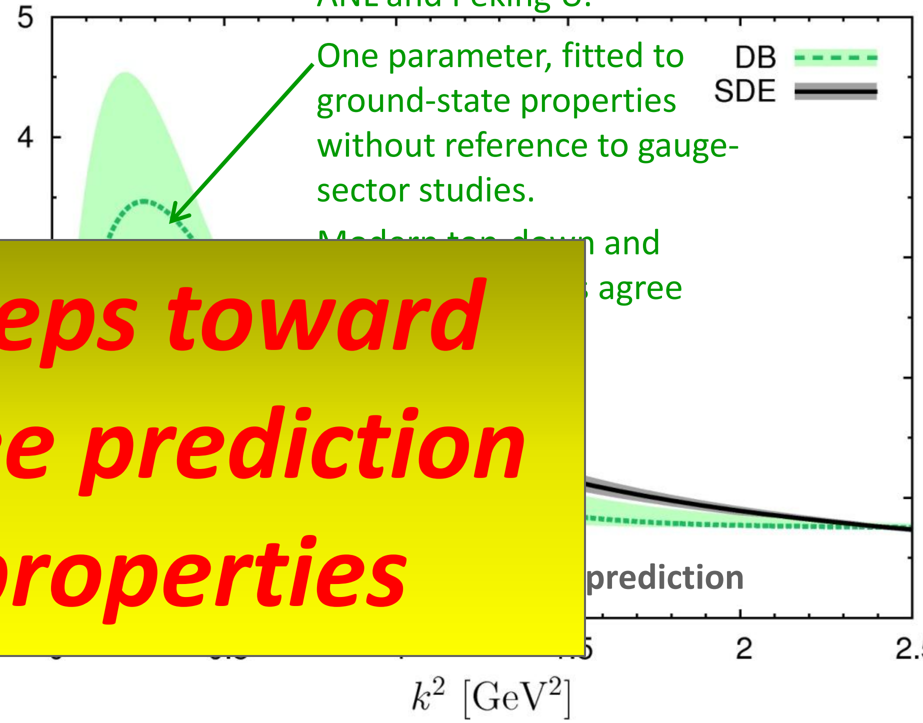
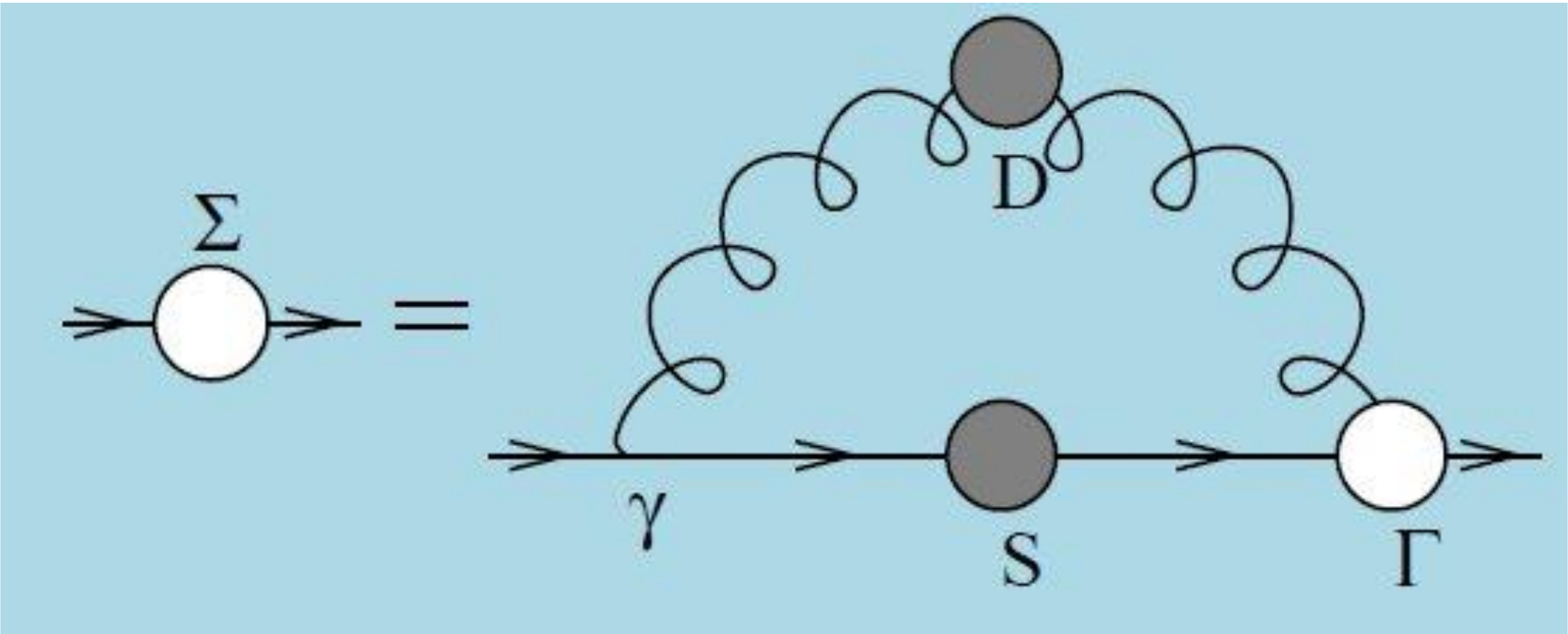
Modern kernels and interaction, developed at ANL and Peking U.



Significant steps toward parameter-free prediction of hadron properties

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Quark Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Dynamical Chiral Symmetry Breaking

- 1963 & 1964 … Baker, Johnson & Willey

  Gap equation in relativistic quantum field theory supports a nonperturbatively-generated, momentum-dependent nonzero solution for $M(p^2)$
- Reproduced time and time-again, in models with varying degrees of separation from QCD
- Result was robust; but ignored by a generally sceptical community
- Until …

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Dynamical Chiral Symmetry Breaking

Information | References (26) | Citations (75) | Files | Plots

Quark propagator in Landau gauge

Jon Ivar Skullerud (Adelaide U. & DESY) , Anthony G. Williams (Adelaide U.)

Jul 2000 - 32 pages

Phys.Rev. D63 (2001) 054508
DOI: 10.1103/PhysRevD.63.054508
ADP-00-40-T423, DESY-00-099
e-Print: hep-lat/0007028 | PDF

Abstract
The Landau gauge quark propagator in momentum space is investigated using the O(a)-improved Sheikholeslami--Wohlert (SW) quark action with a tree-level mean-field improved coefficient c_sw. We have studied the unimproved definition of the quark propagator, as well as two different tree-level O(a)-improved propagators. The ultraviolet behavior of the free lattice propagator is studied for each of these in order to establish which of them provides the most reliable description of the quark propagator up to the medium momentum regime. A general method of tree-level correction is introduced. This exploits asymptotic freedom and removes much of the trivial lattice artifacts at medium to high momenta. We obtain results for the quark propagator which are qualitatively similar to those typically used in quark models. A simple extrapolation of the infrared quark mass M(p^2=0) to the chiral limit gives (298+/-8+/-30) MeV, which is consistent with phenomenological expectations.

Note: 18 pages, 17 figures, uses RevTeX4. Some changes to figures, added further discussion of systematic uncertainties. Version accepted for publication in Phys.Rev.D Report-no: ADP-00-40/T423; DESY 00-099
Keyword(s): INSPIRE: gauge field theory: SU(3) | fermion: lattice field theory | lattice field theory: action | Landau gauge | quark: propagator | quark: mass | effect: finite size | tree approximation | numerical calculations: Monte Carlo
Record added 2000-07-21, last modified 2015-12-22

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Dynamical Chiral Symmetry Breaking

- Next five years – owing to DSE- and lattice-QCD analyses – saw widespread acceptance of the fact that QCD supports a nonzero dressed-quark mass-function in the chiral limit
- Highlight in the USA's 2007 Long Range Plan for Nuclear Science
  - DSE: Bhagwat *et al.*
  - lQCD: CSSM Lattice-QCD (Bowman *et al.*, incl. Derek and Tony W.)

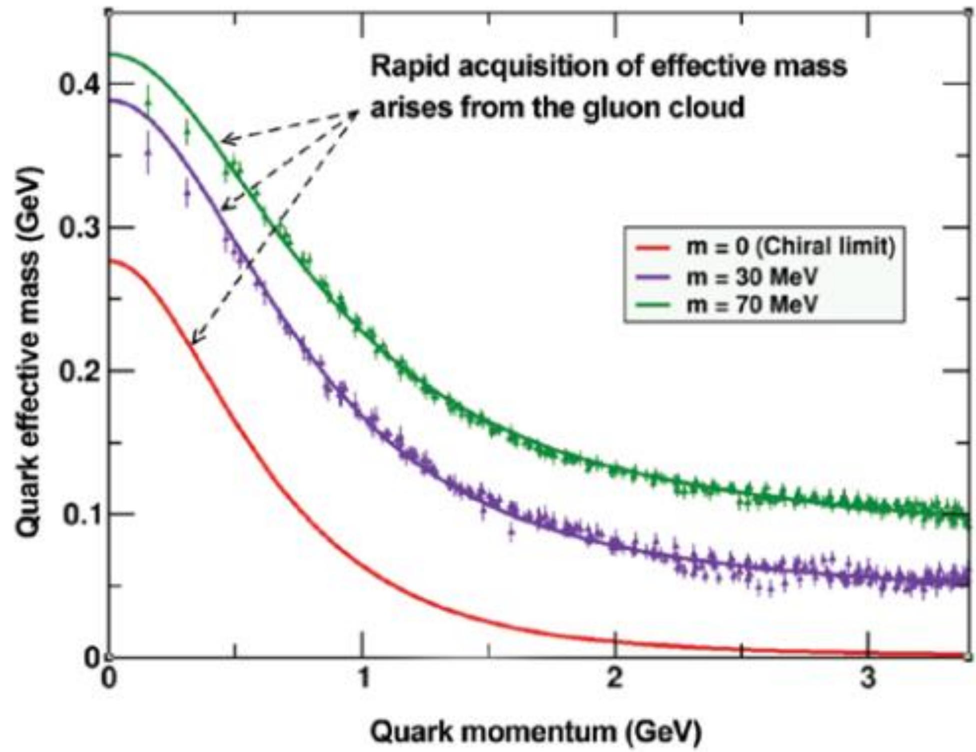
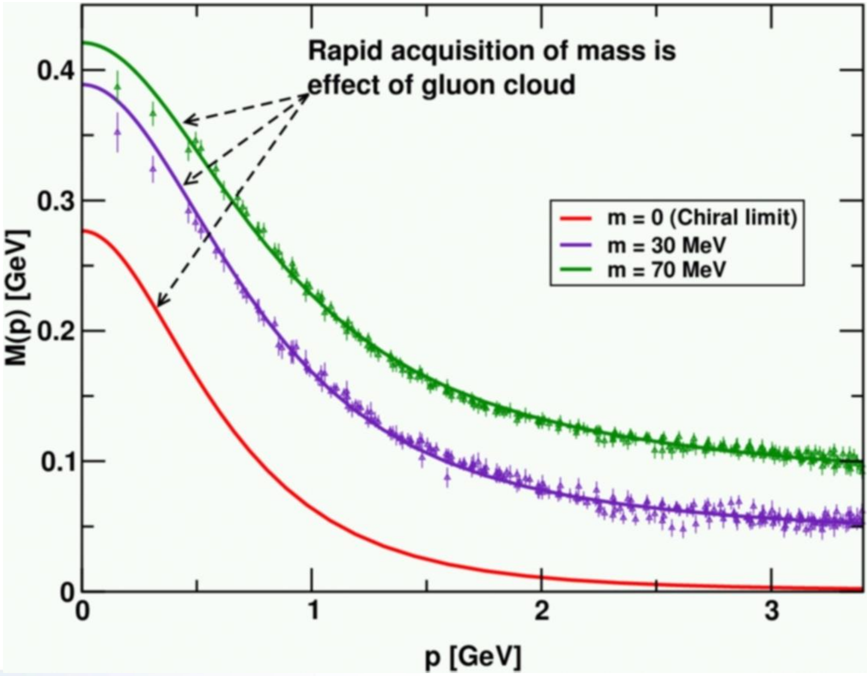


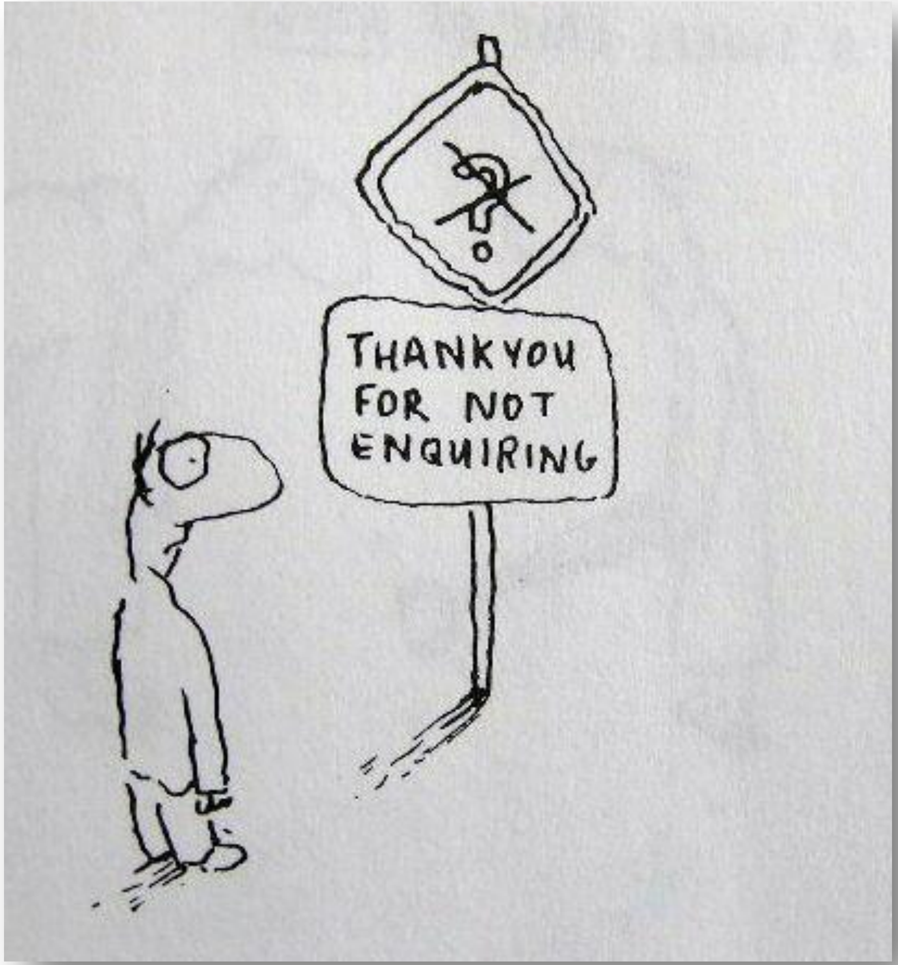
Figure 2.1: Mass from nothing. In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m = 0, red curve) acquires a large constituent mass at low energies.

DCSB

- Dynamical chiral symmetry breaking (DCSB) is a crucial emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that it is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- ***Dynamical***, not spontaneous
  - Add nothing to *QCD* ,

    No Higgs field, nothing! Effect achieved purely through quark+gluon dynamics.





Enigma of Mass

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot PF_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \frac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

$$f_\pi E_\pi(p^2) = B(p^2)$$

This is why $m_\pi=0$ in the absence of a Higgs mechanism

$$f_\pi E_\pi(p^2) = B(p^2)$$

But $m_\pi=0$ is insufficient … QCD predicts $m_\pi^2 \propto m_u+m_d$

If this is missing, then it's not QCD

$$f_\pi E_\pi(p^2) = B(p^2)$$

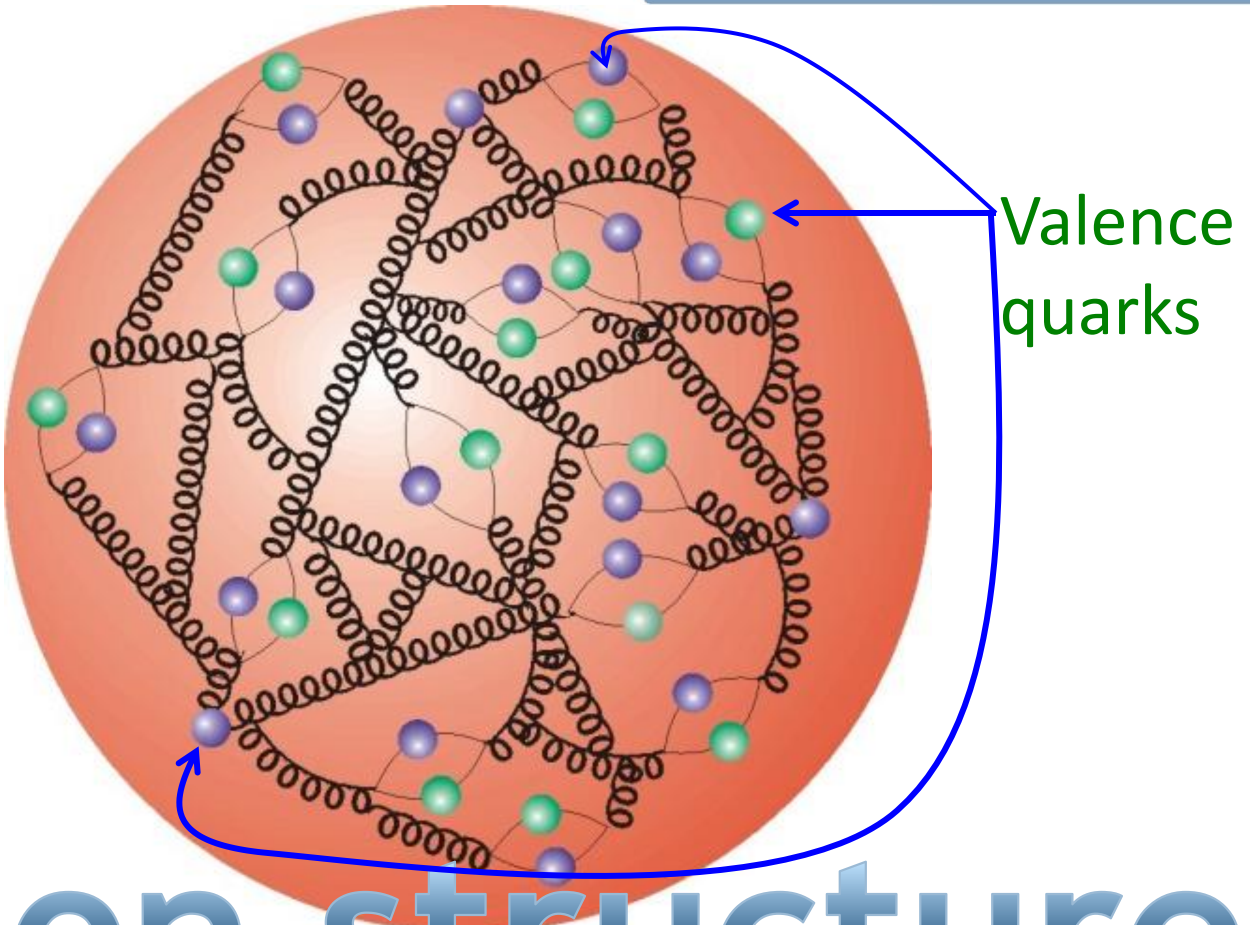
Enigma of mass

This algebraic identity is why QCD's pion is massless in the chiral limit

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

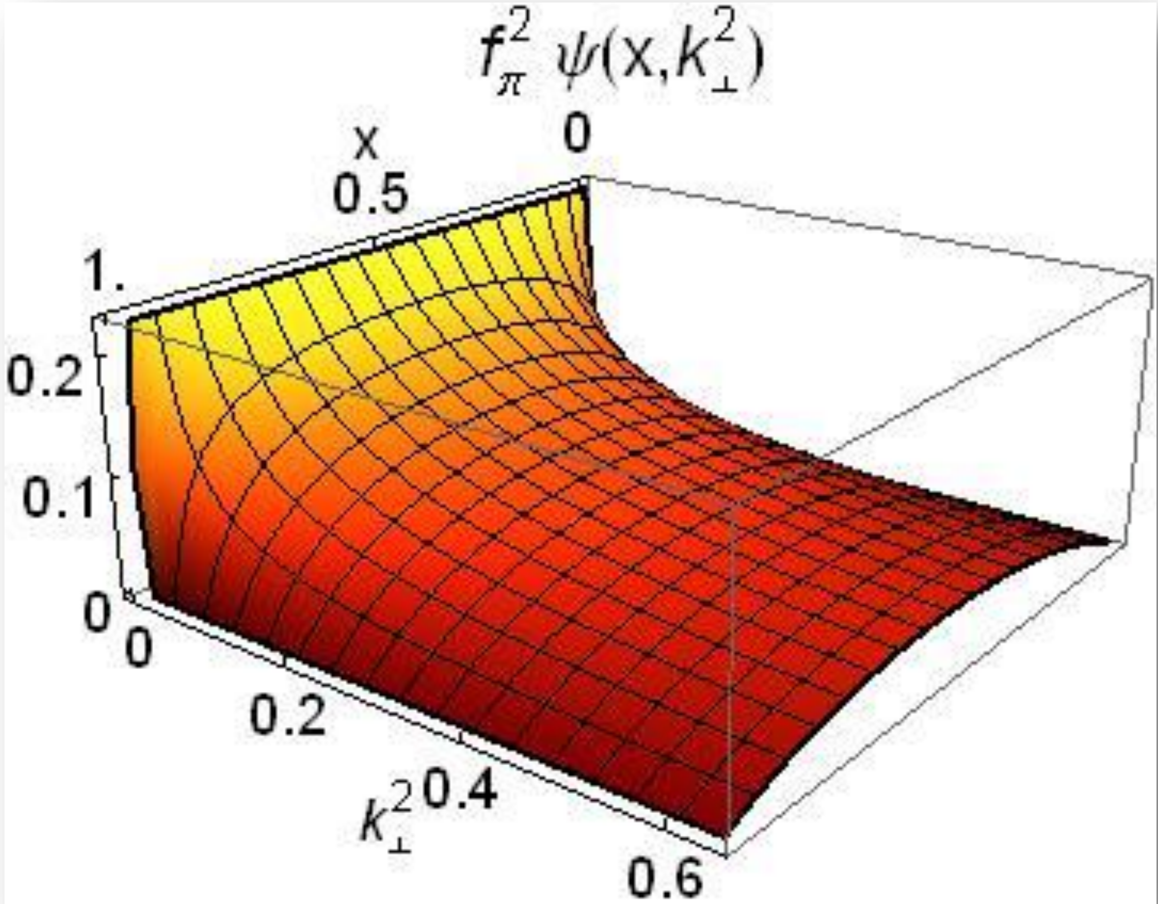
- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.



Parton structure of hadrons

Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it's how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC
- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*



Pion's Wave Function

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Since 2012, methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5 \gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

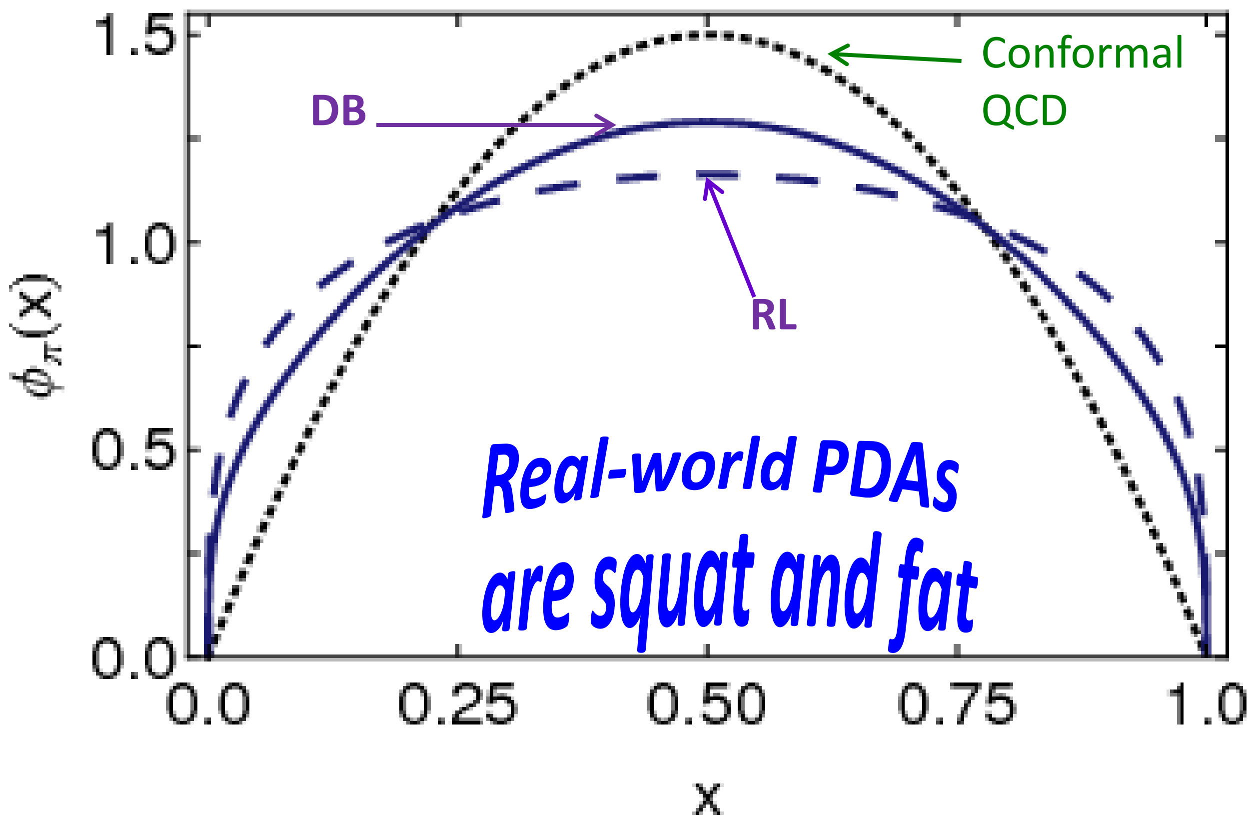
$x^\alpha\,(1-x)^\alpha$, with $\alpha\approx 0.5$

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Continuum-QCD prediction:

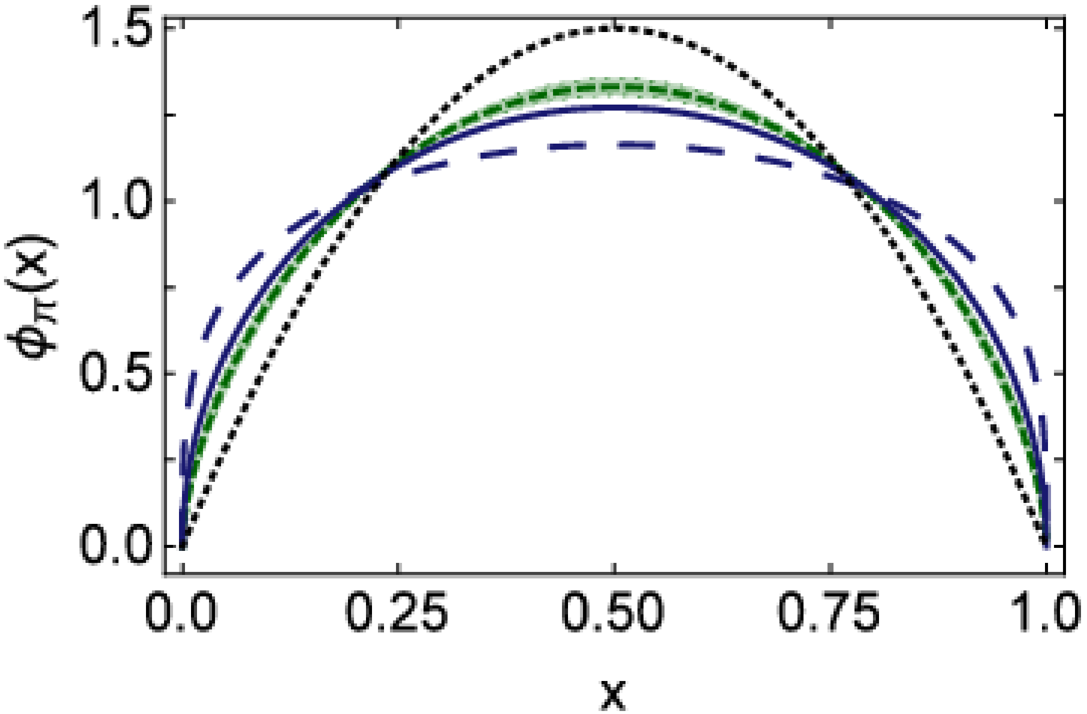
  marked broadening of $\varphi_\pi(x)$, which owes to DCSB



Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi, Lei Chang, Craig D. Roberts, Sebastian M. Schmidt, Peter C. Tandy and Hong-Shi Zong, arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518

Lattice-QCD & Pion's valence-quark PDA

- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Blue solid curve = DSE prediction obtained with DB kernel
- Precise agreement between DSE & lQCD predictions

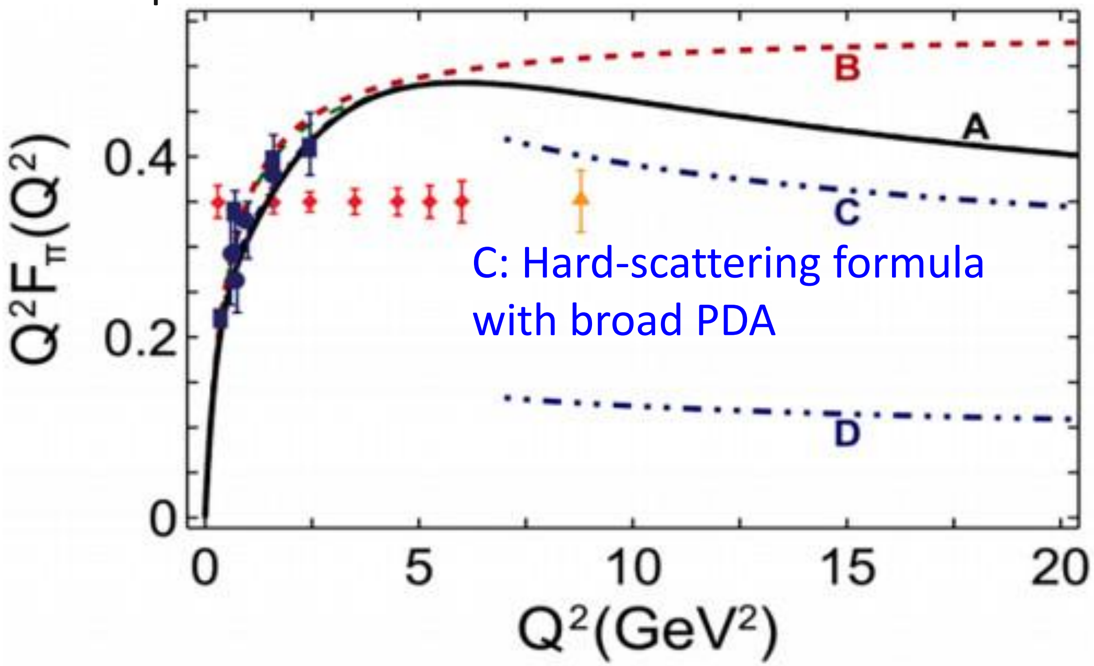


Pion electromagnetic form factor at spacelike momenta
L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt and P. C. Tandy,
arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Highlight in the USA's 2015 Long Range Plan for Nuclear Science

A: Internally-consistent DSE prediction



C: Hard-scattering formula with broad PDA

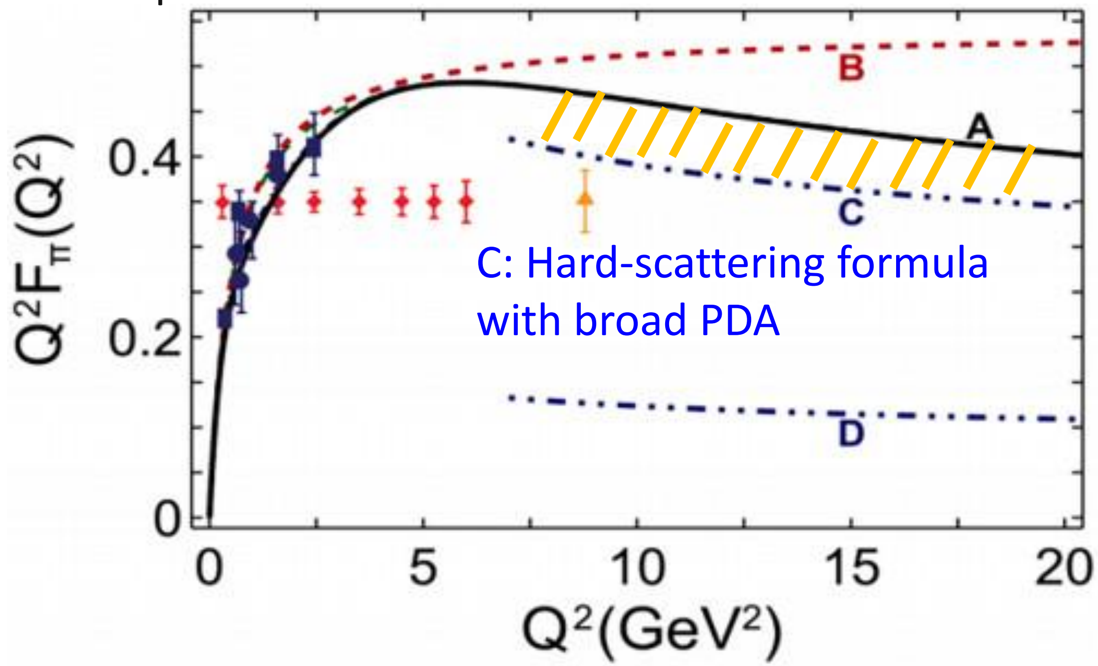
Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pion electromagnetic form factor at spacelike momenta
L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt and P. C. Tandy,
arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Pion's electromagnetic form factor

- Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Highlight in the USA's 2015 Long Range Plan for Nuclear Science
  - Appears that JLab12 is within reach of first verification of a QCD hard-scattering formula

A: Internally-consistent DSE prediction



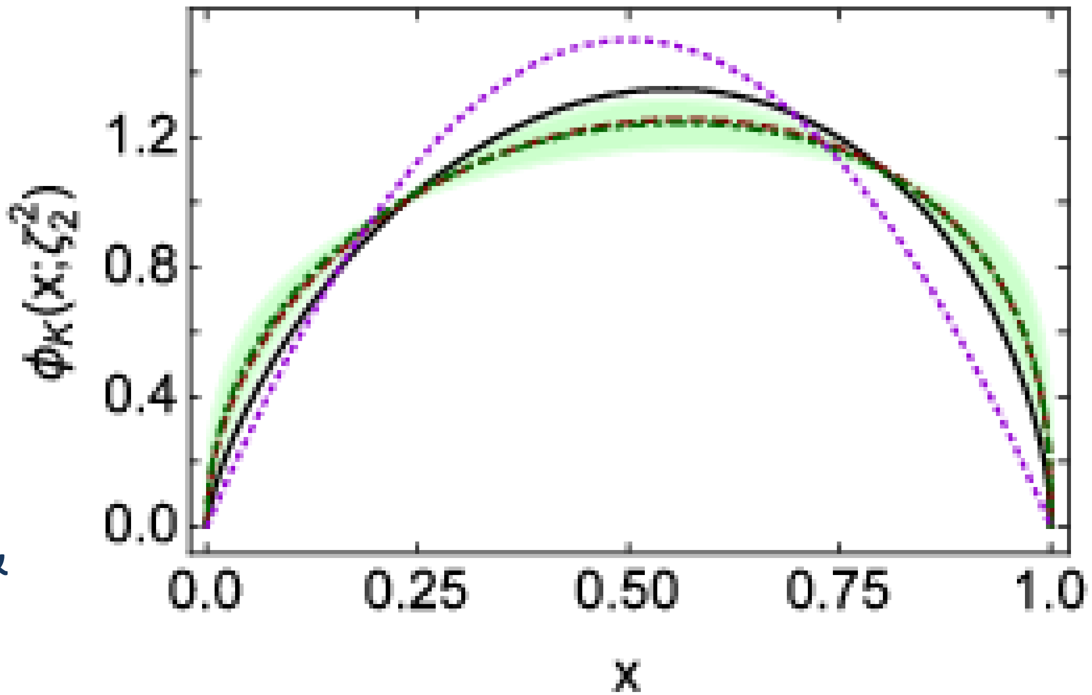
C: Hard-scattering formula with broad PDA

Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Flavour symmetry breaking in the kaon parton distribution amplitude, Chao Shi, Lei Chang, Craig D. Roberts, Sebastian M. Schmidt, Peter C. Tandy and Hong-Shi Zong, arXiv:1406:3353 [nucl-th], Phys. Lett. B 738 (2014) pp. 512–518

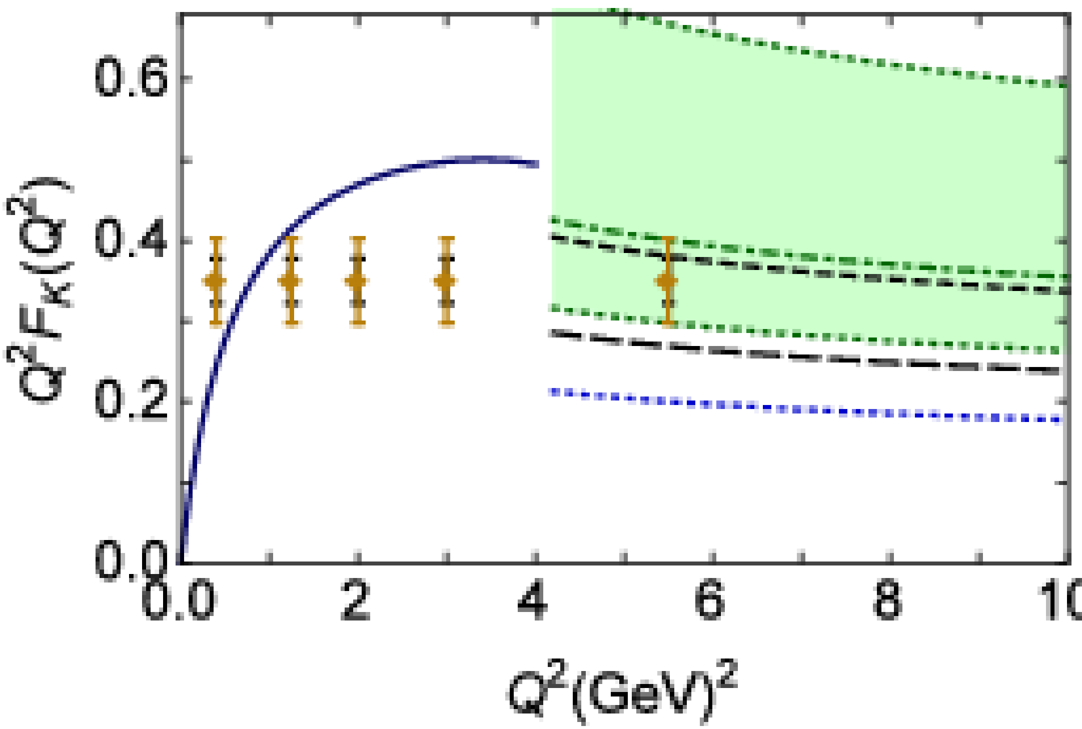
Lattice-QCD & kaon's valence-quark PDA

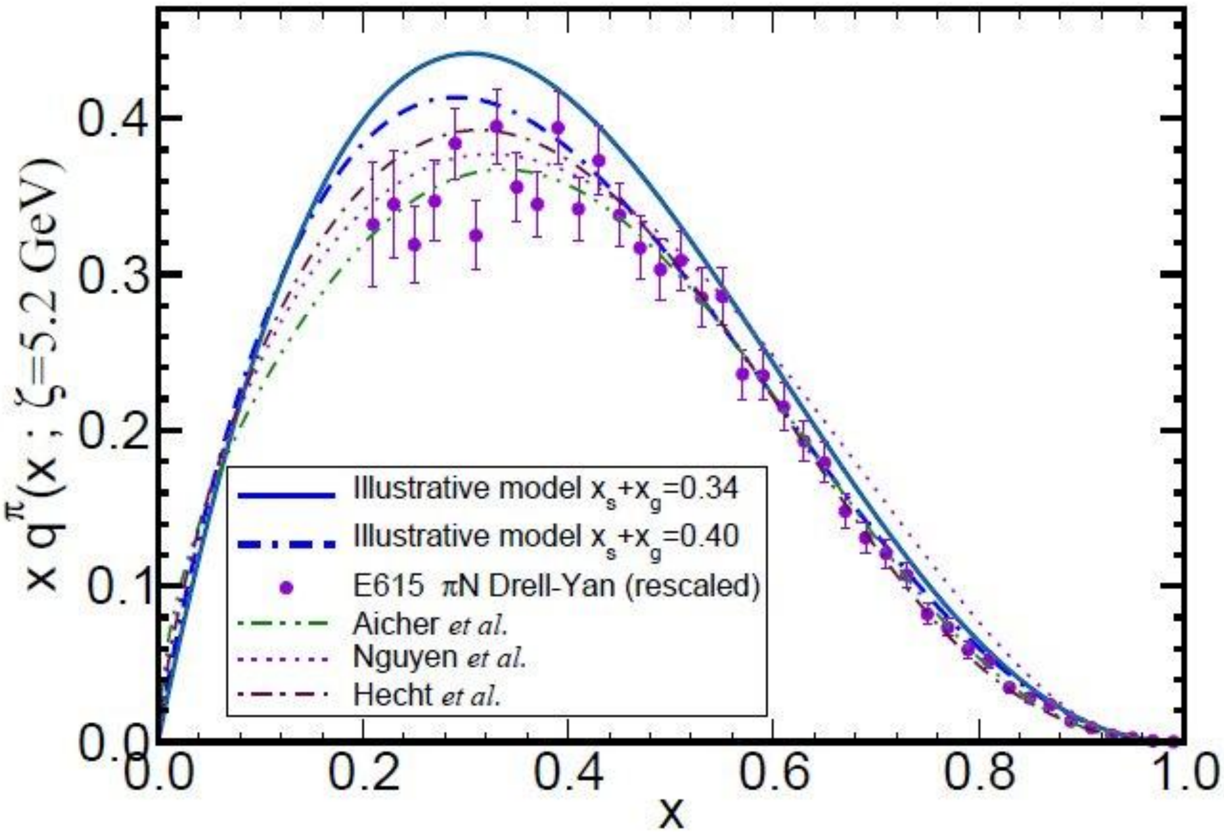
- Isolated dotted curve = conformal QCD
- Green curve & band = result inferred from the single pion moment computed in lattice-QCD
- Black solid and red dashed curves = band of DSE predictions
- Agreement between DSE & lQCD predictions, within errors



Kaon's electromagnetic form factor

- Solid blue curve = Maris-Tandy prediction
- Hard-scattering formula
  - Short- and long-dashed curves = DSE prediction for PDA yields result within this area
  - Green band = broad, skewed lQCD PDA
- Skewing is not the issue: 12%-15%, DSE- and lattice-QCD agree
- It's extent of the broadening that generates the uncertainty
- JLab 12 has potential to settle the issue … Meantime, extend $F_\pi(Q^2)$ analysis on entire spacelike domain → $F_K(Q^2)$





π & K Valence-quark Distribution Functions

π & K PDFs

- Experimental data on π & K PDFs obtained in mesonic Drell-Yan scattering from nucleons in heavy nuclei; but it's old: 1980-1989
- Newer data would be welcome:
  - persistent doubts about the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - single modest-quality measurement of $u^K(x)/u^\pi(x)$ cannot be considered definitive.
- Approved experiment, using tagged DIS at JLab 12, should contribute to a resolution of pion question; and a similar technique might also serve for the kaon.
- Furthermore:
  - new mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs,
  - as could two-jet experiments at the large hadron collider;
  - and an EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.
- Gribov-Lipatov reciprocity (crossing symmetry) entails connection between PDFs and fragmentation functions on $z \simeq 1$ ($z \geq 0.75$)

$$D_{H/q}(z) \approx z\, q^H(z)$$

Reliable information on meson fragmentation functions is critical if the worldwide programme aimed at determing TMDs is to be successful

Pion's Valence-Quark Distribution Function

- Hadron PDAs are not directly measurable; but experiments place constraints on their Mellin moments
- One of the earliest predictions of the QCD parton model (1974,1975):

  $q^\pi(x) \sim (1\text{-}x)^{2}$

- Owing to the validity of factorisation in QCD,
  $q^\pi(x)$ is directly measurable in πN Drell-Yan experiments
- E615 @ FNAL (Conway:1989fs): leading-order analysis of πN Drell-Yan

  $q^\pi(x) \sim (1\text{-}x)^{1}$

  Apparent *disagreement* with QCD.

  Nevertheless used to produce "modern" PDF fits

E615 Controversy

$q^{\pi}(x) \sim (1\text{-}x)^{1}$

Numerous "explanations"

- Nambu – Jona-Lasinio model, translationally invariant regularisaion

  $q^{\pi}(x) \sim (1\text{-}x)^{0}$,

  which becomes one after evolving from a low resolution scale
- NJL models with a hard cutoff & also some duality arguments:

  $q^{\pi}(x) \sim (1\text{-}x)^{1}$
- Relativistic constituent quark models:

  $q^{\pi}(x) \sim (1\text{-}x)^{0\ldots2}$

  depending on the form

  of model wave function
- Instanton-based models

  $q^{\pi}(x) \sim (1\text{-}x)^{1\ldots2}$

Any power you might like in models possessing no connection with QCD.

What can be learnt from this?

Is QCD threatened?

Basic features of the pion valence-quark distribution function,
Lei Chang, Cédric Mezrag, Hervé Moutarde, Craig D. Roberts, Jose Rodríguez-Quintero and Peter C. Tandy,
arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang, Craig D. Roberts, Shaolong Wan and Hong-Shi Zong,
arXiv:1602.01502 [nucl-th]

Valence-quark PDFs within mesons

- Compute PDFs from imaginary part of virtual-photon – pion forward Compton scattering amplitude:

  $\gamma\, \pi \to \gamma\, \pi$

- Handbag diagram is insufficient. Doesn't even preserve global symmetries. Exists a class of leading-twist corrections that remedies this defect ⇒

Projection onto light-front

$$u_V^\pi(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta^\pi)$$

Partial derivative wrt relative momentum

$$\times\, \partial_{k_\eta^\pi} \left[\Gamma_\pi(k_\eta^\pi, -P_\pi) S(k_\eta^\pi)\right] \Gamma_\pi(k_{\bar\eta}^\pi, P_\pi)\, S(k_{\bar\eta}^\pi)\,,$$

Similar expressions for $u_V^K(x)$, $s_V^K(x)$

Valence-quark PDFs within mesons

- Formulae guarantee that valence-quark PDFs satisfy, independent of model details:

$$\langle x \rangle_u^0 = \int_0^1 dx\, x\, u_V^0(x) = \frac{1}{2}$$

$$\int_0^1 dx\, x[u_V^K(x) + \bar{s}_V^K(x)] = 1$$

- Ensure, too, that at an hadronic scale $\zeta \approx 0.5$ GeV

$$q_V^M(x \simeq 1) \propto (1-x)^{2n}$$

in any theory with $(1/k^2)^n$ vector-boson exchange interaction

Valence-quark PDFs within mesons

- Algebraic analysis
  - $\Delta_f(k^2)=1/[k^2+M_f^2]$
  - $M_R=M_{us}^2/[M_u+M_s]$
  - $M_{us}^2 = M_u\, M_s$
  - M_u , M_s = dressed-quark mass-scales: M_u=0.4GeV and $M_s=1.2M_u$ from most sophisticated DSE analyses
  - β = 1 is kaon asymmetry parameter, fixed to reproduce best DSE result for first moment of kaon PDA
  - $\Lambda_{\pi,K}$ = widths of meson Bethe-Salpeter amplitudes: chosen to ensure $f_{\pi,K}$ reproduce empirical values

$$S_f(k) = [-i\gamma\cdot k + M_f]\Delta_{M_f}(k^2)\,,$$

$$\Gamma_\pi(k_{\bar\eta/\eta};\pm P) = i\gamma_5 \frac{M_u}{n_\pi}\frac{3}{4}\int dz \times (1-z^2)M_u^2\Delta_{\Lambda_\pi}(k_z^2)\,,$$

$$\Gamma_K(k_{\bar\eta/\eta};\pm P) = i\gamma_5 \frac{M_R}{n_K}\frac{3}{4}\int dz \times (1-z^2)(1+\beta z)M_{us}^2\Delta_{\Lambda_K}(k_z^2)\,,$$

Could use sophisticated DSE input, like Tandy et al. *arXiv:1102.2448 [nucl-th]; but, as demonstrated in arXiv:1602.01502 [nucl-th], nothing material is gained*

Predictions of algebraic framework are indistinguishable and provide additional, novel insights

arXiv:1602.01502 [nucl-th]

Valence-quark PDFs within mesons

- Pointwise results obtained via reconstruction from (arbitrarily many) computed PDF moments
- Peak in kaon PDFs shifted 17% away from x=½,

  i.e. scale of flavour symmetry breaking is set by DCSB (M_s/M_u=1.2), here as in all other nonperturbative quantities
- $[u_V^K(x)+s_V^K(x)]/2$ must be symmetric, owing to momentum sum rule.

  Similar but not identical to $u_V^\pi(x)$

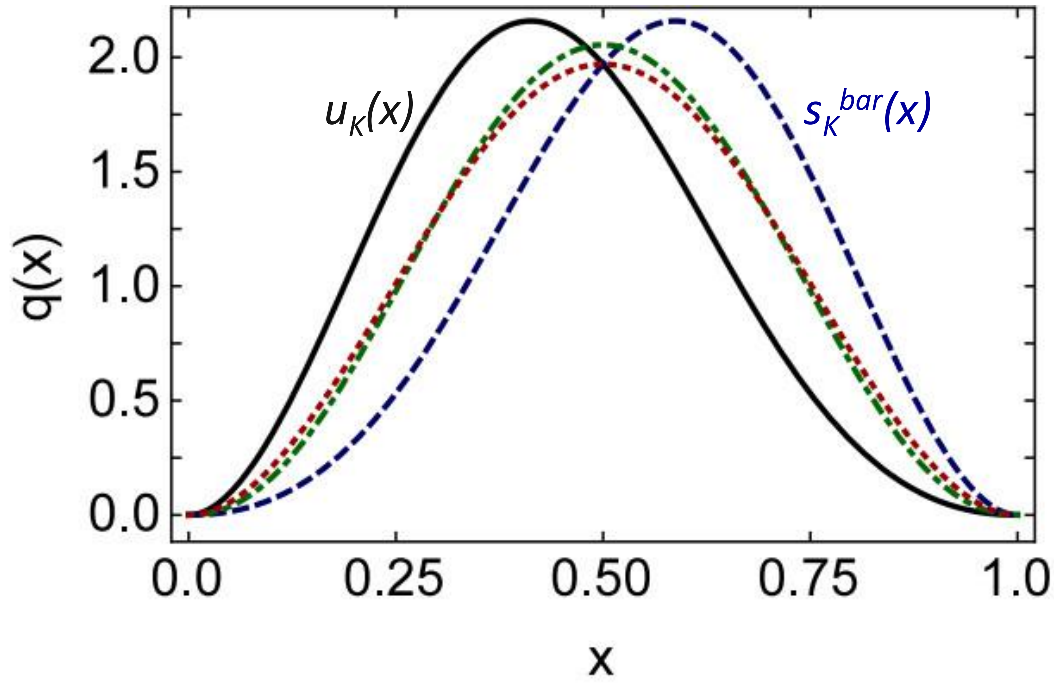


FIG. 1. Valence-quark PDFs at the hadronic scale, ζ_H, defined by Eqs. (14), (17): $u_V^K(x)$, solid (black) curve; $\bar{s}_V^K(x)$, dashed (blue) curve; $u_V^\pi(x)$, dot-dashed (green) curve; and $[u_V^K(x) + \bar{s}_V^K(x)]/2$ dotted (red) curve.

Valence-quark PDFs within mesons

- *x=1* values of ratios of PDFs are invariant under DGLAP evolution

  Consequently, they're a scale invariant discriminator between competing pictures of hadron structure

$$\left.\frac{u_V^K(x)}{u_V^\pi(x)}\right|_{x\to 1} = 0.37\,, \quad \left.\frac{u_V^\pi(x)}{\bar{s}_V^K(x)}\right|_{x\to 1} = 0.29$$

- On the other hand

$$\lim_{x\to 0}\frac{u^K(x;\zeta)}{u^\pi(x;\zeta)} \stackrel{\Lambda_{\rm QCD}/\zeta\simeq 0}{\to} 1$$

  Owes to inexorable growth in both mesons' gluon and sea-quark content driven by pQCD splitting mechanisms.

  Analogous to the convergence of all meson PDAs to conformal form as $\Lambda_{QCD}/\zeta \to 0$

Building realistic distributions

- In order to sensibly evolve PDFs, one must include sea and glue content at hadronic scale, ζ_H
- Pion distribution at ζ_H
  - DSE prediction, following from analysis of leptonic decay: π contains 5% sea
  - Assume GRV analysis of πN Drell-Yan is reliable, then 30% of π momentum carried by glue

  Adopting standard PDF parametrisations, this additional information is sufficient to completely fix realistic $u_\pi(x;\zeta_H) = valence + sea + glue$
- Kaon distribution at ζ_H
  - Owing to heavier mass of intermediate states that can introduce sea-quarks, safe to assume sea-quark content of kaon is effectively zero
  - Treat momentum fraction carried by glue as a parameter to be used in understanding $u^K(x)/u^\pi(x)$

    … owing to heavier mass of s-quark, expect $\langle x\rangle_g^K < \langle x\rangle_g^\pi$; but how much less?

Pion PDF

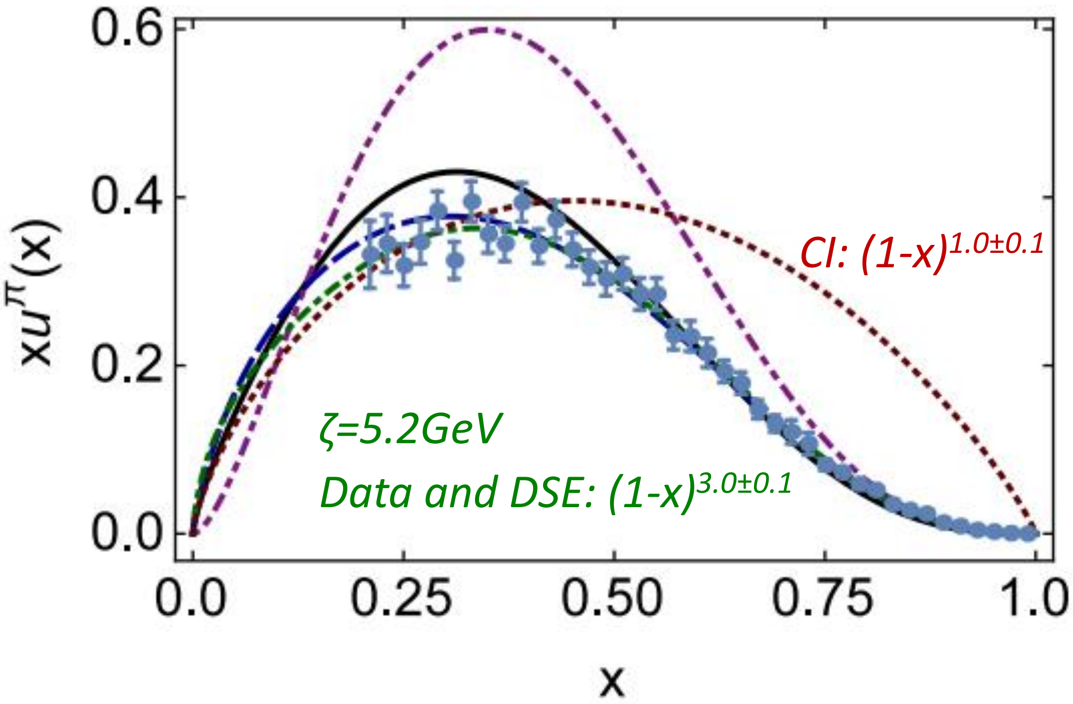


FIG. 3. $xu^{\pi}(x;\zeta_{5.2})$. Solid (black) curve, our prediction, expressed in Eqs. (32), (33); dot-dot-dashed (purple) curve, result obtained when sea-quark and gluon contributions are neglected at ζ_H, *i.e.* using $u_V^{\pi}(x)$ from Eqs. (14), (17); dashed (blue) curve first DSE prediction [38]; and data, Ref. [4], rescaled according to the reanalysis described in Ref. [40], from which the dot-dashed (green) curve is drawn. The dotted (red) curve is the result obtained using a Poincaré-covariant regularisation of a contact interaction, Eq. (36).

- Purple dot-dot-dash = prediction at ζ_H
- Data = modern reappraisal of E615: NLO analysis plus soft-gluon resummation (ASV)
- Solid black curve, prediction evolved to ζ=5.2GeV, the scale associated with the experiments
- Blue dashed curve = first DSE prediction, in 2000 (ζ=5.2GeV)
- Dotted red curve = result obtained with momentum-independent gluon exchange (contact interaction, ζ=5.2GeV)

Pion:
DSE comparison with lQCD moments

- All lQCD studies agree with each other, within errors
- DSE and lQCD agree, within errors; and DSE at level of 4% with lQCD-average
- [66]: Brommel et al. (2007)
  [67]: Best *et al*. (1997)
  [68]: Detmold *et al*. (2003)
- On light-front, just 52% of the pion's momentum is carried by valence-quarks at ζ_2 = 2GeV, down from 65% at ζ_H= 0.51GeV

moments. Such results are available for $u^\pi(x)$, *e.g.* a contemporary simulation [66], using two dynamical fermion flavours, $m_\pi \gtrsim 0.34$ GeV and nonperturbative renormalisation at $\zeta_2 = 2$ GeV, produces the first row here:

	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
[66]	0.27(1)	0.13(1)	0.074(10)
[67]	0.28(8)	0.11(3)	0.048(20)
[68]	0.24(2)	0.09(3)	0.053(15)
average	0.26(8)	0.11(4)	0.058(27)
herein	0.26	0.11	0.052

(37)

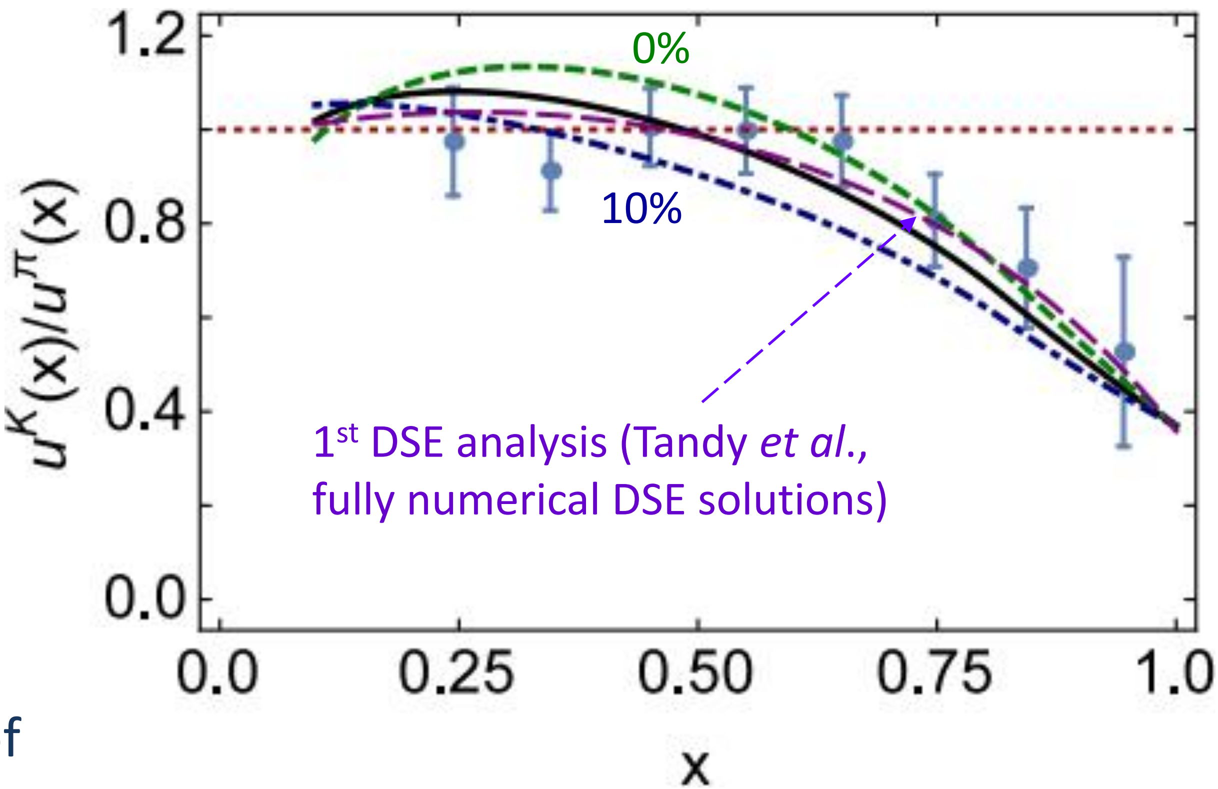
The results in Ref. [66] agree with those obtained in earlier estimates based on simulations of quenched lQCD [67, 68] and are consistent with the values obtained from our computed distribution, which are reported in the last row of Eq. (37).

Gluon content of kaon

- $\langle x\rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- Evolved to ζ_2

q	$\langle x\rangle_q^K$	$\langle x^2\rangle_q^K$	$\langle x^3\rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum



π & K PDFs

- Dressed-quark basis and symmetry-preserving (beyond-handbag) expressions used to analyse π & K valence-quark PDFs … guarantee that at hadronic scale

  $q_V(x;\zeta_H) \propto (1-x)^2$ on $x \simeq 1$

- Flavour-dependence of DCSB modulates the strength of SU(3)-flavour symmetry breaking in meson PDFs, as it does in every other nonperturbative quantity
- At ζ_H:
  - valence dressed-quarks carry roughly two-thirds of pion's light-front momentum, with the bulk of the remainder carried by glue … sea-quarks carry roughly 5%
  - contrast, valence dressed-quarks carry approximately 95% of the kaon's light-front momentum, with the remainder lying in the gluon distribution … sea-quarks carry $\simeq$ 0 %
    - heavier *s*-quarks radiate soft gluons less readily than lighter quarks and momentum conservation subsequently constrains gluons associated with the kaon's *u*-quark

π & K PDFs

- Evolving distributions to scale characteristic of meson-nucleon Drell-Yan experiments, ζ=5.2 GeV
  - extant data reproduced
    - *$x \simeq 1$: $q(x) \propto (1-x)^3$*
  - ratio $u^K(x)/u^\pi(x)$ explained by vastly different gluon content of π & K
- Distributions evolved the distributions to ζ_2 = 2 GeV, which is typically used in numerical simulations of lattice-regularised QCD:
  - Valence-quarks carry roughly half the pion’s light-front momentum but two-thirds of the kaon’s momentum.
- *Next steps*
  - *Do not anticipate any improvement over these results using continuum methods in QCD*
    - *Can lQCD say anything novel? (As yet, nothing from lattice on kaon PDF)*
  - *Empirical verification of the predictions is essential (equivalent to verifying hard-scattering formulae)*
  - *Extend analysis to GPDs, TMDs and Fragmentation Functions*

Epilogue

Epilogue

- Conformal anomaly ... *gluons and quarks acquire momentum-dependent masses*, values are large in the infrared $m_g \propto 500$ MeV & $M_q \propto 350$ MeV ... underlies DCSB and has numerous observable consequences
- In a Universe with light quarks, confinement is a dynamical phenomenon ... *no linear potentials, no tower of linear, nonintersecting "Regge" trajectories*
- Top-down and bottom-up DSE analyses agree on RGI interaction in *continuum-QCD ⇒ parameter-free prediction of hadron properties*
- Not many good reasons left which justify using rainbow-ladder truncation ... pointwise forms of interaction and propagators are simply wrong
- *π & K PDAs and PDFs are understood* ... framework exists for computation of all related distribution functions
- Numerous other applications in hadron physics ... also learnt much about baryons

  meson cloud does not alter level ordering in baryon spectrum

  Nucleon → Nucleon ... *Nucleon → Δ* ... *Nucleon → Roper* ... understood

Congratulations Tony!