Collaboration in aspects of non-perturbative QFT

- Studies of Non-perturbative propagators, vertices in QED and QCD
- Dynamical Chiral Symmetry Breaking and criticality of the running coupling using DSE
- Quark-gluon vertex on the Lattice

An excited Journey from understanding the existence of visible matter to invisible matter…

Happy belated birthday Tony…

Non-perturbative Interactions of Quarks and Gluons

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Anthony G. Williams (Adelaide)

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

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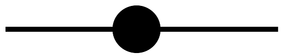
Centre for Subatomic Structure of Matter

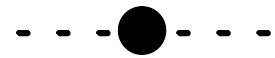
Components of QCD

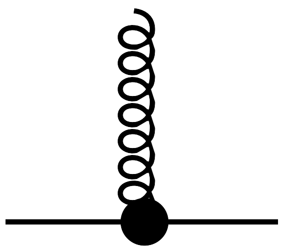
Gluon Propagator

Quark Propagator

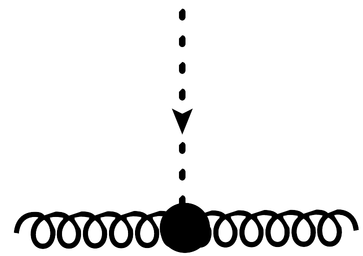


Ghost Propagator



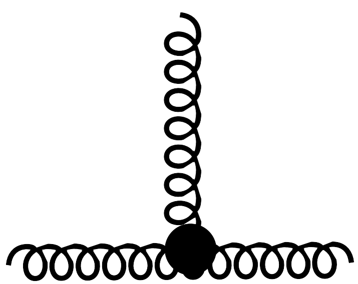


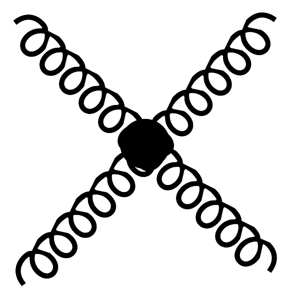
QCD



Quark-Gluon Vertex

Ghost-Gluon Vertex

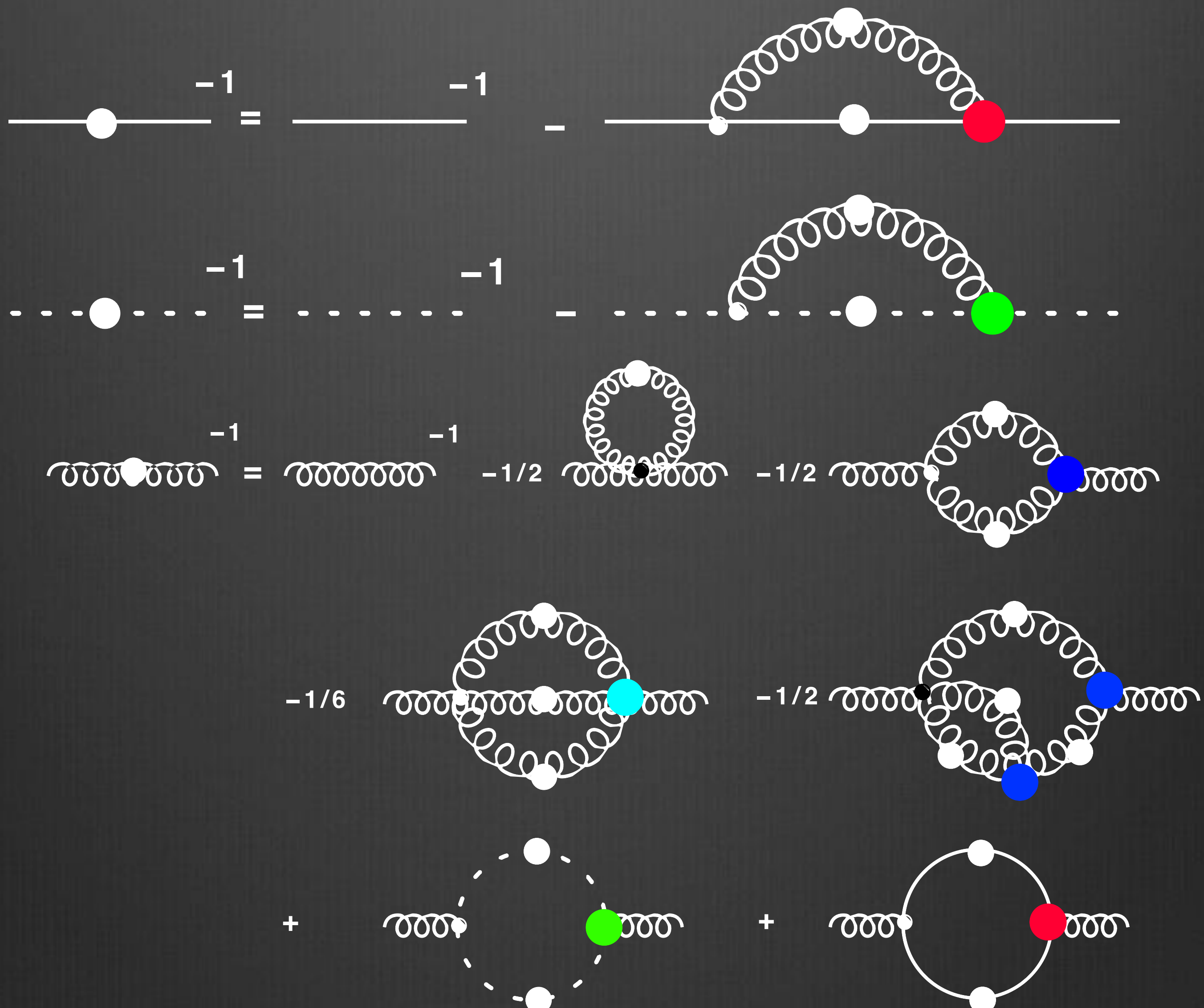




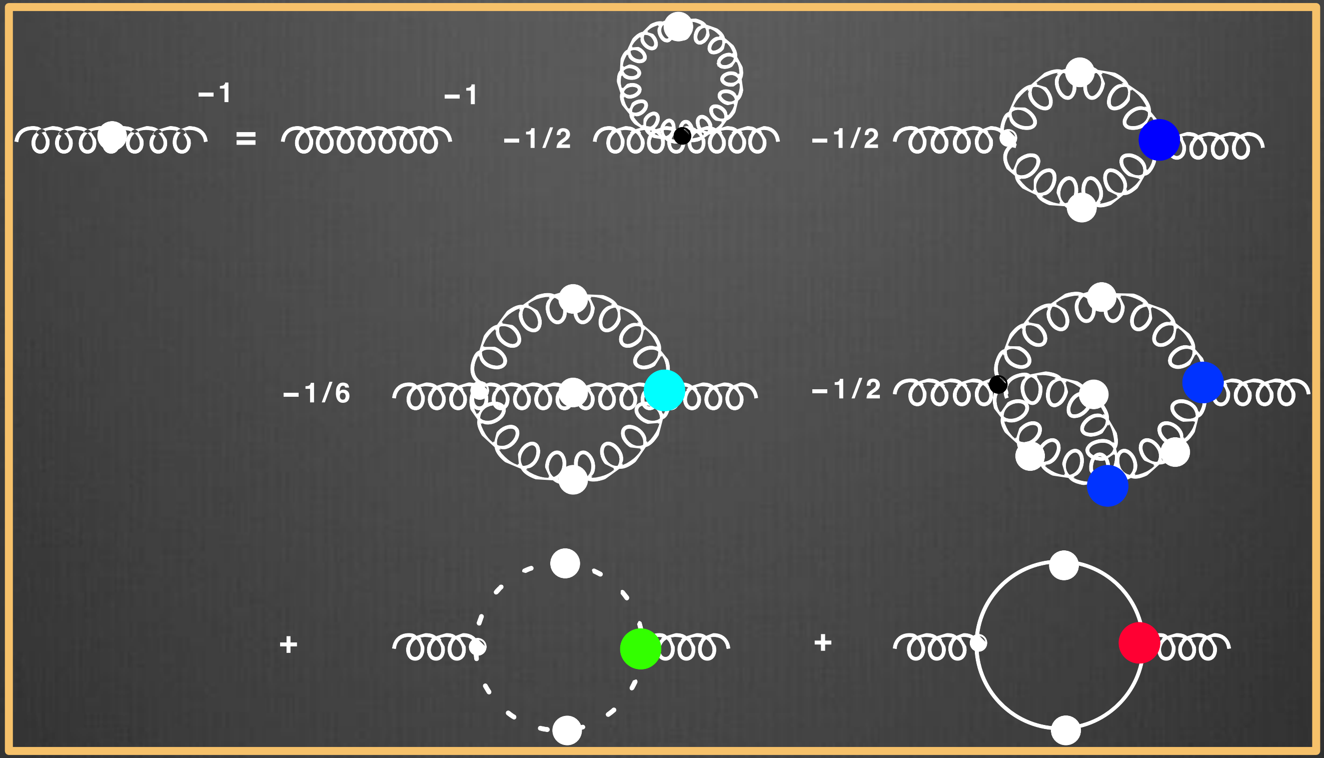
3-Gluon Vertex

4-Gluon Vertex

Schwinger-Dyson Equations



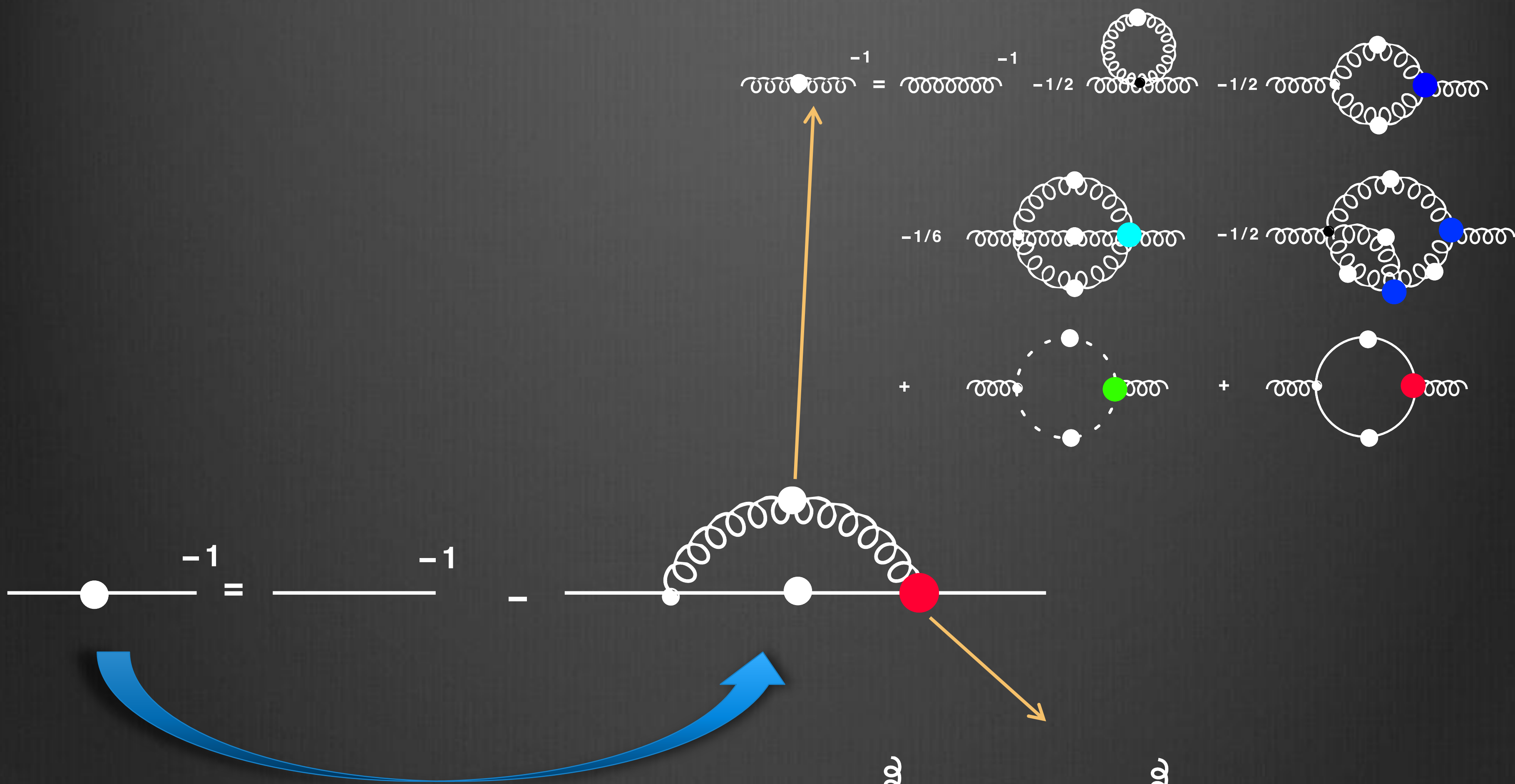
Gluon Propagator



$$D_{\mu\nu}(q) = \frac{-1}{q^2}\left[G(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \xi \frac{q_\mu q_\nu}{q^2}\right]$$

Ghost Propagator

$$D_G(q) = -\frac{G(q)}{q^2}$$



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Quark Propagator in Landau Gauge

In Continuum

$$S_F(p) = \frac{Z(p^2)}{\not{p} - M(p^2)} = \frac{1}{A(p^2)\ \not{p} - B(p^2)}$$

In Discrete

$$S^L(pa) = \frac{Z^L(pa)}{ia\,\not{K}(p) + aM^L(pa)}$$

$$K_\mu(p) \equiv \frac{1}{a} sin(p_\mu a)$$

$Z \equiv 1/A$ Quark (Fermion) Wave-Function Renormalisation Z^L

$M \equiv B/A$ Mass Function M^L

Improved Wilson, Clover Fermion Action (Sheikholeslami-Wohlert) in Discrete

$$S_{SW} = S_W - i\frac{a}{4}g_0 c_{SW} \sum_x \sum_{\mu\nu} \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

Continuum Limit

$$\overline{\psi}(x)\left(i \not{D} + m\right)\psi(x) + \overline{\psi}(x)D^2\psi(x) + \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

Simulation Parameters:

β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	N_{cfg}
5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900
5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900
5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908
			$64^3 \times 64$	290		314
5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900

Other fermion actions such as Staggered, overlap fermions are used in different studies…

Howeever continuum limit must stay the same

$$S^L(pa) = Z^L(p^2)\,\frac{-ia\,\not{K}(p) + aM^{(}pa)}{a^2K^2(p) + a^2M^2(p)}$$

$$Z(p^2) \approx 1 - \frac{a^2k^2}{16} + \mathcal{O}(a^2)$$

$$aM(p^2) \approx am\left(1 - \frac{am}{2} + \frac{a^2k^2}{16} + \frac{k^2 + 4m^2 - 3k^4/m^2}{k^2 + m^2} - \frac{a^2k^2}{16}\right) + \mathcal{O}(a^3)$$

The quark propagator using state-of-the art gauge configurations with $N_f = 2$ flavors of O(a) improved Wilson fermion.

SW action, an O(a) improved quark propagator

$$S(x,y) \quad \equiv \quad < \psi(x)\overline{\psi}(y) >$$

O(a) improvement of quark wave function

$$\psi \quad \rightarrow \quad (1 + b_q am)(1 - c_q a \not{D})\psi$$
$$\overline{\psi} \quad \rightarrow \quad \overline{\psi}(1 + b_q am)(1 + c_q a \overleftarrow{\not{D}})$$

$$S(x,y) \quad = \quad < (1 + b_q am)^2(1 - c_q a \not{D}(x))S_0(x,y;U)(1 + c_q a \not{D}(y)) >$$

tree level values:

$$c_q = b_q = 1/4$$

tree level quark propagator

$$S^L(pa) = \frac{Z^L(pa)}{ia\, \not{K}(p) + aM^L(pa)}$$

The O(a) improved quark propagator suffers from large tree-level lattice artefacts, which complicate the determination of the physical momentum dependence of the form factors.

$$Z(ap) = \frac{Z^L(ap)}{Z^{(0)}(ap)} \qquad aM(p) = \frac{M^L(pa) - a\Delta M^{(-)}(pa)}{Z_m^{(+)}}$$

tree-level quark wave function

Need to remove lattice artefacts:

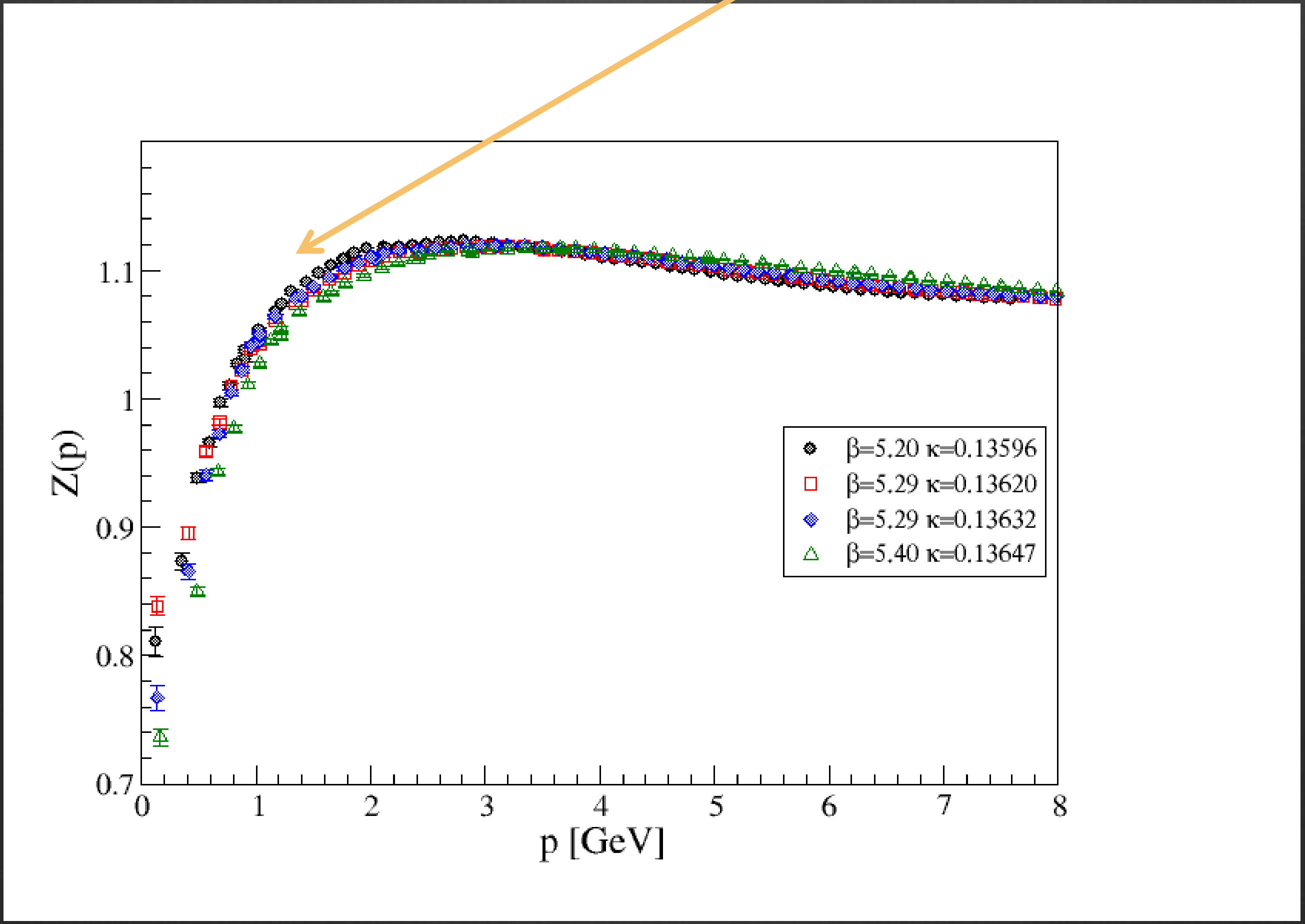
- momentum cuts (cylinder cut)
  select momenta close to the diagonal in 4-momentum space, which have the smallest hypercubic artefacts.
- tree level correction

Quark wave function renormalisation

$(m_\pi = 422MeV\ ;\ m_q = 17.0MeV)$
$(m_\pi = 280MeV\ ;\ m_q = 6.2MeV)$
$(m_\pi = 295MeV\ ;\ m_q = 8.0MeV)$
$(m_\pi = 426MeV\ ;\ m_q = 18.4MeV)$

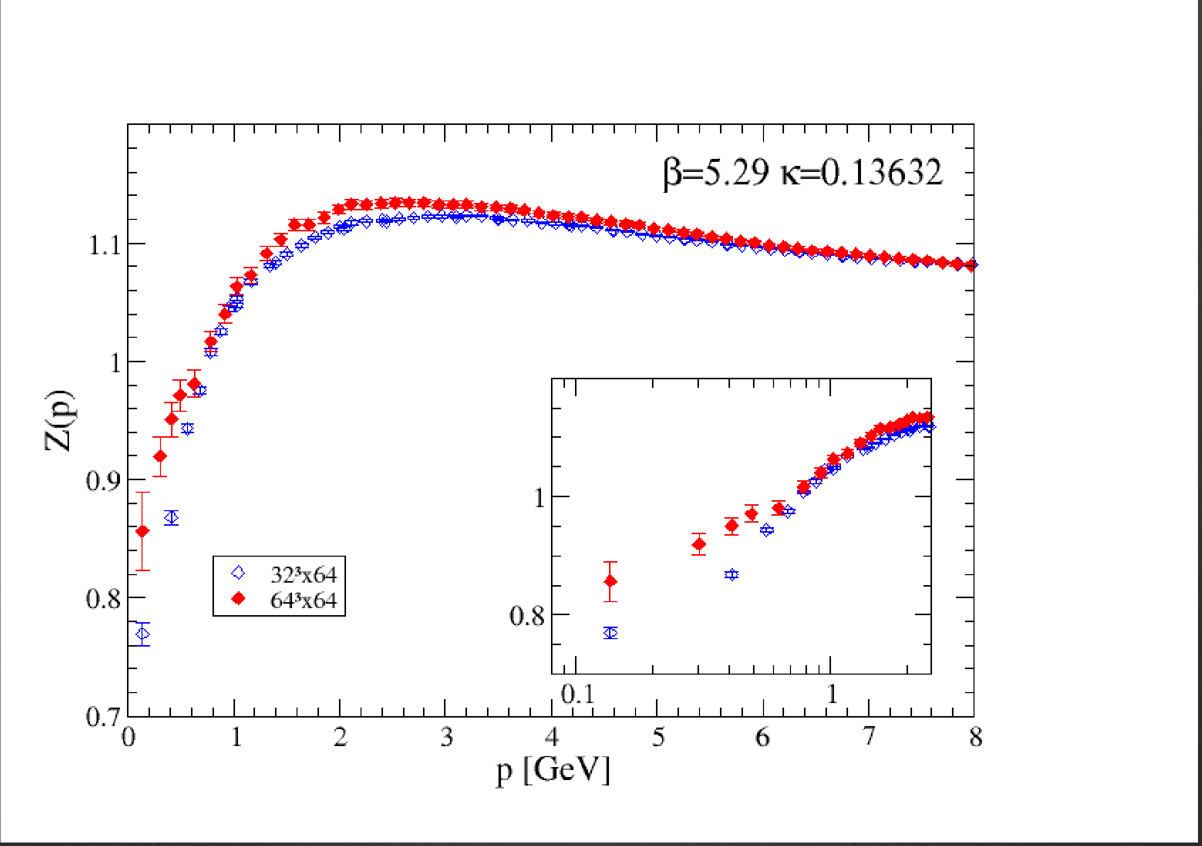
All 4 ensembles on the 32^3 x 64 volume, using the tree-level corrected Z(p)

very slight dependence on the lattice spacing



Finite Volume Effects

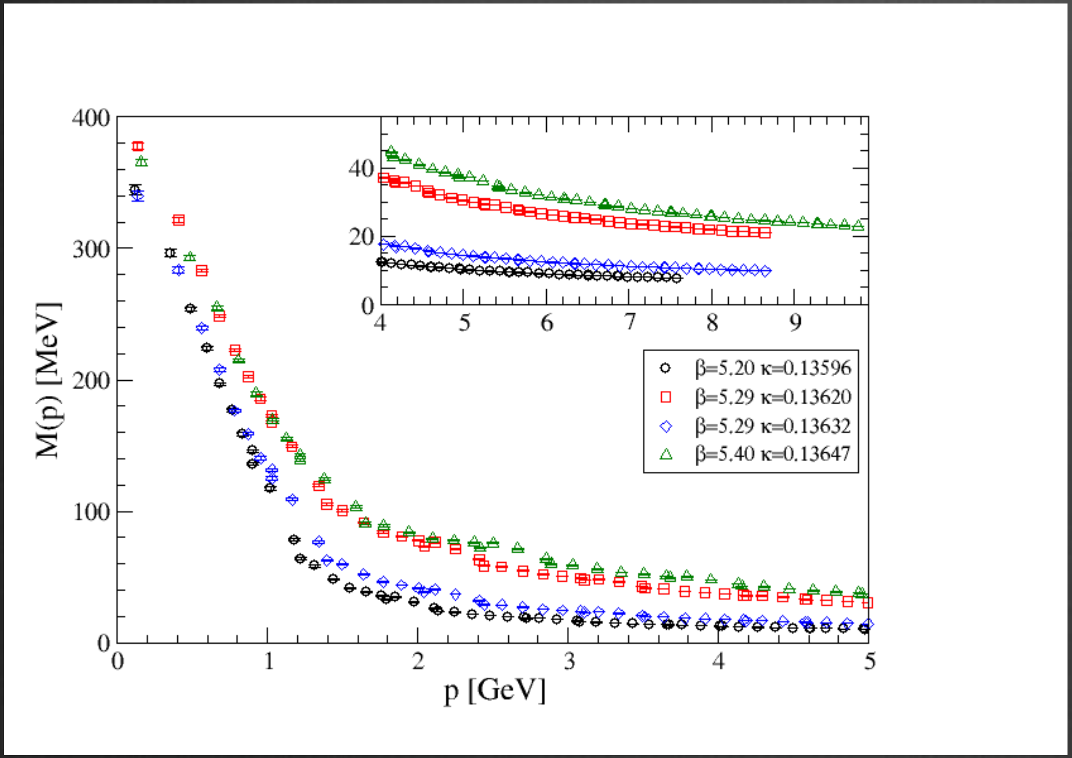
Quark Wave-function for two lattice volumes at β = 5.29, κ =0.13632



Mass function

$$(m_\pi = 422MeV\,;\, m_q = 17.0MeV)$$
$$(m_\pi = 280MeV\,;\, m_q = 6.2MeV)$$
$$(m_\pi = 295MeV\,;\, m_q = 8.0MeV)$$
$$(m_\pi = 426MeV\,;\, m_q = 18.4MeV)$$

All 4 ensembles on the 32^3 x 64 volume, using the hybrid correction scheme for M(p).

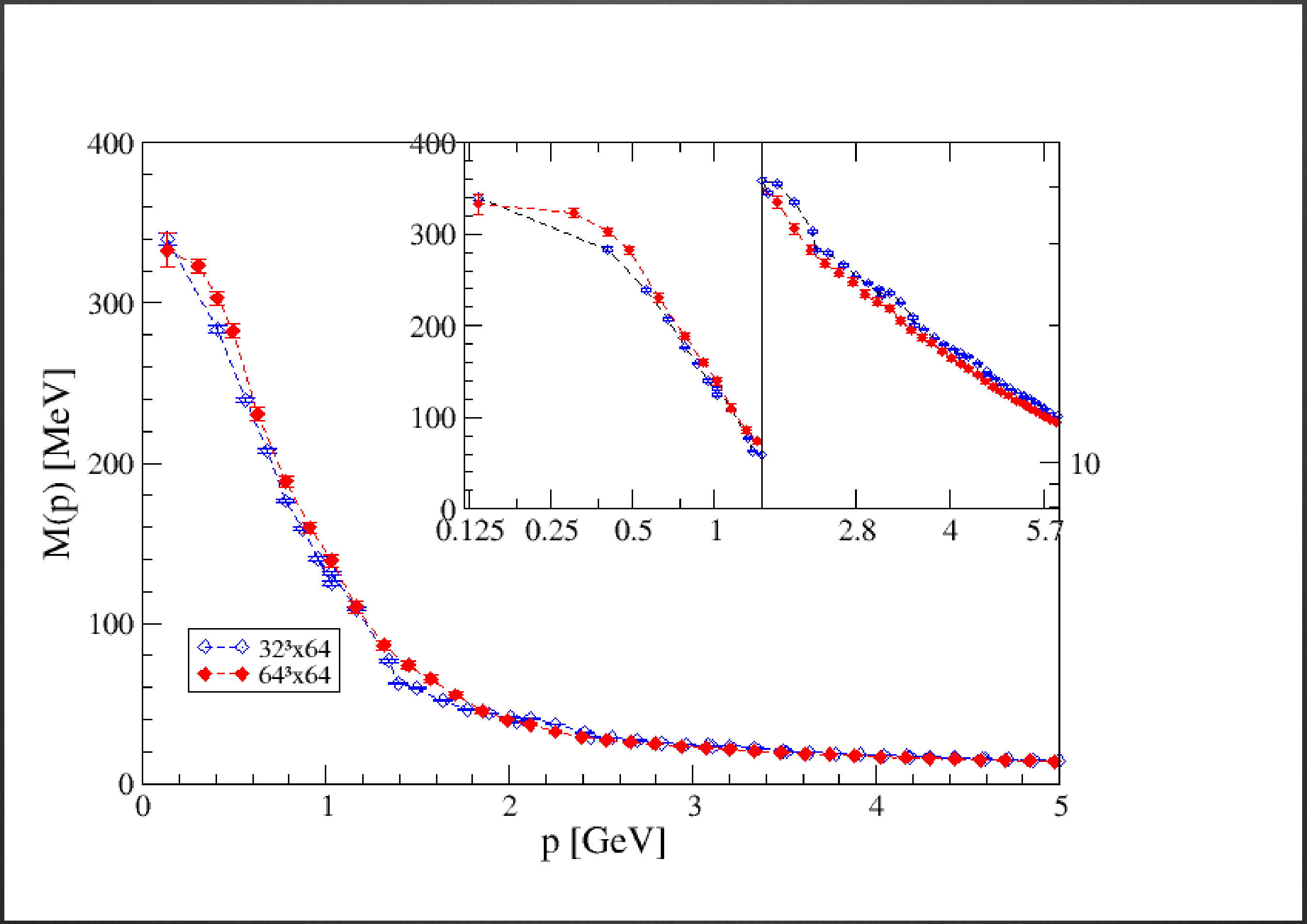


$$(m_\pi = 426MeV\,;\, m_q = 18.4MeV)$$
$$(m_\pi = 422MeV\,;\, m_q = 17.0MeV)$$
$$(m_\pi = 295MeV\,;\, m_q = 8.0MeV)$$
$$(m_\pi = 280MeV\,;\, m_q = 6.2MeV)$$

The mass function M(p) shows the usual dynamically generated “constituent” quark mass of 300–400 MeV in the infrared, and approaches the bare quark mass m_q in the ultraviolet.

Finite Volume Effects

Mass function for two lattice volumes at $\beta = 5.29$, $\kappa = 0.13632$



Summary

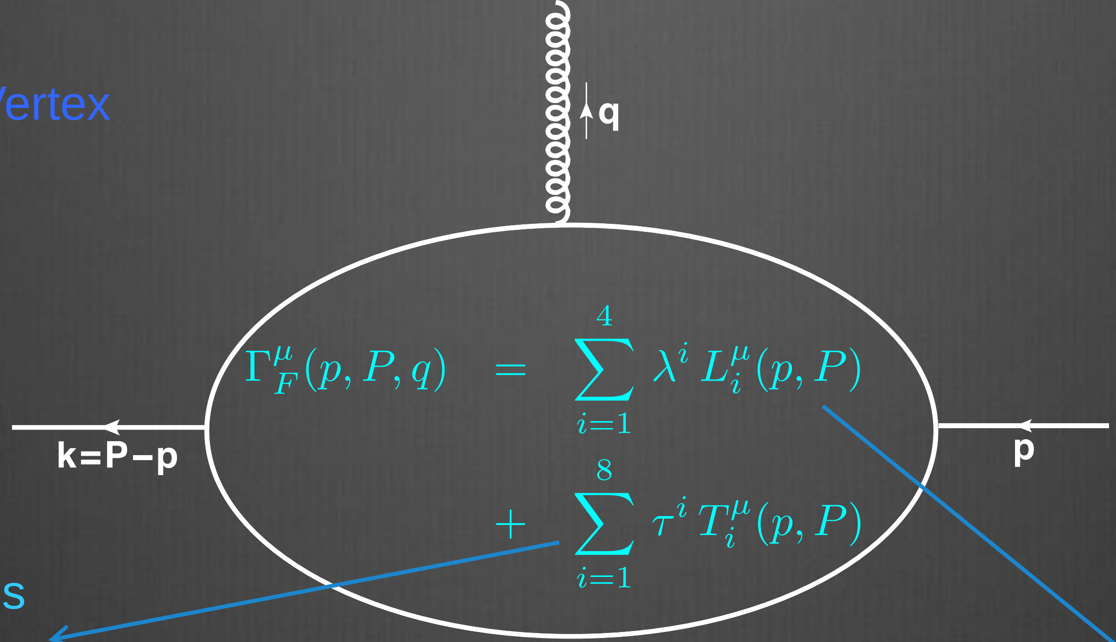
- $M(0) \backsimeq 339.9 \pm 3.7$ MeV

- lattice artefacts under control for Z(p2) but not for for $M(p^2)$

- mild effects on $Z(p^2)$

- quark wave function seems to be lattice space independent at UV but not in the IR

Quark Gluon Vertex



$$\Gamma_F^\mu(p,P,q) = \sum_{i=1}^{4} \lambda^i L_i^\mu(p,P) + \sum_{i=1}^{8} \tau^i T_i^\mu(p,P)$$

Transverse Basis

$$
\begin{aligned}
T_{1_\mu} &= -i\,[(pq)k_\mu - (kq)p_\mu] \\
T_{2_\mu} &= -\,\not{P}\,[(pq)k_\mu - (kq)p_\mu] \\
T_{3_\mu} &= \not{q}\,q_\mu - q^2\gamma_\mu \\
T_{4_\mu} &= -i\left[q^2\boxed{\sigma_{\mu\nu}}P_\nu + 2q_\mu\boxed{\sigma_{\nu\lambda}}p_\nu k_\lambda\right] \\
T_{5_\mu} &= -i\,\boxed{\sigma_{\mu\nu}}q_\nu \\
T_{6_\mu} &= (qP)\gamma_\mu - \not{q}\,P_\mu \\
T_{7_\mu} &= -\frac{i}{2}(qP)\boxed{\sigma_{\mu\nu}}P_\nu - i\,P_\mu\boxed{\sigma_{\nu\lambda}}p_\nu\,k_\lambda \\
T_{8_\mu} &= -\gamma_\mu\boxed{\sigma_{\nu\lambda}}p_\nu\,k_\lambda - \not{p}\,k_\mu + \not{k}\,p_\mu
\end{aligned}
$$

Longitudinal Basis

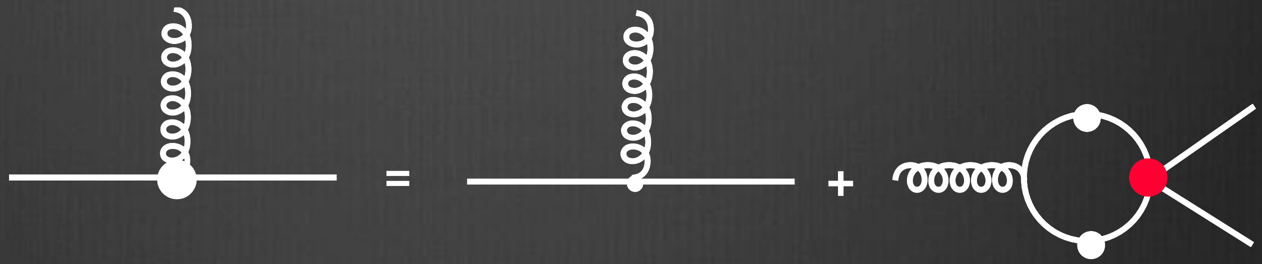
$$
\begin{aligned}
L_{1_\mu} &= \gamma_\mu \\
L_{2_\mu} &= -\,\not{P}P_\mu \\
L_{3_\mu} &= -i\,P_\mu \\
L_{4_\mu} &= -i\boxed{\sigma_{\mu\nu}}P_\nu
\end{aligned}
$$

Determining Quark-Gluon Vertex

1-Determining qqg vertex perturbatively:

2- Nonperturbatively in continuum by solving qqg Vertex DSE :

a-Either by solving DSE for the vertex itself



b-By solving coupled DSE for the propagators self consistently by imposing constraints on them

J.S. Ball and T.W. Chiu, Phys.Rev. D22 (1980) 2542

J.S. Ball and T.W. Chiu, Phys.Rev. D22 (1980) 2550,

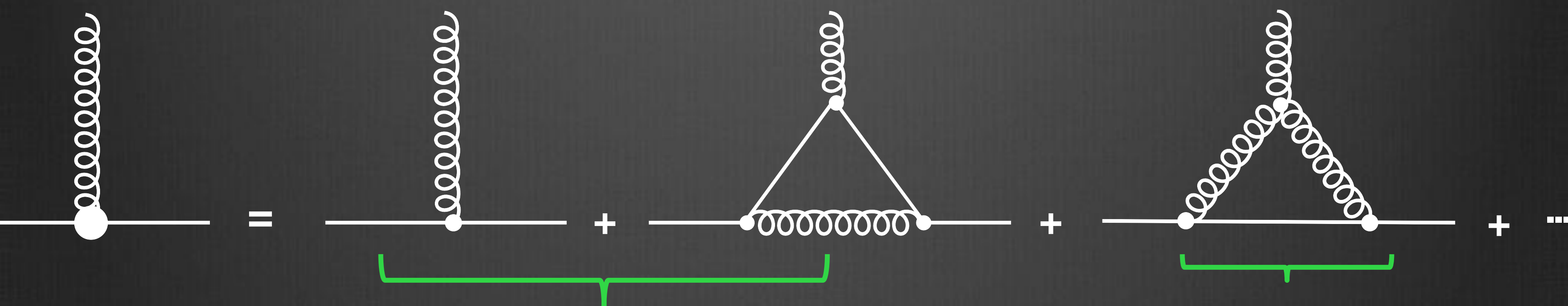
c-using Ward-Green-Takahashi identities…

S-X Qin ,C. D. Roberts, S. M. Schmidt ,Phys.Lett. B733 (2014) 202-208

3-Non-perturbatively in discreet space by using latice gauge field theory

In Perturbation theory:

$$M(p^2) \sim m_0\ [\ldots.]$$



$$\left(C_F - \frac{C_A}{2}\right) \qquad (C_A)$$

$$\boxed{\lambda_i^{\mathbf{QCD}} = \left(\mathbf{C_F} - \frac{\mathbf{C_A}}{\mathbf{2}}\right)\lambda_i^{\mathbf{a}} + \mathbf{C_A}\lambda_i^{\mathbf{b}}}$$

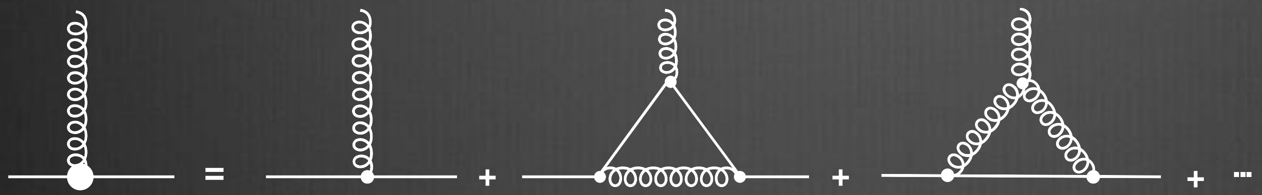
(Color factors in Fundamental and Adjoint Rep.) $\mathbf{C_A = N,\ C_F = \frac{N^2-1}{2N}}$

Quark-Gluon Vertex

Ghost-Quark Scattering Kernel

<u>Slavnow Taylor Identity</u>

$$\mathbf{q}^{\mu}\mathbf{\Gamma}_{\mu}(\mathbf{p},\mathbf{q}) = \mathbf{G_P}(\mathbf{q^2})\ \left[\bar{\mathbf{H}}(\mathbf{k},-\mathbf{p},-\mathbf{q})\mathbf{S}^{-1}(\mathbf{k}) - \mathbf{S}^{-1}(\mathbf{p})\mathbf{H}(-\mathbf{p},\mathbf{k},-\mathbf{q})\right]$$



Zeroth Order:

$$q_{\mu}\Gamma^{\mu} = \left[\mathbf{S}^{-1}(\mathbf{k}) - \mathbf{S}^{-1}(\mathbf{p})\right]^{(\mathbf{0})}$$

One-loop Abelian:

$$+\left(\mathbf{C_F} - \frac{\mathbf{C_A}}{\mathbf{2}}\right)\mathbf{C_F^{-1}}\left[[\mathbf{S}^{-1}(\mathbf{k})]^{(\mathbf{1})} - [\mathbf{S}^{-1}(\mathbf{p})]^{(\mathbf{1})}\right]$$

One-loop Non- Abelian :

$$+ \quad \frac{C_A}{2}C_F^{-1}\left[[S^{-1}(k)]^{(1)}H^{(0)} - H^{(0)}[S^{-1}(p)]^{(1)}\right]$$

$$+ \quad \left[[S^{-1}(k)]^{(0)}H^{(1)} - H^{(1)}[S^{-1}(p)]^{(0)}\right]$$

$$+ \quad G_P^{(1)}\ \left[S^{-1}(k)]H - HS^{-1}(p)\right]^{(0)}$$

Perturbative

J.S.Ball and T.W. Chiu, Phys.Rev.D22, 2542(1980)
A.Kizilersu,M.Reenders and M.R.Pennington, Phys.Rev.D52,1242 (1995). (QED)
A.I.Davydychev, P.Osland and L.Saks, Phys.Rev.D63,014022 (2001) (QCD)
J.A.Gracey, Phys.Rev.D90,025014(2014) (QCD)

Dressed quark-quark-gluon vertex in continuum

Longitudinal Part:

$$\Gamma_L^\mu(p,k) = \lambda_1 \left[\gamma^\mu\right] + \lambda_2 \left[(\not k + \not p)(k+p)^\mu\right] + \lambda_3 \left[(k+p)^\mu\right] + \lambda_4 \left[(k^\nu + p^\nu)\sigma^{\mu\nu}\right]$$

Longitudinal Part of the Abelian Vertex : KNOWN AS BALL-CHIU VERTEX

$$\lambda_1^M(p^2,k^2) = \frac{1}{2}\left[A(k^2) + A(p^2)\right] , \qquad [A = 1/Z]$$

$$\lambda_2^M(p^2,k^2) = \frac{1}{2(k^2-p^2)}\left[A(k^2) - A(p^2)\right] ,$$

$$\lambda_3^M(p^2,k^2) = \frac{-1}{k^2-p^2}\left[M(k^2)\,A(k^2) - M(p^2)\,A(p^2)\right] ,$$

$$\lambda_4^M(p^2,k^2) = 0$$

Transverse Part of the dressed quark-gluon Vertex:

In Continuum…
Non-Perburbative (DSE)

Ball_Chiu, Curtis-Pennington, C.D. Roberts, P. Tandy, P. Maris, Haeri,
H. Matevosyan,-A. Thomas, A.Bashir, Kizilersu-Pennington,
R. Williams.,J. Pawlowski, C. Fischer
C. Aguilar, D. Binosi, D. Ibanez, J. Papavassiliou,
El Bennich, Pelaez, Tissier
…

In discrete…
Non-Perturbative (Lattice)

J. Skullerud, A.Kizilersu, T. Williams, D.Leinweber, P.Bowman
M. Pelaez, M. Tissier and N. Wschebor, (one-loop)

J. Skullerud and A.Kizilersu, JHEP09(2002)013
J. Skullerud, P.O.Bowman, A.Kizilersu,D.B. Leinweber, A.G.Williams, JHEP04(2003)047

$$\mathbf{V}^{\mathbf{a}}_{\mu}(\mathbf{x},\mathbf{y},\mathbf{z})^{\mathbf{ij}}_{\alpha\beta} =< \mathbf{\Psi}^{\mathbf{i}}_{\mathbf{a}}(\mathbf{x})\bar{\mathbf{\Psi}}(\mathbf{z})\mathbf{A}^{\mathbf{a}}_{\mu}(\mathbf{y}) > = << \mathbf{S}^{\mathbf{ij}}_{\alpha\beta}(\mathbf{x},\mathbf{z})\mathbf{A}^{\mathbf{a}}\mu(\mathbf{y}) >>$$

$$\mathbf{\Lambda}^{\mathbf{a,lat.}}_{\mu}(\mathbf{p},\mathbf{q}) = \mathbf{S}(\mathbf{p})^{-\mathbf{1}}\mathbf{V}^{\mathbf{a}}_{\nu}(\mathbf{p},\mathbf{q})\mathbf{S}(\mathbf{p}+\mathbf{q})^{-\mathbf{1}}\mathbf{D}(\mathbf{q})^{-\mathbf{1}}_{\nu\mu}$$

$$(\mathbf{\Lambda}^{\mathbf{a}}_{\mu})^{\mathbf{ij}}_{\alpha\beta} \;=\; -\mathbf{g_0}\mathbf{t}^{\mathbf{a}}_{\mathbf{ij}}(\mathbf{\Gamma}_{\mu})_{\alpha\beta}$$

Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\Gamma^T_\mu(p,k,q) = P^T_{\mu\nu}(q)\Gamma_\mu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Gamma_\mu(p,k,q)$$

We study transverse projected vertex:

$$\mathbf{\Lambda^{a,P}_\mu(p,q) \equiv P_{\mu\nu}\Lambda^{a,lat.}_\nu(p,q) = S(p)^{-1}V^a_\nu(p,q)S(p+q)^{-1}D(q^2)^{-1}}$$

$$P^T_{\mu\nu}\,L^1_\nu = -\frac{1}{q^2}\,T_{3,\mu}$$

$$P^T_{\mu\nu}\,L^2_\nu = -\frac{2}{q^2}\,T_{2,\mu}$$

$$P^T_{\mu\nu}\,L^3_\nu = -\frac{2}{q^2}\,T_{1,\mu}$$

$$P^T_{\mu\nu}\,L^4_\nu = \frac{1}{q^2}\,T_{4,\mu}$$

$$\Gamma^{T\,\mu} = -\frac{1}{q^2}\left[\Lambda_1\,T^\mu_3 + \Lambda_2\,T^\mu_2 + \Lambda_3\,T^\mu_1 + \Lambda_4\,T^\mu_4\right] + \tau_5\,T^\mu_5 + \tau_6\,T^\mu_6 + \tau_7\,T^\mu_7 + \tau_8\,T^\mu_8$$

$$\boxed{\begin{array}{ll} \Lambda_1 = \lambda_1 - q^2\,\tau_3 & \Lambda_2 = \lambda_2 - \dfrac{q^2}{2}\,\tau_2 \\ \Lambda_3 = \lambda_3 - \dfrac{q^2}{2}\,\tau_1 & \Lambda_4 = \lambda_4 + q^2\,\tau_4 \end{array}}$$

General Kinematics:

$$\Lambda_1 = \lambda_1 - q^2\,\tau_3 \qquad \Lambda_2 = \lambda_2 - \frac{q^2}{2}\,\tau_2$$

$$\Lambda_3 = \lambda_3 - \frac{q^2}{2}\,\tau_1 \qquad \Lambda_4 = \lambda_4 + q^2\,\tau_4$$

$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] = i\,\Lambda_3\,\frac{1}{q^2}\left[(q\cdot P)q_\mu - q^2\,P_\mu\right]$$

$$Tr_4[\gamma_\nu\,\Gamma^T_\mu]$$

$$= \Lambda_1\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \Lambda_2\,\frac{2}{q^2}\,P_\mu\,\ell_\nu + \tau_6\,(q\cdot P)\left(\delta_{\mu\nu} - \frac{q_\mu P_\nu}{qP}\right)$$

$$Tr_4[\gamma_5\gamma_\alpha\Gamma^T_\mu] \quad = \tau_8\,\epsilon_{\alpha\mu\kappa\lambda}k_\kappa p_\lambda$$

$$Tr_4[\sigma_{\alpha\beta}\Gamma^T_\mu]$$

$$= \qquad i\left[\Lambda_4 + \frac{1}{2}(q\cdot P)\,\tau_7\right](\delta_{\alpha\mu}P_\beta - \delta_{\beta\mu}P_\alpha)$$

$$+ \quad i\left[\Lambda_4\frac{q_\mu}{q^2} + \frac{1}{2}\,\tau_7 P_\mu\right](P_\alpha q_\beta - P_\beta q_\alpha) + i\,\tau_5\,(\delta_{\alpha\mu}q_\beta - \delta_{\beta\mu}q_\alpha)$$

General Kinematics:

$$\boxed{P_\mu = p_\mu + k_\mu = 2p_\mu + q_\mu}$$

1) $$Tr_4[\mathbf{I}\,\Gamma^T_\mu] = i\,\Lambda_3\,\frac{1}{q^2}\left[(q\cdot P)q_\mu - q^2 P_\mu\right]$$

In Continuum:

$$\mathbf{\Lambda_3} \equiv -\left(\lambda_3 - \frac{\mathbf{q^2}}{\mathbf{2}}\tau_1\right) = -\mathbf{i}\frac{\mathbf{q^2}}{[(\mathbf{q}\cdot\mathbf{P})^2 - \mathbf{q^2P^2}]}\mathbf{Tr_4}\,[\mathbf{I}\,\mathbf{\Gamma}_\mu]\,\mathbf{P}_\mu$$

$$\boxed{\begin{aligned} K_\mu(p) &= \frac{1}{a}\sin p_\mu a \\ Q_\mu(p) &= \frac{2}{a}\sin\left(\frac{p_\mu a}{2}\right) \\ C_\mu(p) &= \cos(p_\mu a) \end{aligned}}$$

On the lattice:

Tree-level correction:

$Q_\mu(P)$ or $K_\mu(P)$

$$\begin{aligned}
\frac{\mathrm{Tr}_4\left(I\,\Gamma^T_\mu\right)}{R} &= -2c_q\,\frac{i}{Q^2(q)}\left[K(q)\cdot K(P)Q_\mu(q) - Q^2(q)\bar{C}_\mu(q)K_\mu(P)\right] \\
&+ \frac{1}{2}\,\frac{i}{Q^2(q)}\left[\left(1-c_q^2\,K^2(p)\right)\left(1-c_q^2\,K^2(k)\right) - 4c_q^2K(p)\cdot K(k)\right]\left(Q(q)\cdot Q(P)Q_\mu(q) - Q^2(q)Q_\mu(P)\right] \\
&+ 4ic_q^2c_{sw}\,\bar{C}_\mu(q)\left[K(k)\cdot K(q)K_\mu(p) - K(p)\cdot K(q)K_\mu(k)\right] \\
&+ \frac{c_q^3}{2}\,i\left[K(P)\cdot K(q)\left\{2\bar{c}_\mu^2 - 1 + \frac{Q^2(P)}{Q^2(q)}\right\}Q_\mu(q) - 2\left[K^2(k)+K^2(p)\right]\bar{C}_\mu(q)K_\mu(P)\right]
\end{aligned}$$

On the lattice :

$$\mathbf{X}_\mu(\mathbf{P}) = \lambda_{\mathbf{1}}^{\mathbf{1a,lat.}} \mathbf{K}_\mu(\mathbf{P}) + \lambda_{\mathbf{1}}^{\mathbf{2a,lat.}} \mathbf{Q}_\mu(\mathbf{P})$$

$$\boxed{\begin{aligned} \frac{Q_\mu(P)}{Q^2(P)} X_\mu(P) &= \lambda_1^{1a,lat.} \frac{K(P)\cdot Q(P)}{Q^2(P)} + \lambda_1^{2a,lat.} \\ \frac{K_\mu(P)}{K^2(P)} X_\mu(P) &= \lambda_1^{1a,lat.} + \frac{K(P)\cdot Q(P)}{K^2(P)} \lambda_1^{2a,lat.} \end{aligned}}$$

$$\boxed{\frac{1}{M} \sum_{i=1}^{N} h_i \lambda_1^{ia,lat} \longrightarrow \lambda_1^{ia,cont.}}$$

In Continuum:

$$\Lambda_3 \equiv -\left(\lambda_3 - \frac{q^2}{2}\,\tau_1\right) = -i\,\frac{q^2}{[(q\cdot P)^2 - q^2 P^2]}\, Tr_4\,[I\,\Gamma_\mu]\, P_\mu$$

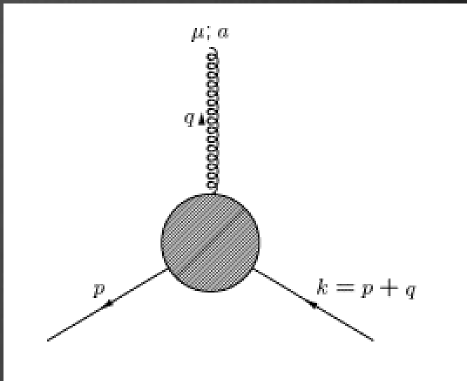
Tree level lattice expression :

$$\begin{aligned}
\Lambda_3 \;&=\; -i\,\frac{Q^2(q)}{D_L^2}\left[\left(\frac{\mathrm{Tr}_4\,\Gamma_\mu^T}{R}\right) Q_\mu(P)\right] \\
&=\; -i\,\frac{Q^2(q)}{D_L^2}\Bigg\{ 2ic_q\,\frac{1}{Q^2(q)}\,\left[Q^2(q)\,K(P)\cdot\{K(k)+K(p)\} - \{Q(P)\cdot Q(q)\}\,\{K(P)\cdot K(q)\}\right] \\
&-\; i\,\frac{1}{2Q^2(q)}\,\left[\left(1-c_q^2\,K^2(p)\right)\left(1-c_q^2\,K^2(k)\right) - 4c_q^2 K(k)K(p)\right]\left[Q^2(P)Q^2(q) - \{Q(P)\cdot Q(q)\}^2\right] \\
&+\; 2ic_q^2 c_{sw}\left[\{K(k)+K(p)\}^2\; K(q)\cdot[K(k)-K(p)] - \{K(q)\cdot K(P)\}\; K(q)\cdot(K(k)+K(p))\right] \\
&-\; i\,\frac{c_q^3}{2Q^2(q)}\Big[\{Q^2(q)-Q^2(P)\}\;\{K(P)\cdot K(q)\}\;\{Q(P)\cdot Q(q)\} \\
&\qquad +4Q^2(q)\,\left[K^2(p)\;\{K(P)\cdot K(k)\} + K^2(k)\;\{K(P)\cdot K(p)\}\right]\;\Big]\;\Bigg\}
\end{aligned}$$

$$D_L^2 \equiv (Q(q)\cdot Q(P))^2 - Q(q)^2 Q(P)^2$$

Quark-Gluon Vertex in Special Kinematics

- Soft Gluon Kinematics : $q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$

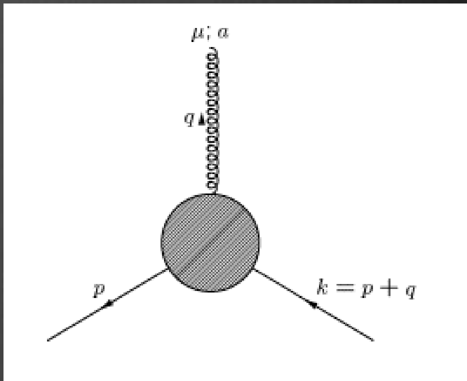


$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] = -\,2\,i\,\lambda_3\,p_\mu$$

$$Tr_4[\gamma_\mu\,\Gamma^T_\mu] = \lambda_1\delta_{\mu\nu} - 4\,\lambda_2 p_\mu p_\nu$$

$$Tr_4[\sigma_{\alpha\beta}\Gamma^T_\mu] = -\,2\,i\,\lambda_4\,(p_\alpha\delta_{\beta\mu} - p_\beta\delta_{\alpha\mu})$$

Quark-Gluon Vertex in Special Kinematics



- Soft Gluon Kinematics : $q_\mu = 0 \qquad k_\mu = p_\mu = \dfrac{P_\mu}{2}$

$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] = -2\,i\,\lambda_3\,p_\mu$$

$$\lambda_3(P, q=0) = \frac{1}{Q^2(P)}\; iQ_\mu(P) \left(\frac{\mathrm{Tr}_4\left(\Gamma^T_\mu\right)}{R} \right) \Bigg|_{q=0}$$

Tree-level correction :

$$\lambda_3(P, q=0) = \frac{1}{2} \left\{ \left(1 - \frac{c_q^2}{4}\,Q^2(P)\right)^2 - c_q^2 Q^2(P) - 4c_q \left(1 - \frac{1}{4}c_q^2 Q^2(P)\right) \frac{Q(P)\cdot K(P)}{Q^2(P)} \right\}$$

Lattice Simulation Parameters:

- We thank the Regensburg Collaboration(RQCD) for allowing us to use their configurations in this project

  G. S. Bali et all, Phys Rev D91, 054501 (2014) [arXiv:1412.7336]

- The calculations (gauge fixing , propagators) where performed using the HLRN (Germany) for supercomputing

  https://www.hlrn.de/home/view/Service

- Fermion Action is Clover Rotated
- 900 configurations

$$32^3 \times 64 \quad lattice$$

$$\beta = 5.29$$

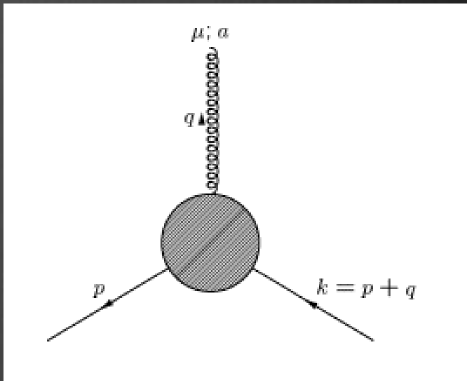
$$k = 0.13632$$

$$a = 0.07 fm$$

$$m_\pi = 290 MeV$$

Quark-Gluon Vertex in Special Kinematics

- Soft Gluon Kinematics : $q_\mu = 0 \qquad k_\mu = p_\mu = \dfrac{P_\mu}{2}$



$$Tr_4[\gamma_\alpha \, \Gamma^T_\mu] \; = \lambda_1 \delta_{\alpha\mu} - 4 \, \lambda_2 p_\alpha p_\mu$$

$$\boxed{\lambda_1(P, q=0) \quad = \quad \frac{1}{3} \left\{ \left(\frac{\mathrm{Tr}_4 \left(\gamma_\mu \Gamma^T_\mu \right)}{R} \right) \Bigg|_{q=0} \quad - \frac{Q_\mu(P) Q_\alpha(P)}{Q^2(P)} \left(\frac{\mathrm{Tr}_4 \left(\gamma_\alpha \Gamma^T_\mu \right)}{R} \right) \Bigg|_{q=0} \right\}}$$

Tree Level Expression:

$$\lambda_1(P, q=0) \quad = \quad \frac{1}{3} \left[4 - \frac{\overline{Q}^2(P)}{8} - \frac{K(P) \cdot Q(P)}{Q^2(P)} \right] \left[\left(1 - \frac{c_q^2}{4} Q^2(P) \right)^2 + c_q^2 Q^2(P) \right]$$

$$\lambda_1 \Big|_{\substack{q=0, P_\mu=0, \alpha=\mu \\ \text{no sum over } \mu}} \quad = \quad \left(\frac{\mathrm{Tr}_4 \left(\gamma_\alpha \Gamma^T_\mu \right)}{R} \right) \Bigg|_{\substack{q=0,, \alpha=\mu \\ \text{no sum over } \mu}}$$

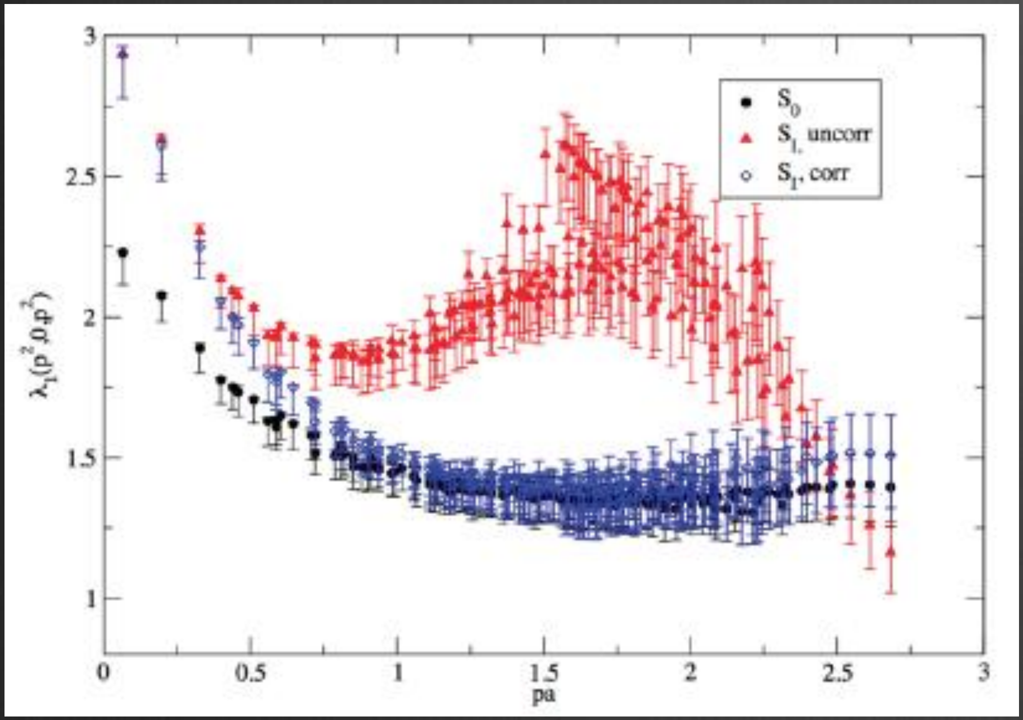
Tree Level Expression:

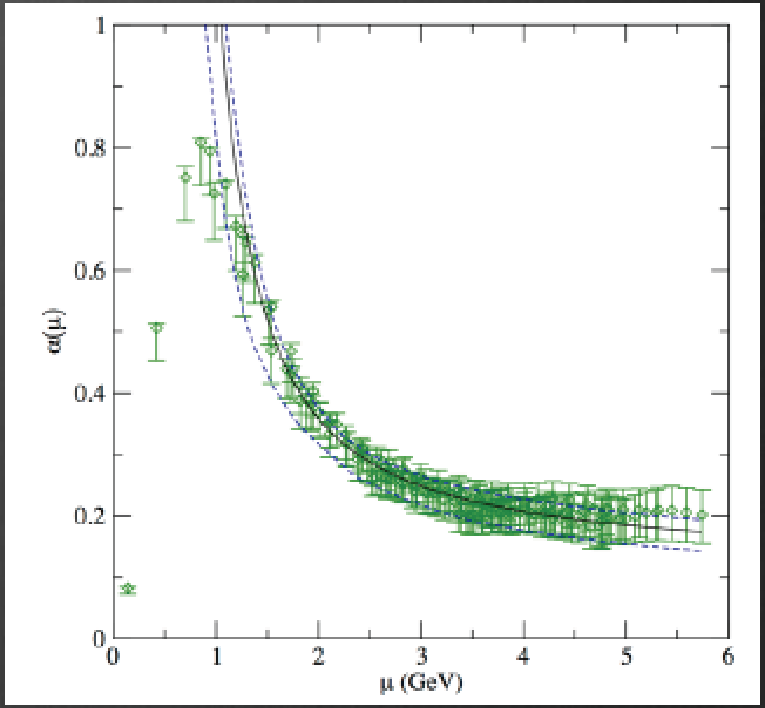
$$\lambda_1 \Big|_{\substack{q=0, P_\mu=0, \alpha=\mu \\ \text{no sum over } \mu}} \quad = \quad \left(1 - \frac{1}{4} c_q^2 Q^2(P) \right)^2 + c_q^2 Q^2(P)$$

Quenched Calculation

$$\beta = 6.0 \qquad , 16^3 \times 48 \, , \quad a = 0.09 fm$$

$$m_q = 122 MeV \ , (1.5 fm)^3 \times (4.5 fm)$$

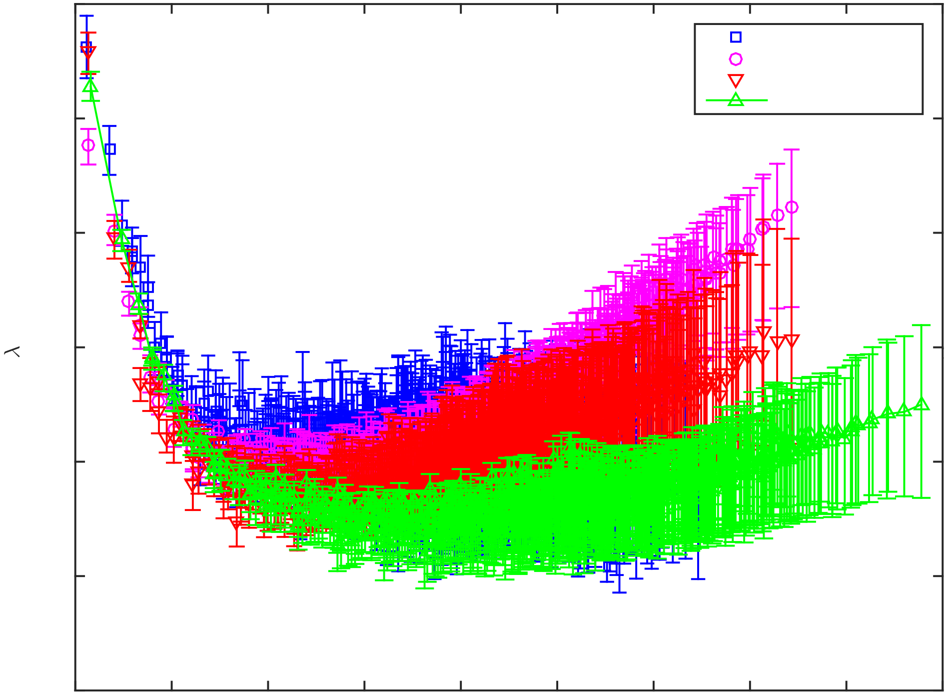


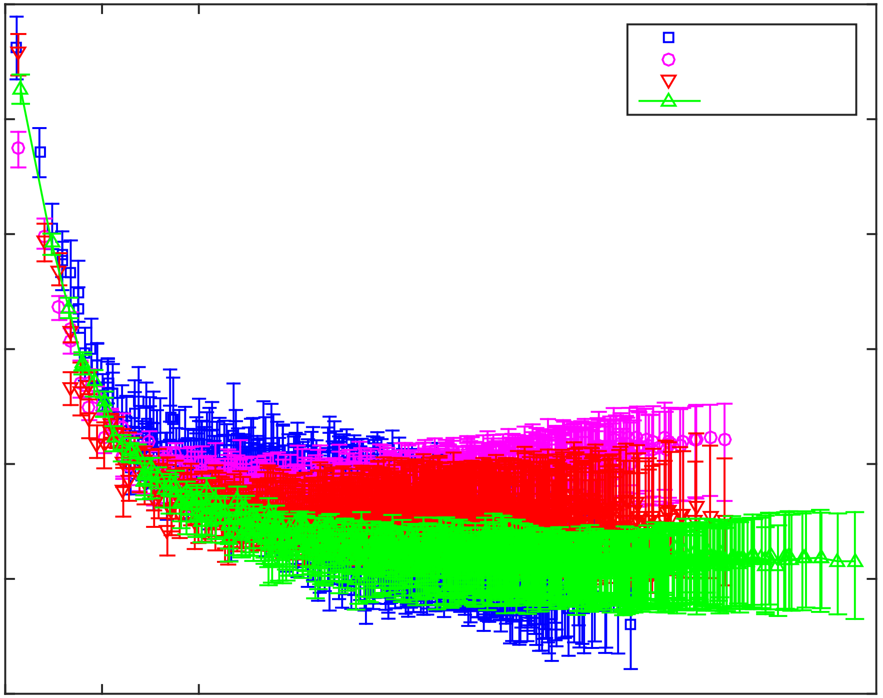


J. Skullerud and A.Kizilersu, JHEP09(2002)013

Soft gluon Kinematics (Asymmetric) -Unquenched

$$N_f = 2 : \ \beta = 5.40, \kappa = 0.13647, 32^3 \times 64$$

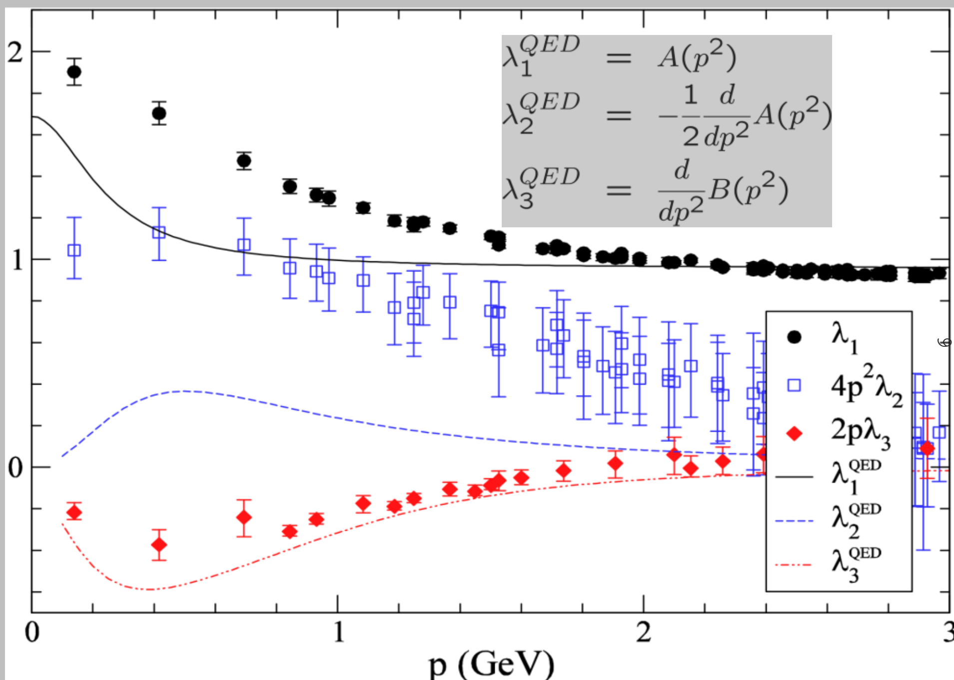


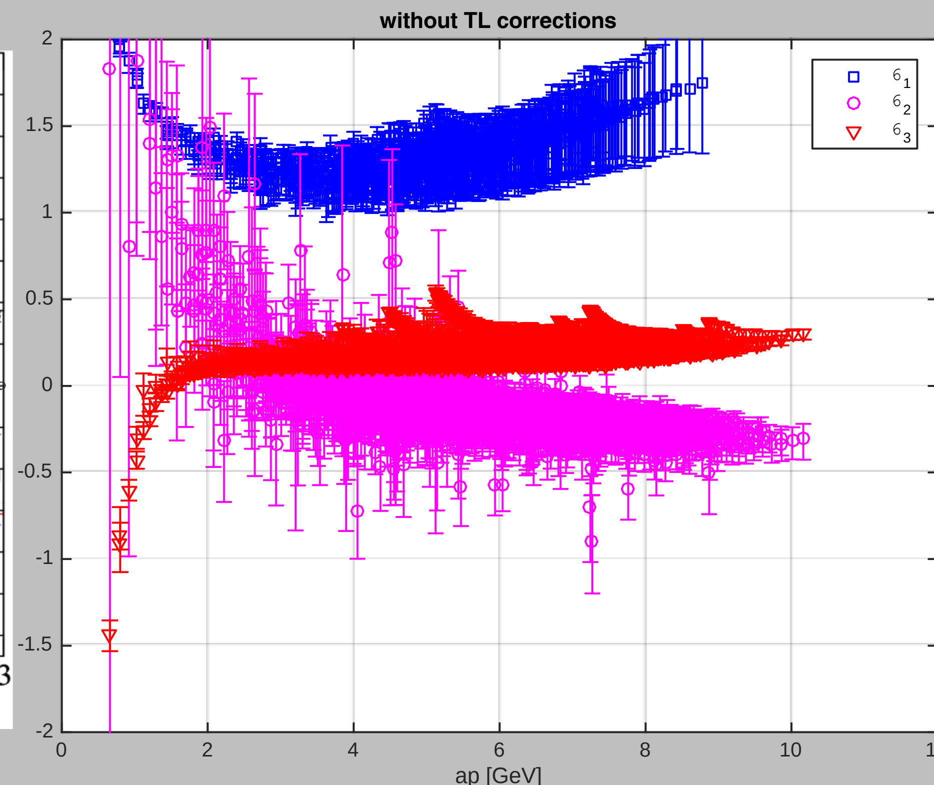


Soft Gluon Kinematics :

$N_f = 0$, Quenched

$N_f = 2$, Unquenched





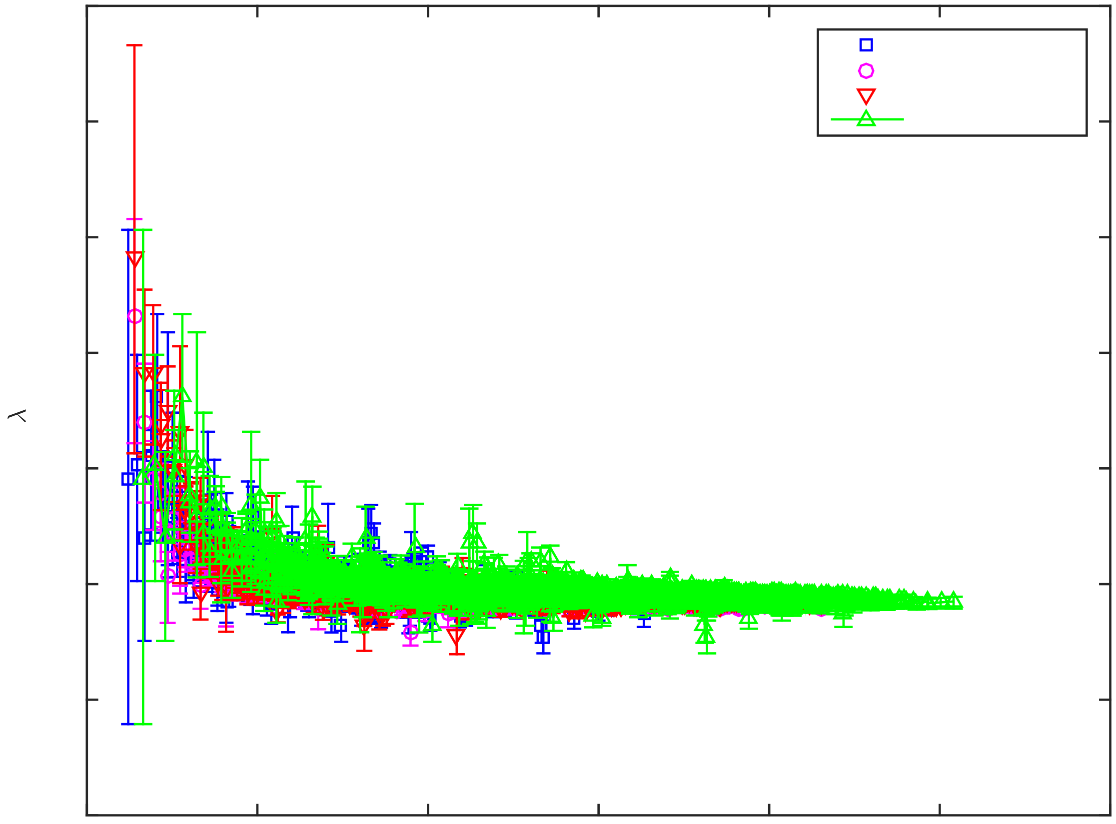
Renormalised form factors $\lambda_1, \lambda_2, \lambda_3$ versus p. Mean-field improved SW action, $\beta = 0.6$ and $16^3 \times 48$ lattice, $\mu = 2GeV$, 495 configuration, $m = 115MeV$.

Form factors $\lambda_1, \lambda_2, \lambda_3$ versus pa. $\beta = 5.4$, $a = 0.06fm$ and $32^3 \times 64$, 900 configuration,

J. Skullerud,P. Bowman, A.Kizilersu, D.Leinweber,A.Williams, JHEP04(2003)047

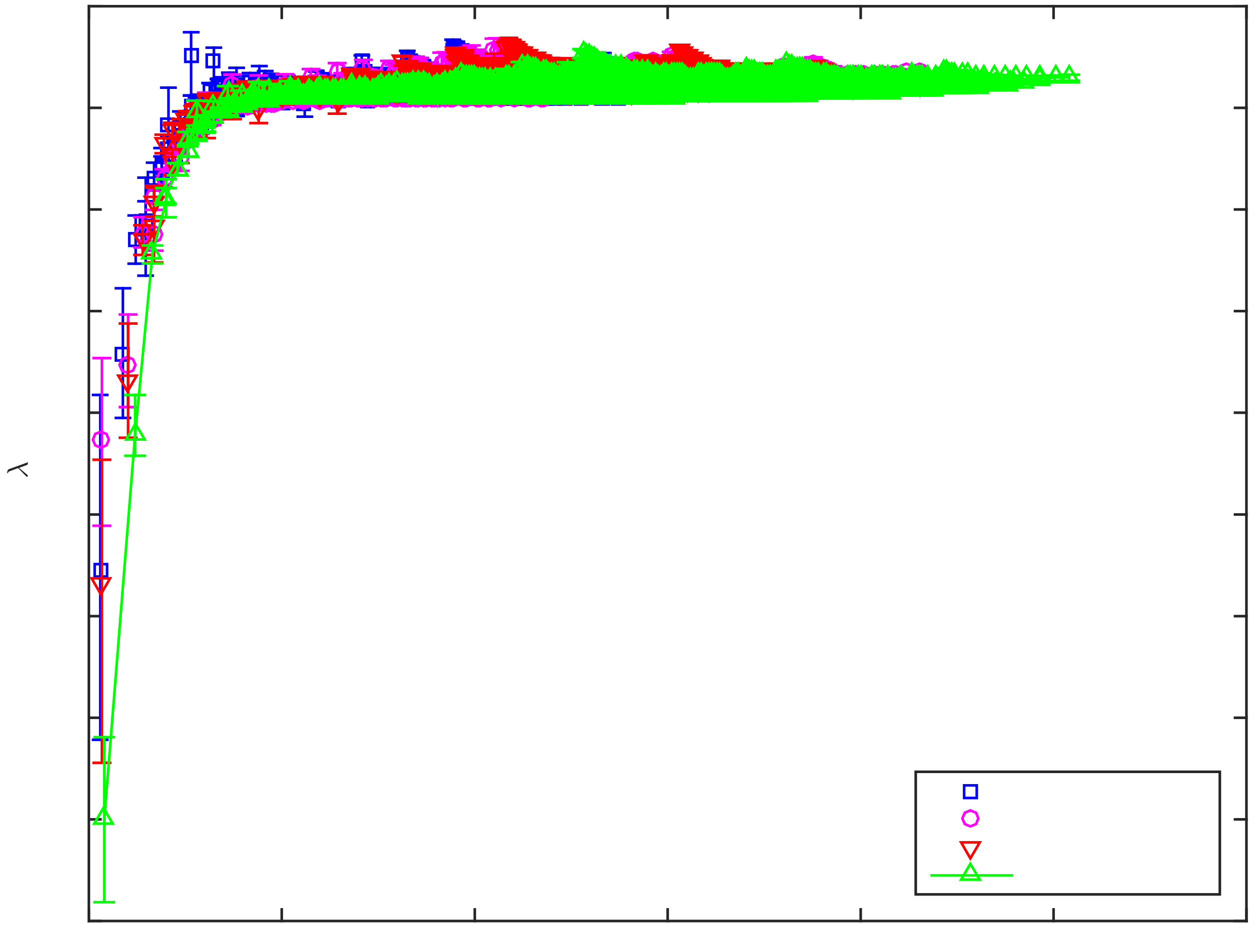
Soft gluon Kinematics (Asymmetric) -Unquenched

$$N_f = 2: \ \beta = 5.40, \kappa = 0.13647, 32^3 \times 64$$



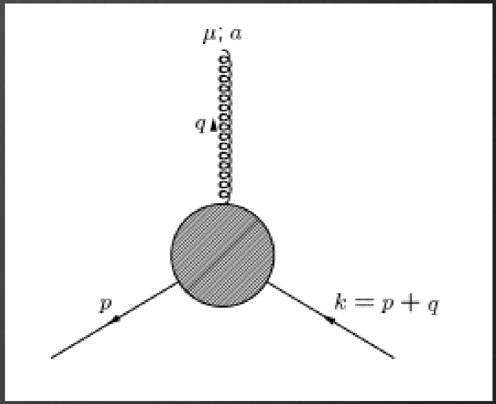
Soft gluon Kinematics (Asymmetric) -Unquenched

$$N_f = 2: \;\; \beta = 5.40, \kappa = 0.13647, 32^3 \times 64$$



Quark-Gluon Vertex in Special Kinematics

- Hard Reflection Kinematics : $P_\mu = 0 \qquad k_\mu = -p_\mu = \dfrac{q_\mu}{2}$



$$Tr_4[\gamma_\mu \, \Gamma^T_\mu] = \underbrace{\left(\lambda_1 - q^2 \tau_3\right)}_{\Lambda_1} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

$$\boxed{\Lambda_1(P=0,q) = \frac{1}{3}\left(\frac{\mathrm{Tr}_4\left(\gamma_\mu \Gamma^T_\mu\right)}{R}\right)\Bigg|_{P=0}}$$

$$Tr_4[\sigma_{\alpha\beta}]\,\Gamma^T_\mu = -i\,\tau_5\left(q_\alpha \delta_{\beta\mu} - q_\beta \delta_{\alpha\mu}\right)$$

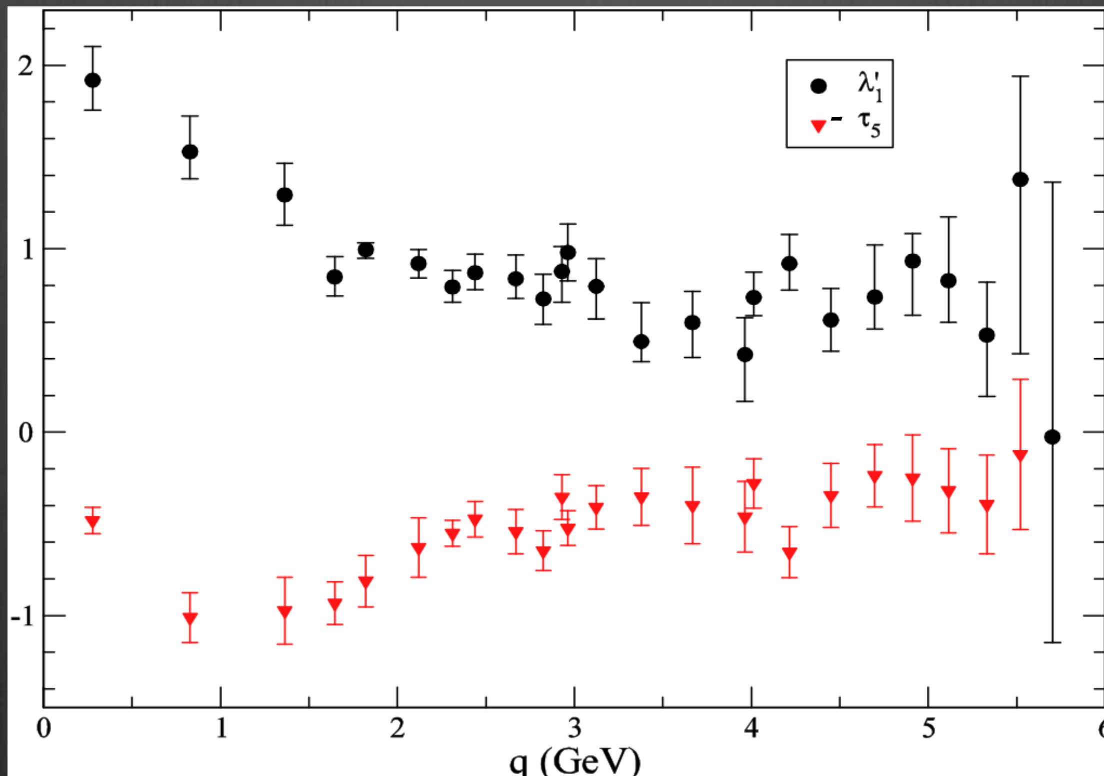
$$\boxed{\tau_5(P=0,q) = \frac{i}{3\,Q^2(q)} \sum_{\alpha,\beta,\mu} Q_\alpha(q)\,\delta_{\beta\mu}\left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta}\Gamma^T_\mu\right)}{R}\right)\Bigg|_{P=0}}$$

Quenched results

➢ Hard Reflection Kinematics : $P_\mu = 0 \qquad k_\mu = -p_\mu = \dfrac{q_\mu}{2}$

$\boxed{N_f = 0, \text{ Quenched}}$

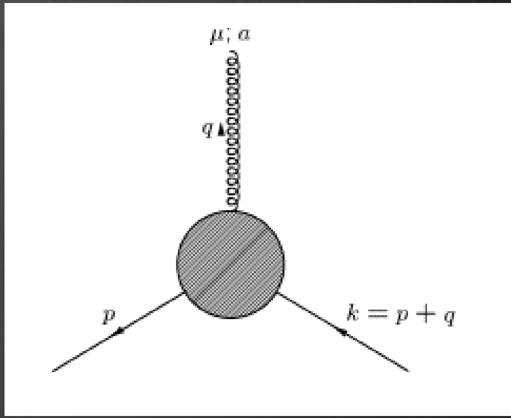
λ'_1 and $t_5\,(q^2/4, q^2, q^2/4)$ at (Quark Reflection Point)



Renormalised form factors Λ_1, τ_5 versus p.
Mean-field improved SW action, $\beta = 0.6$ and
$16^3 \times 48$ lattice, $\mu = 2GeV$, 495 configuration, $m = 115MeV$.

Quark-Gluon Vertex in Special Kinematics

- Orthogonal Kinematics : $q \cdot P = 0 \qquad k^2 = p^2$



$$Tr_4[\mathbf{I}\,\Gamma^T_\mu] \qquad = -i\Lambda_3 P_\mu$$

$$Tr_4[\gamma_\mu\,\Gamma^T_\mu] \ = \Lambda_1\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) - P_\mu P_\nu \Lambda_2$$

$$Tr_4[\sigma_{\alpha\beta}\Gamma^T_\mu]$$
$$= i\,\Lambda_4\left(\delta_{\alpha\mu}P_\beta - \delta_{\beta\mu}P_\alpha\right) + i\left[\Lambda_4\,\tfrac{q_\mu}{q^2} + \tfrac{1}{2}\,\tau_7 P_\mu\right]\left(P_\alpha q_\beta - P_\beta q_\alpha\right)$$
$$+\, i\,\tau_5\ \left(\delta_{\alpha\mu}q_\beta - \delta_{\beta\mu}q_\alpha\right)$$

Orthogonal Kinematics : $\quad q \cdot P = 0 \qquad k^2 = p^2$

$$\Lambda_1 = \frac{1}{2}\left\{ \left(\frac{\mathrm{Tr}_4\left(\gamma_\mu \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} - \frac{Q_\mu(P) Q_\alpha(P)}{Q^2(P)} \left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} \right\}$$

$$\Lambda_2 = -\frac{1}{4Q^4(P)}\left\{ Q^2(P) \left(\frac{\mathrm{Tr}_4\left(\gamma_\mu \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} - 3Q_\mu(P) Q_\alpha(P) \left(\frac{\mathrm{Tr}_4\left(\gamma_\alpha \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} \right\}$$

$$\Lambda_3 = \frac{1}{Q^2(P)}\; Q_\mu(P) \left(\frac{\mathrm{Tr}_4\left(\Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0}$$

$$\Lambda_4 = i\frac{1}{2Q^2(P)}\left\{ Q_\mu(P)\delta\beta\mu \left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta} \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0}\right.$$

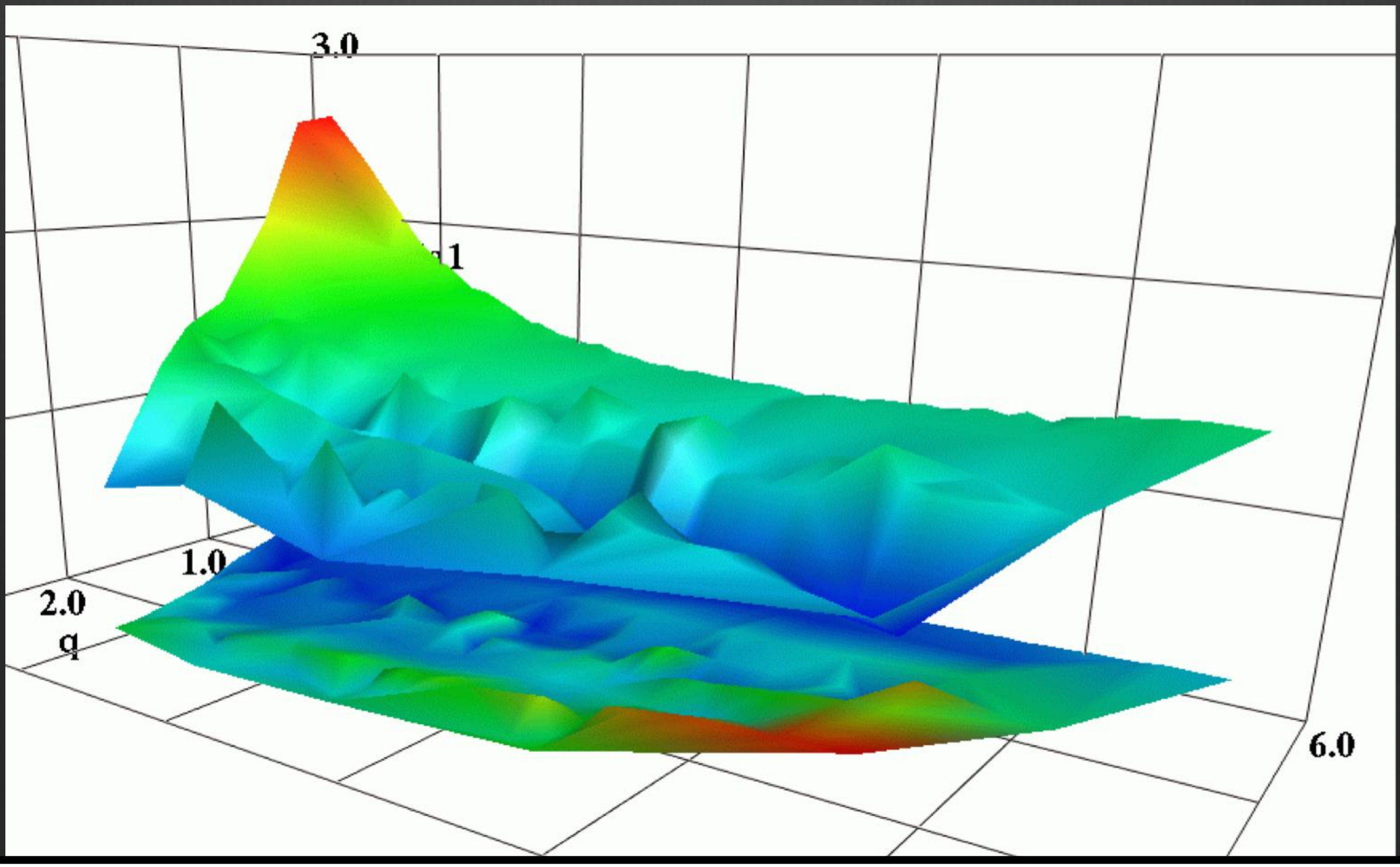
$$\tau_5 = -i\frac{1}{4Q^2(P)Q^2(q)}\left\{ Q^2(P)Q_\alpha(q)\delta\beta\mu \left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta} \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} - Q_\alpha(q)Q_\mu(P)Q_\beta(P) \left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta} \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=}\right.$$

$$\tau_7 = i\frac{1}{2Q^4(P)Q^2(q)}\left\{ Q^2(P)\delta\beta\mu \left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta} \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} - Q_\alpha(q)Q_\beta(P)Q_\mu(P) \left(\frac{\mathrm{Tr}_4\left(\sigma_{\alpha\beta} \Gamma^T_\mu\right)}{R}\right)\Bigg|_{q\cdot P=0} \right\}$$

- Orthogonal Kinematics : $q \cdot P = 0 \qquad k^2 = p^2$

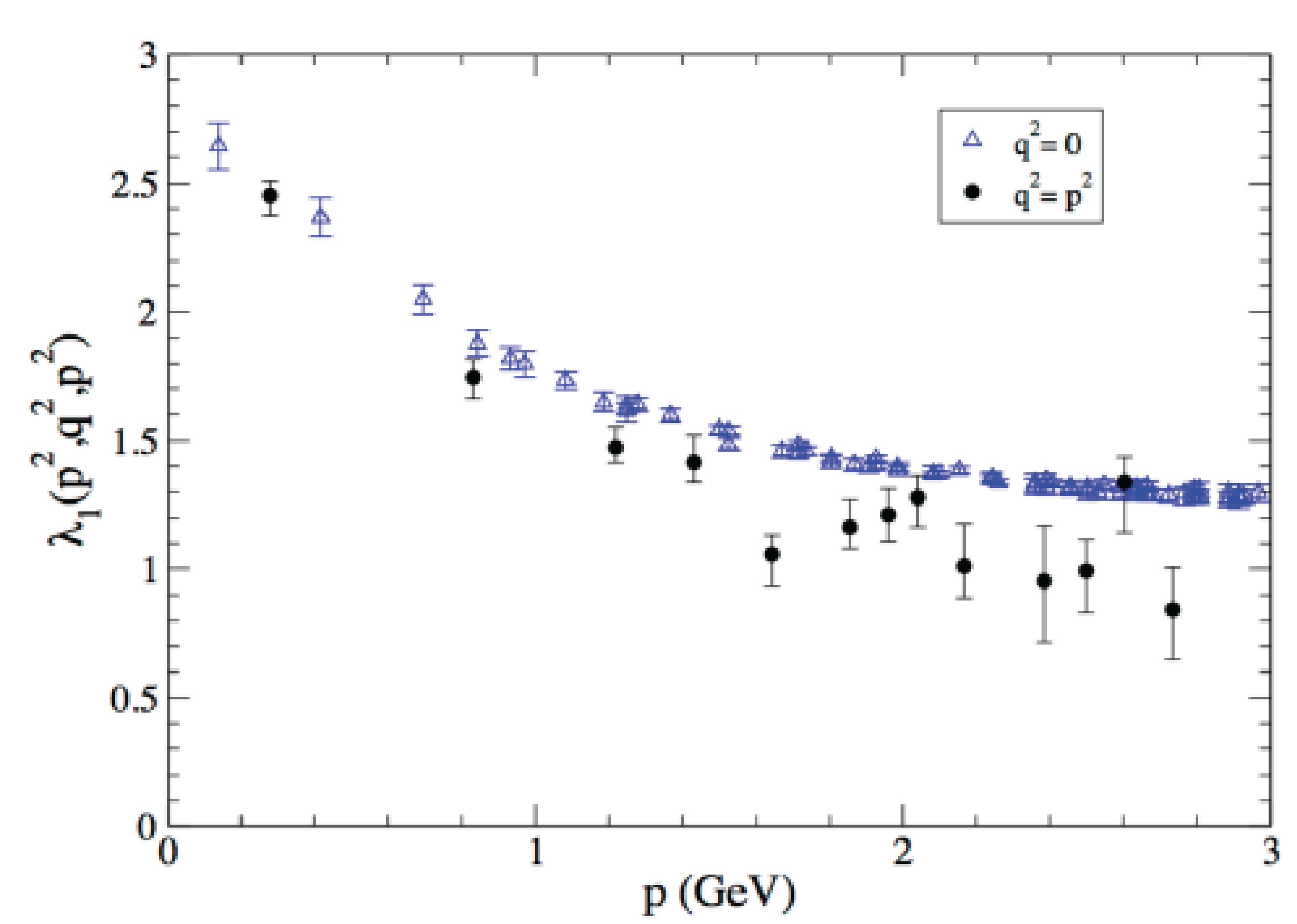
$\boxed{N_f = 0, \text{ Quenched}}$

$$\Lambda_1 = \frac{1}{\left(1 - \frac{q_\mu^2}{q^2}\right)} \left(\frac{\mathrm{Tr}_4 \left(\gamma_\alpha \Gamma_\mu^T \right)}{R} \right) \Bigg|_{\substack{\alpha=\mu \\ q\cdot P=0, P_\mu=0}}$$



Form factor $\lambda_1'(p^2, k^2, q^2)$ versus p and q for $(k^2 = p^2)$. Quenched, $\beta = 6.0$ clover (SW) fermion action with mean field improvement coefficient, $m_q = 115 MeV$.

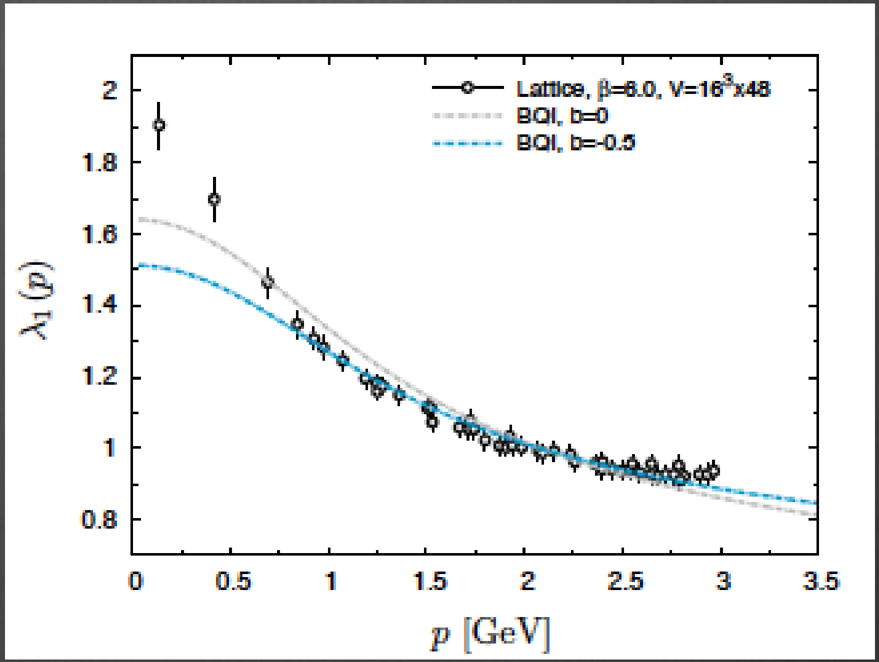
General Kinematics

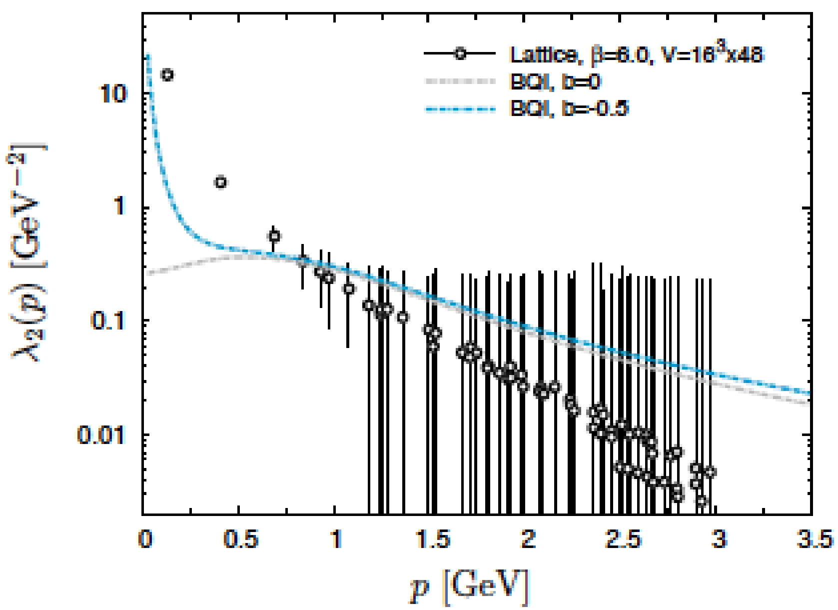


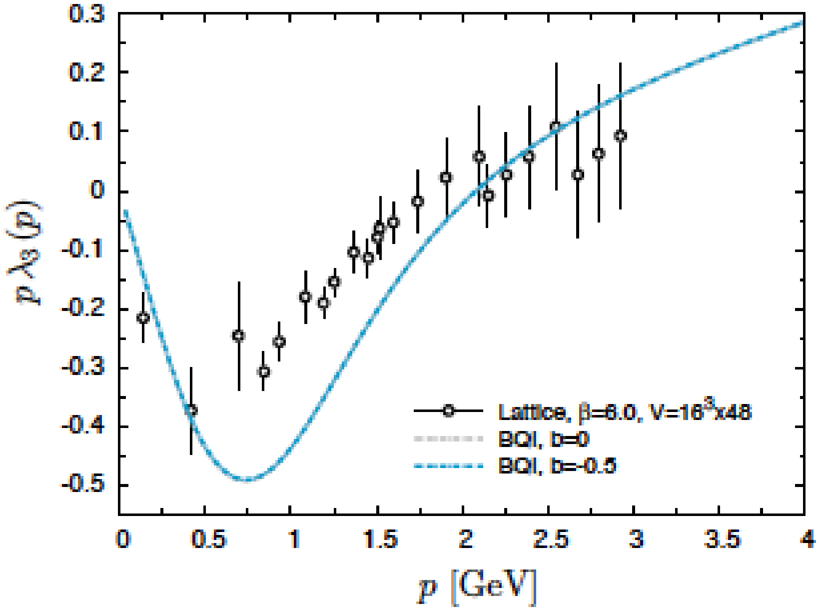
A. Kizilersu, D.B. Leinweber, J.I. Skullerud, A.G. Williams, Eur. Phys. J. C50, 871 (2007)

A.C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys.Rev.D90,(2014),065027

Backround Field Method

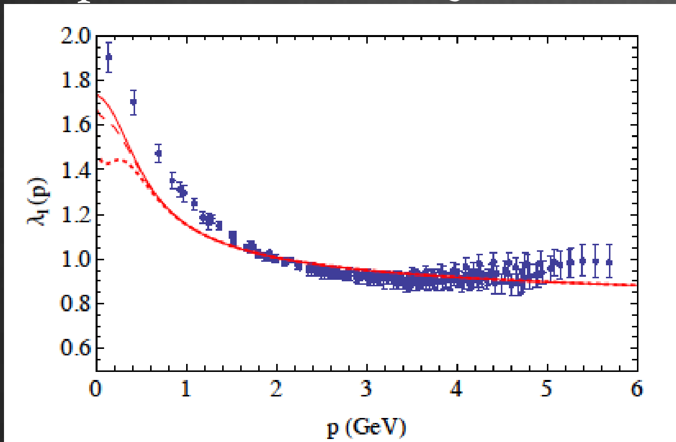


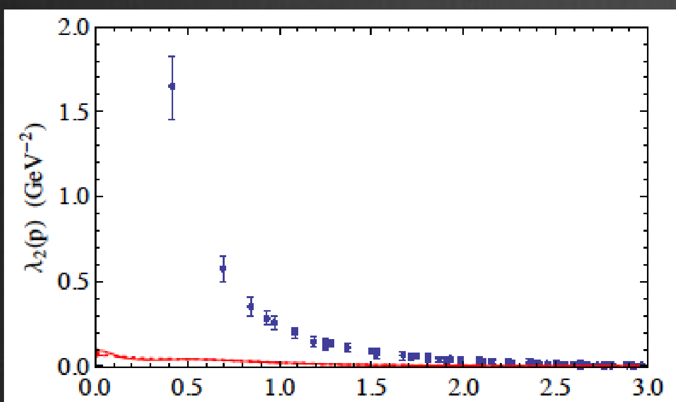


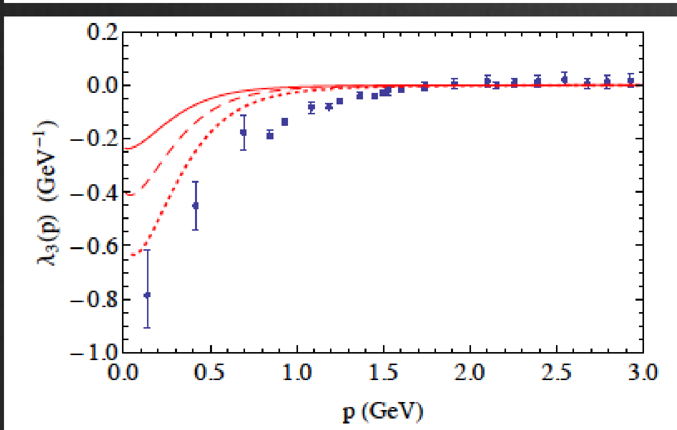


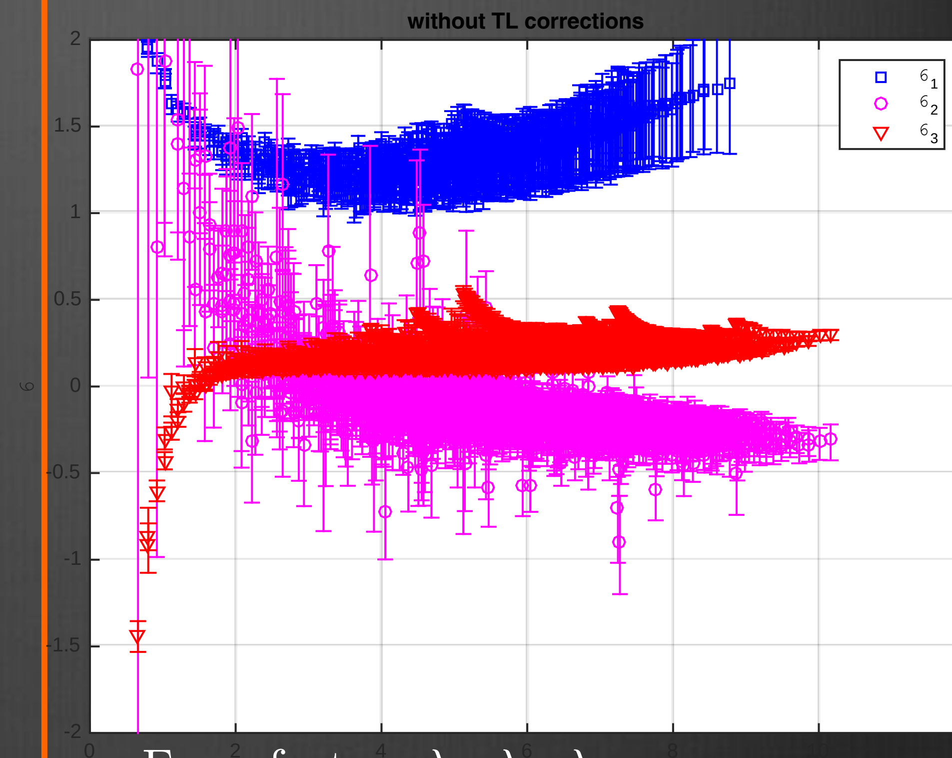
Form Factors in Soft Gluon Kinematics :

Comparison with our Quenched Calc.









Form factors $\lambda_1, \lambda_2, \lambda_3$ versus pa.
$\beta = 5.9$, $a = 0.07 fm$ and
$32^3 \times 64$, m_π $290 MeV$ config.,
532 configuration,

Curci-Ferrari Model (One loop)

M. Pelaez, M.Tissier and N. Wschebor, Hep-th-1504.05157v2(2015)
J. Skullerud,P. Bowman, A.Kizilersu,
D.Leinweber,A.Williams, JHEP04(2003)047

Comparison with model vertices

Maris-Tandy

$$\lambda_1(s) = \frac{4\pi^2}{\omega^6} D\, s\, e^{-s/\omega^2} + \frac{8\,\pi^2\,\gamma_m\,\mathcal{F}(s)}{\ln\left[\tau + (1 + s/\Lambda_{QCD}^2)^2\right]}$$

$$\mathcal{F}(s) = \frac{1 - e^{-s/4m_t^2}}{s} \qquad m_t = 0.5\ \mathrm{GeV}$$

$$\gamma_m = \frac{12}{33 - 2\,N_f} \qquad \Lambda_{QCD} = 0.234\ \mathrm{GeV}$$

$$\tau = e^2 - 1$$

P. Maris, P. C. Tandy, Phys. Rev. C60, 055214 (1999)

ground states

$$\omega = 0.4\ \mathrm{GeV}$$
$$\omega D = (0.8\ \mathrm{GeV})^3$$

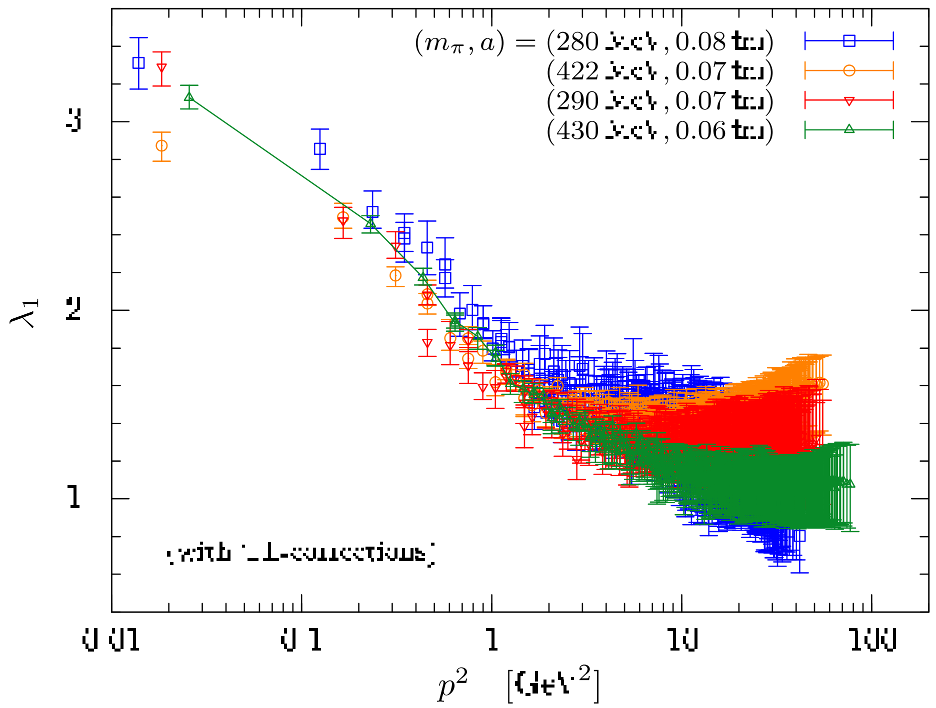
Model 1

$$\omega = 0.6\ \mathrm{GeV}$$
$$\omega D = (1.1\ \mathrm{GeV})^3$$

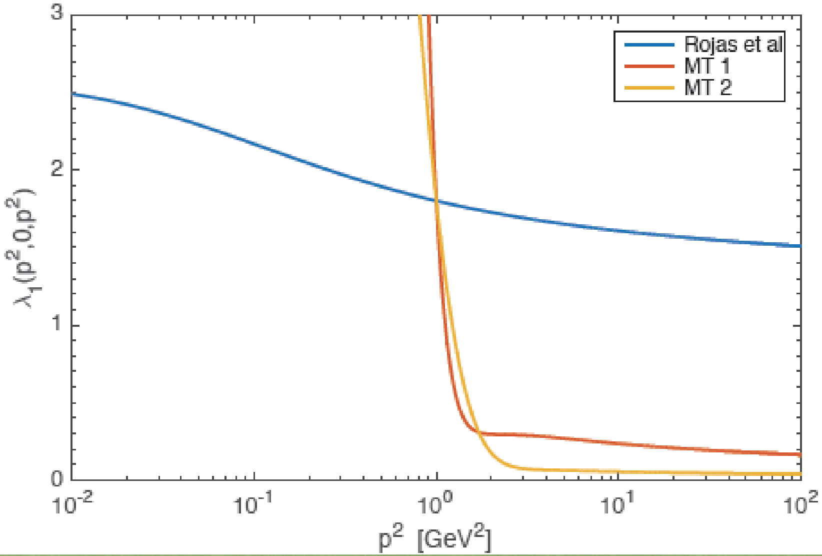
Model 2

excited states

E. Rojas, B. El-Bennich, J.P.B.C.Melo, Phys. Rev. D90, 074025 (2014)



$(m_\pi, a) = (280$ MeV, 0.08 fm)
(422 MeV, 0.07 fm)
(290 MeV, 0.07 fm)
(430 MeV, 0.06 fm)
λ_1
p^2 [GeV^2]



Rojas et al
MT 1
MT 2
$\lambda_1(p^2,0,p^2)$
p^2 [GeV^2]

Conclusion: **Quark-Gluon Vertex in Special Kinematics**

- Soft Gluon Kinematics : $q_\mu = 0 \qquad k_\mu = p_\mu = \frac{P_\mu}{2}$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0$$

- Hard Reflection Kinematics : $P_\mu = 0 \qquad k_\mu = -p_\mu = \frac{q_\mu}{2}$

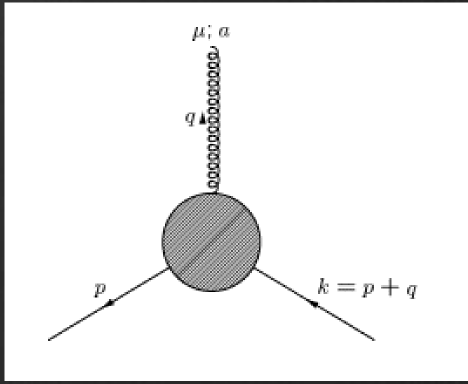
$$\boldsymbol{\Lambda_1, \tau_5}$$

- Orthogonal Kinematics : $q \cdot P = 0 \qquad k^2 = p^2$

$$\boldsymbol{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_7}$$

- General Kinematics :

$$\boldsymbol{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_5, \tau_6, \tau_7, \tau_8}$$



Summary and Outlook

• high statistics quark propagator and quark-gluon vertex computation closer to the physical point
•λ_1 IR enhanced
• qualitative agreement with quark-gluon models
• explore further kinematics + form factors of the vertex
• need to have a better understanding of the lattice artefacts