Hadron Structure using the Feynman-Hellmann Theorem

Alexander Chambers
Ross Young & James Zanotti
QCDSF-UKQCD/CSSM Collaborations

University of Adelaide

7th March 2016

Overview

Want to demonstrate a Feynman-Hellmann approach to tackling two important problems in lattice QCD

Want to demonstrate a Feynman-Hellmann approach to tackling two important problems in lattice QCD

Disconnected contributions to matrix elements

Overview

Want to demonstrate a Feynman-Hellmann approach to tackling two important problems in lattice QCD

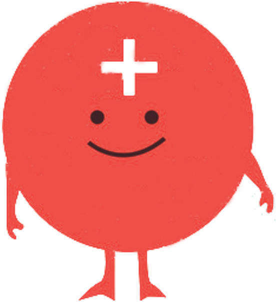
Disconnected contributions to matrix elements

High-momentum form factors

Disconnected Contributions - Proton Spin Structure

Disconnected Contributions - Proton Spin Structure

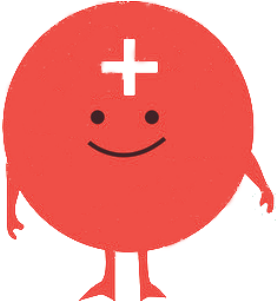
The proton is a spin-$\frac{1}{2}$ baryon



Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

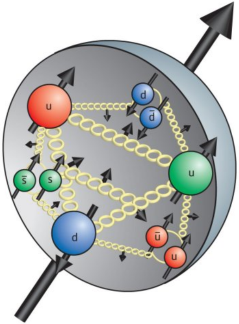
Where does this spin come from?



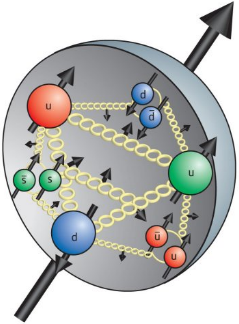
Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



The proton is a complex composite object



The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?

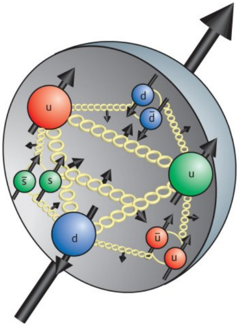
The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)

Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



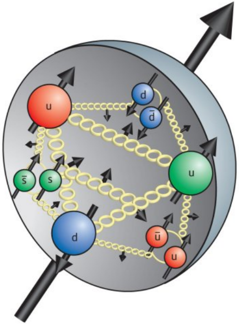
The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons

Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



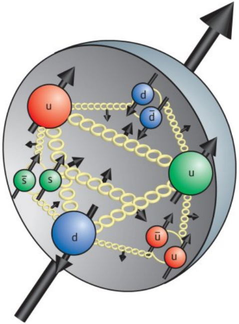
The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

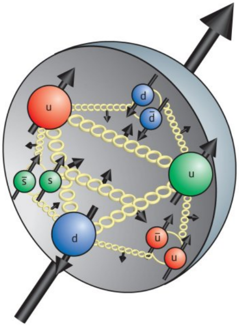
We can break up the total spin into 3 contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



The proton is a complex composite object

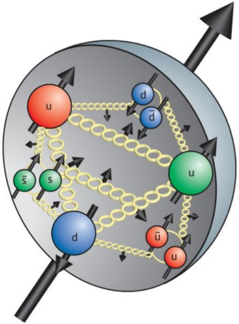
- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

We can break up the total spin into 3 contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- $\Delta\Sigma \longrightarrow$ quark spin

Disconnected Contributions - Proton Spin Structure



The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?

The proton is a complex composite object

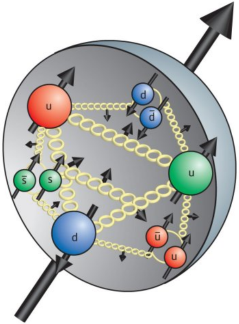
- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

We can break up the total spin into 3 contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- $\Delta\Sigma \longrightarrow$ quark spin
- $L_q \longrightarrow$ quark orbital angular momentum

Disconnected Contributions - Proton Spin Structure



The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?

The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

We can break up the total spin into 3 contributions

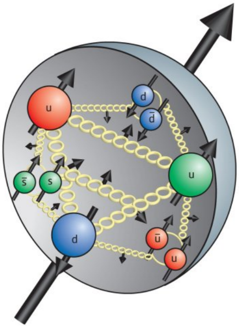
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- $\Delta\Sigma \longrightarrow$ quark spin
- $L_q \longrightarrow$ quark orbital angular momentum
- $J_g \longrightarrow$ gluon angular momentum

Disconnected Contributions - Proton Spin Structure

The proton is a spin-$\frac{1}{2}$ baryon

Where does this spin come from?



The proton is a complex composite object

- Bound state of 3 spin-$\frac{1}{2}$ quarks (u, u, d)
- Quarks interact via spin-1 gluons
- These all have orbital angular momentum

We can break up the total spin into 3 contributions

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

- $\Delta\Sigma \longrightarrow$ quark spin
- $L_q \longrightarrow$ quark orbital angular momentum
- $J_g \longrightarrow$ gluon angular momentum

Can we calculate these spin fractions using lattice QCD?

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Yes and no . . .

Lattice has seen varying success in these calculations

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%,

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

- This is a purely <u>disconnected</u> contribution

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

- This is a purely disconnected contribution
- Difficult to calculate in lattice

Disconnected Contributions - Proton Spin Structure

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

- This is a purely <u>disconnected</u> contribution
- Difficult to calculate in lattice

Gluon Angular Momentum J_g

Disconnected Contributions - Proton Spin Structure

Yes and no ...

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

- This is a purely disconnected contribution
- Difficult to calculate in lattice

Gluon Angular Momentum J_g

- Also purely disconnected

Yes and no . . .

Lattice has seen varying success in these calculations

Quark Spin $\Delta\Sigma$

- Experimentally around 30%, lattice gets around 50-60%

Are there still unresolved systematic errors?

- Chiral extrapolation, volume dependence, etc.

Is the strange contribution Δs large?

- This is a purely disconnected contribution
- Difficult to calculate in lattice

Gluon Angular Momentum J_g

- Also purely disconnected

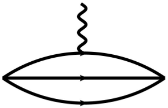
For complete calculations we require access to disconnected contributions!

Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

What are connected/disconnected contributions to matrix elements?

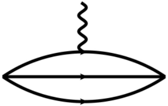
Connected



Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

Connected

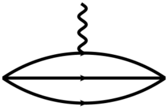


$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted

What are connected/disconnected contributions to matrix elements?

Connected

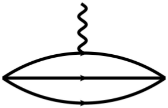


$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate

What are connected/disconnected contributions to matrix elements?

Connected



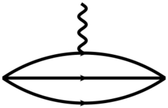
$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

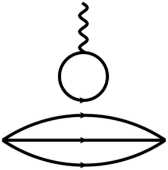
Connected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

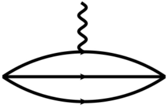
Disconnected



Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

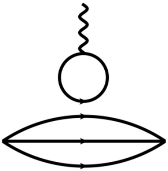
Connected



$$\langle H' \,|\, \mathcal{O} \,|\, H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Disconnected



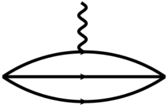
$$\langle H' \,|\, \mathcal{O} \,|\, H \rangle$$

- Fields in operator contracted

Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

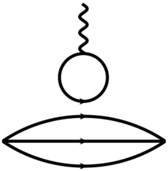
Connected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Disconnected

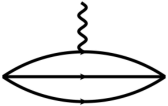


$$\langle H' | \mathcal{O} | H \rangle$$

- Fields in operator contracted
- All-to-all propagators unfeasible

What are connected/disconnected contributions to matrix elements?

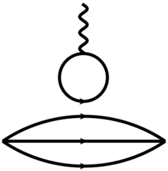
Connected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Disconnected



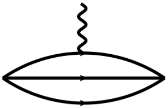
$$\langle H' | \mathcal{O} | H \rangle$$

- Fields in operator contracted
- All-to-all propagators unfeasible
- Use stochastic estimation

Connected vs. Disconnected Contributions

What are connected/disconnected contributions to matrix elements?

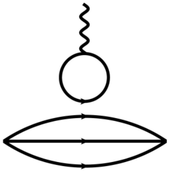
Connected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Disconnected



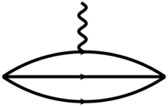
$$\langle H' | \mathcal{O} | H \rangle$$

- Fields in operator contracted
- All-to-all propagators unfeasible
- Use stochastic estimation

Much more difficult!

What are connected/disconnected contributions to matrix elements?

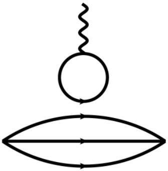
Connected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fermion fields in interpolators and current are contracted
- Straightforward to calculate
- Established methods

Disconnected



$$\langle H' | \mathcal{O} | H \rangle$$

- Fields in operator contracted
- All-to-all propagators unfeasible
- Use stochastic estimation

Much more difficult!

Feynman-Hellmann method can access both contributions

Feynman-Hellmann Recipe (Forward Case)

How does the method work?

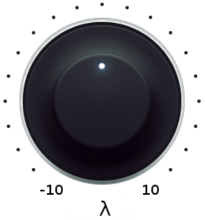
Feynman-Hellmann Recipe (Forward Case)

How does the method work?

Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p}) \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

How does the method work?



Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p}) \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

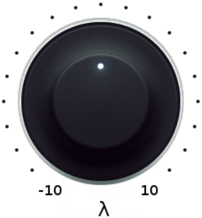
1. Include an extra term in the QCD Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \mathcal{O}$$

How does the method work?

Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p}) \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$



1. Include an extra term in the QCD Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda\mathcal{O}$$

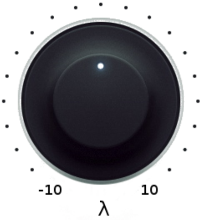
2. Determine energy of your hadron state while changing λ

$$G(\lambda, \vec{p}, \Delta t) = \int \mathrm{d}^3\vec{x}'\, e^{-i\vec{p}\cdot\Delta\vec{x}} \langle\, \chi'_H(x')\chi_H(x) \,\rangle_\lambda \overset{\text{large } \Delta t}{\sim} e^{-E_H(\lambda,\vec{p})\Delta t}$$

How does the method work?

Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p})\,|\,\mathcal{O}(0)\,|\,H(\vec{p})\,\rangle$$



1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L} \rightarrow \mathcal{L} + \lambda\mathcal{O}$$

2. **Determine energy of your hadron state while changing λ**

$$G(\lambda, \vec{p}, \Delta t) = \int \mathrm{d}^3\vec{x}'\, e^{-i\vec{p}\cdot\Delta\vec{x}} \langle\, \chi'_H(x')\chi_H(x)\,\rangle_\lambda \overset{\text{large }\Delta t}{\sim} e^{-E_H(\lambda,\vec{p})\Delta t}$$

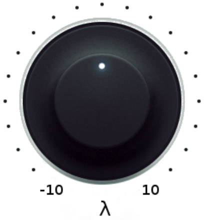
3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda, \vec{p})}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})}\langle\, H(\vec{p})\,|\mathcal{O}(0)|\,H(\vec{p})\,\rangle$$

How does the method work?

Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p}) \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$



1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L} \to \mathcal{L} + \lambda \mathcal{O}$$

2. **Determine energy of your hadron state while changing λ**

$$G(\lambda, \vec{p}, \Delta t) = \int \mathrm{d}^3\vec{x}'\, e^{-i\vec{p}\cdot\Delta\vec{x}} \langle\, \chi'_H(x')\chi_H(x) \,\rangle_\lambda \overset{\text{large } \Delta t}{\sim} e^{-E_H(\lambda,\vec{p})\Delta t}$$

3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})} \langle\, H(\vec{p}) \,|\mathcal{O}(0)|\, H(\vec{p}) \,\rangle$$

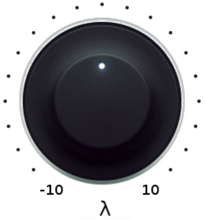
Calculation of matrix element $\to$ hadron spectroscopy

Feynman-Hellmann Recipe (Forward Case)

How does the method work?

Suppose you want to calculate some forward matrix element

$$\langle\, H(\vec{p}) \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$



1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L} \to \mathcal{L} + \lambda \mathcal{O}$$

2. **Determine energy of your hadron state while changing λ**

$$G(\lambda, \vec{p}, \Delta t) = \int \mathrm{d}^3\vec{x}'\, e^{-i\vec{p}\cdot\Delta\vec{x}} \langle\, \chi'_H(x')\chi_H(x) \,\rangle_\lambda \overset{\text{large } \Delta t}{\sim} e^{-E_H(\lambda,\vec{p})\Delta t}$$

3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})} \langle\, H(\vec{p}) \,|\mathcal{O}(0)|\, H(\vec{p}) \,\rangle$$

Calculation of matrix element $\to$ hadron spectroscopy

You only need to calculate two-point functions!

Feynman-Hellmann Recipe (Forward Case)

Where should the Lagrangian be modified?

Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

Feynman-Hellmann Recipe (Forward Case)

Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Feynman-Hellmann Recipe (Forward Case)

Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Quark propagators in $\overline{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$

Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

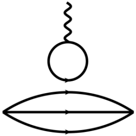
Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Quark propagators in $\overline{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$

Modify to access <u>disconnected</u> contributions



Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

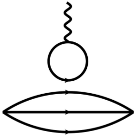
Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Quark propagators in $\overline{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$

Modify to access <u>disconnected</u> contributions



Need new gauge ensembles

Feynman-Hellmann Recipe (Forward Case)

Where should the Lagrangian be modified?

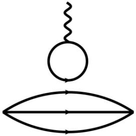
Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Modify to access <u>disconnected</u> contributions

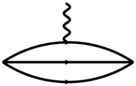


Need new gauge ensembles

Quark propagators in $\overline{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$

Modify to access <u>connected</u> contributions



Where should the Lagrangian be modified?

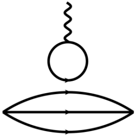
Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overbracket{\mathcal{O}}[A_{(i)}]$$

Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

Modify to access <u>disconnected</u> contributions

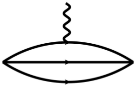


Need new gauge ensembles

Quark propagators in $\overbracket{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$

Modify to access <u>connected</u> contributions



Easy!

Feynman-Hellmann Recipe (Forward Case)

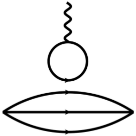
Where should the Lagrangian be modified?

Lattice QCD estimates path-integrals using weighted sums

$$\frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\;\mathcal{O}[A,\bar{\psi},\psi]\,e^{-S[A,\bar{\psi},\psi]} \longrightarrow \frac{1}{N}\sum_{i=1}^{N}\overline{\mathcal{O}}[A_{(i)}]$$

Gauge fields are generated with weighting

$$\det[D(A)]\,e^{-S_g[A]}$$

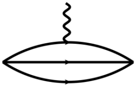


Modify to access <u>disconnected</u> contributions

Need new gauge ensembles

Quark propagators in $\overline{\mathcal{O}}$ are calculated by inverting Dirac matrix

$$S^{ab}_{\alpha\beta}(x,y) = \left[D^{ab}_{\alpha\beta}(x,y)\right]^{-1}$$



Modify to access <u>connected</u> contributions

Easy! - Try this first

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the connected contributions to quark axial charges in the nucleon

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the connected contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0}) \,|\, \bar{q}(0)\gamma_\mu\gamma_5 q(0) \,|\, N_s(\vec{0}) \,\rangle = 2is_\mu \Delta q \qquad q \in (u, d)$$

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the connected contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0})\,|\,\bar{q}(0)\gamma_\mu\gamma_5 q(0)\,|\,N_s(\vec{0})\,\rangle = 2is_\mu \Delta q \qquad q \in (u,d)$$

Include the axial operator in the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q$$

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the <u>connected</u> contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0}) \,|\, \bar{q}(0)\gamma_\mu\gamma_5 q(0) \,|\, N_s(\vec{0}) \,\rangle = 2is_\mu \Delta q \qquad q \in (u,d)$$

Include the axial operator in the Lagrangian

$$\mathcal{L} \rightarrow \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

and choose projection matrices to project spin-up/down states

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the connected contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0}) \,|\, \bar{q}(0)\gamma_\mu\gamma_5 q(0) \,|\, N_s(\vec{0}) \,\rangle = 2is_\mu \Delta q \qquad q \in (u,d)$$

Include the axial operator in the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

and choose projection matrices to project spin-up/down states

Feynman-Hellmann relation gives

$$\left.\frac{\partial E_N(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \pm \Delta q_{\text{conn.}}$$

Quark Axial Charges in the Nucleon (Connected)

We want to calculate the <u>connected</u> contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0}) \,|\, \bar{q}(0)\gamma_\mu\gamma_5 q(0) \,|\, N_s(\vec{0}) \,\rangle = 2is_\mu \Delta q \qquad q \in (u, d)$$

Include the axial operator in the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

and choose projection matrices to project spin-up/down states

Feynman-Hellmann relation gives

$$\left.\frac{\partial E_N(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \pm \Delta q_{\text{conn.}}$$

Energy shifts are proportional to the quark axial charges

We want to calculate the <u>connected</u> contributions to quark axial charges in the nucleon

Need to determine the axial matrix elements

$$\langle\, N_s(\vec{0}) \,|\, \bar{q}(0)\gamma_\mu\gamma_5 q(0) \,|\, N_s(\vec{0}) \,\rangle = 2is_\mu \Delta q \qquad q \in (u, d)$$

Include the axial operator in the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

and choose projection matrices to project spin-up/down states

Feynman-Hellmann relation gives

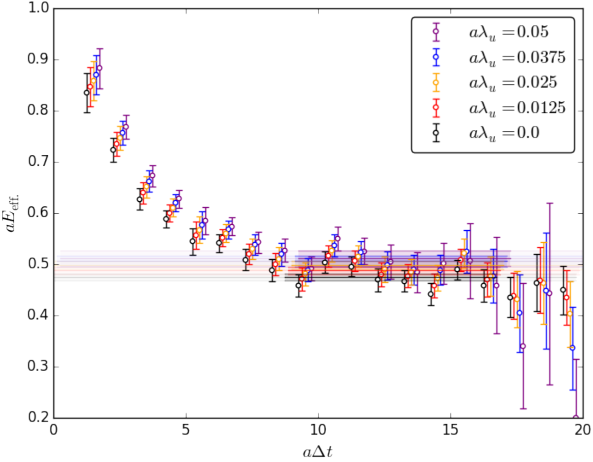
$$\left.\frac{\partial E_N(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \pm \Delta q_{\text{conn.}}$$

Energy shifts are proportional to the quark axial charges

- Note spin-up/down $\to \pm\lambda$

Quark Axial Charges in the Nucleon (Connected)

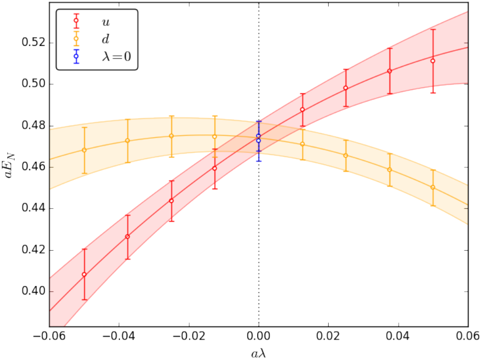
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



$$E_{\text{eff.}}(\lambda, \Delta t + \frac{a}{2}) = \frac{1}{a} \log \left| \frac{G(\lambda, \Delta t)}{G(\lambda, \Delta t + a)} \right| \xrightarrow{\text{large } \Delta t} E(\lambda)$$

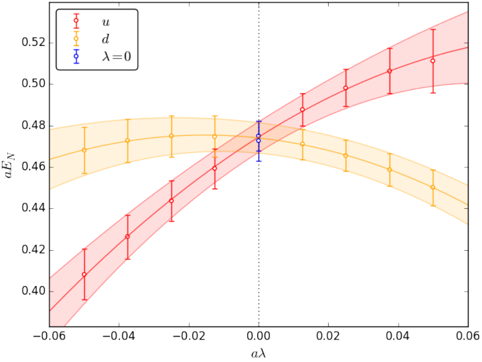
Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



Quark Axial Charges in the Nucleon (Connected)

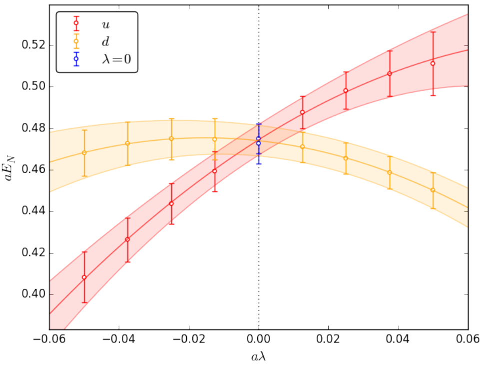
$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



- Make quadratic fit to data

Quark Axial Charges in the Nucleon (Connected)

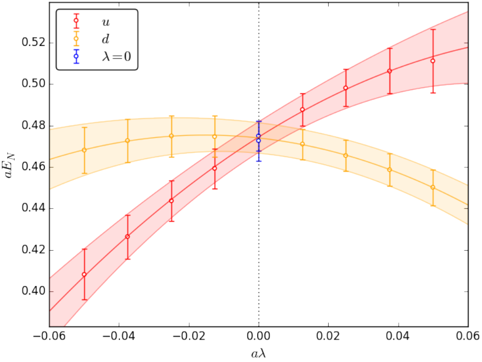
$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$

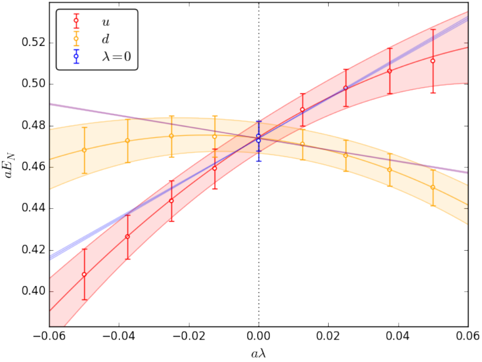


- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

	FH
$\Delta u_{\text{conn.}}$	0.83(13)
$\Delta d_{\text{conn.}}$	-0.22(12)
$\Delta\Sigma_{\text{conn.}}$	1.05(18)
$g_{A\text{conn.}}$	0.61(18)

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$

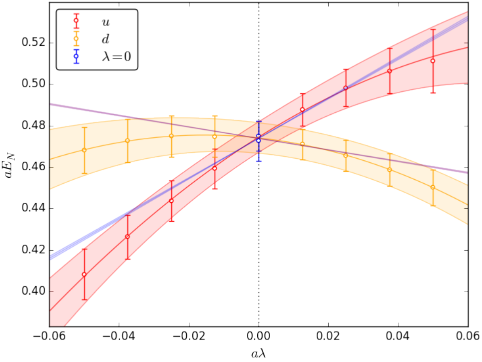


- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.83(13)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.22(12)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	1.05(18)	1.056(15)
$g_{A\text{conn.}}$	0.61(18)	0.586(15)

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



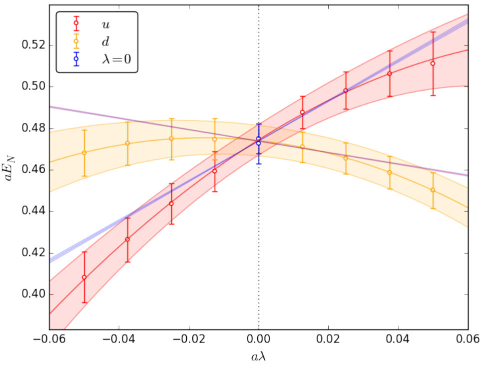
- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.83(13)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.22(12)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	1.05(18)	1.056(15)
$g_{A\text{conn.}}$	0.61(18)	0.586(15)

3-pt. results on same ensemble, but $\times 5$ more configurations

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

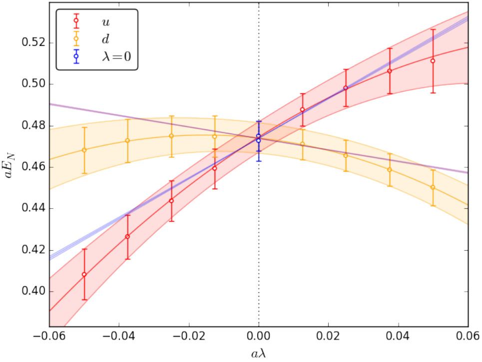
	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.83(13)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.22(12)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	1.05(18)	1.056(15)
$g_{A\text{conn.}}$	0.61(18)	0.586(15)

3-pt. results on same ensemble, but $\times 5$ more configurations

The Feynman-Hellmann approach agrees with existing methods!

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



- Make quadratic fit to data
- Extract linear shift at $\lambda = 0$

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.83(13)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.22(12)	-0.2353(48)
$\Delta \Sigma_{\text{conn.}}$	1.05(18)	1.056(15)
$g_{A\text{conn.}}$	0.61(18)	0.586(15)

3-pt. results on same ensemble, but $\times 5$ more configurations

The Feynman-Hellmann approach agrees with existing methods!

Though the signal could be better...

Quark Axial Charges in the Nucleon (Connected)

Can we improve the signal?

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

- We can extract $E(\lambda)$ from this, however ...

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

- We can extract $E(\lambda)$ from this, however ...

We are only interested in the energy <u>shifts</u>

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

- We can extract $E(\lambda)$ from this, however ...

We are only interested in the energy <u>shifts</u>

We can take ratios of spin-up/down correlators at zero and non-zero λ

$$R(\lambda, \Delta t) = \frac{G_\uparrow(\lambda, \Delta t)}{G_\uparrow(0, \Delta t)} \frac{G_\downarrow(0, \Delta t)}{G_\downarrow(\lambda, \Delta t)} \overset{\text{large } \Delta t}{\sim} e^{-2\Delta E(\lambda)\Delta t}$$

Quark Axial Charges in the Nucleon (Connected)

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

- We can extract $E(\lambda)$ from this, however ...

We are only interested in the energy <u>shifts</u>

We can take ratios of spin-up/down correlators at zero and non-zero λ

$$R(\lambda, \Delta t) = \frac{G_{\uparrow}(\lambda, \Delta t)}{G_{\uparrow}(0, \Delta t)} \frac{G_{\downarrow}(0, \Delta t)}{G_{\downarrow}(\lambda, \Delta t)} \overset{\text{large } \Delta t}{\sim} e^{-2\Delta E(\lambda)\Delta t}$$

Extract energy shifts directly

Can we improve the signal?

The long term behaviour of the correlation functions is given by

$$G(\lambda, \Delta t) \overset{\text{large } \Delta t}{\sim} e^{-E(\lambda)\Delta t}$$

- We can extract $E(\lambda)$ from this, however ...

We are only interested in the energy <u>shifts</u>

We can take ratios of spin-up/down correlators at zero and non-zero λ

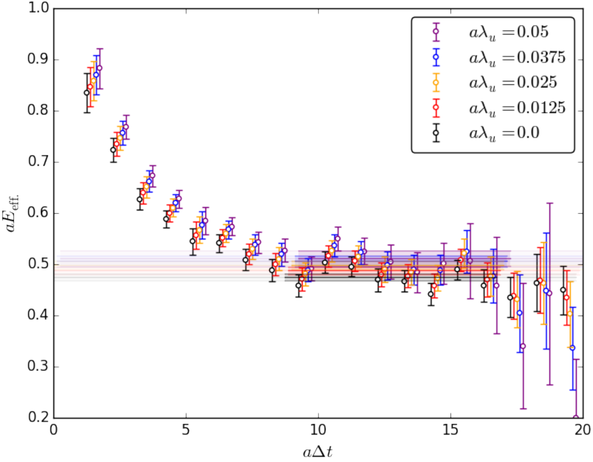
$$R(\lambda, \Delta t) = \frac{G_{\uparrow}(\lambda, \Delta t)}{G_{\uparrow}(0, \Delta t)} \frac{G_{\downarrow}(0, \Delta t)}{G_{\downarrow}(\lambda, \Delta t)} \overset{\text{large } \Delta t}{\sim} e^{-2\Delta E(\lambda)\Delta t}$$

Extract energy shifts directly

Improve signal by allowing statistical fluctuations to cancel

Quark Axial Charges in the Nucleon (Connected)

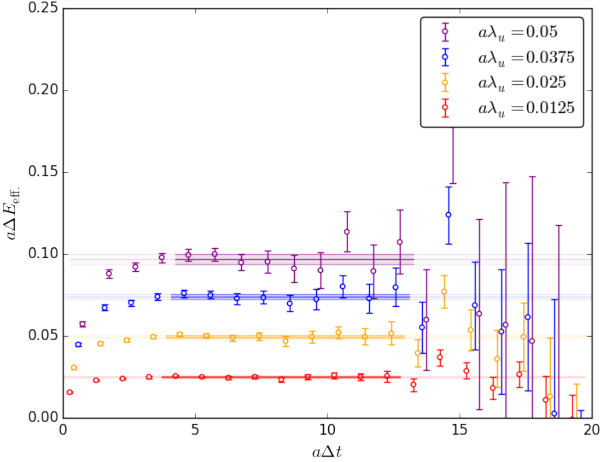
$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



$$E_{\text{eff.}}(\lambda, \Delta t + \frac{a}{2}) = \frac{1}{a}\log\left|\frac{G(\lambda, \Delta t)}{G(\lambda, \Delta t + a)}\right| \xrightarrow{\text{large } \Delta t} E(\lambda)$$

Quark Axial Charges in the Nucleon (Connected)

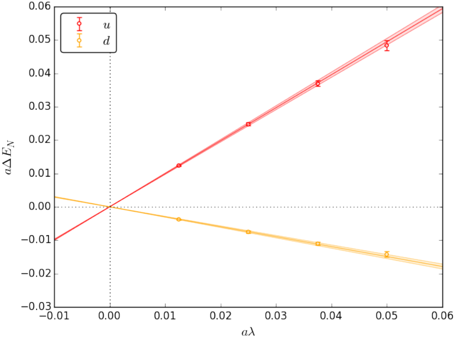
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



$$\Delta E_{\text{eff.}}(\lambda, \Delta t + \frac{a}{2}) = \frac{1}{a} \log \left| \frac{R(\lambda, \Delta t)}{R(\lambda, \Delta t + a)} \right| \xrightarrow{\text{large } \Delta t} \Delta E(\lambda)$$

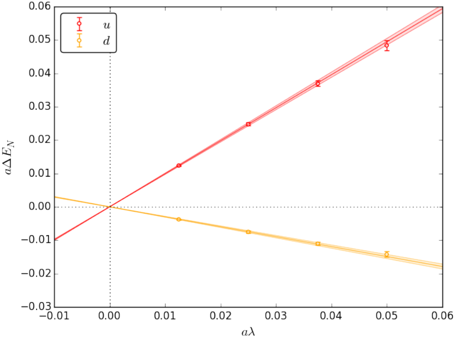
Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



Quark Axial Charges in the Nucleon (Connected)

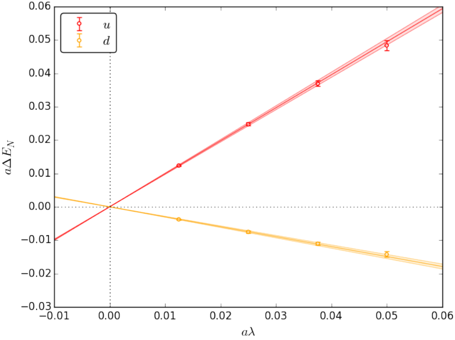
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



- Quadratic terms in λ are completely removed

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$

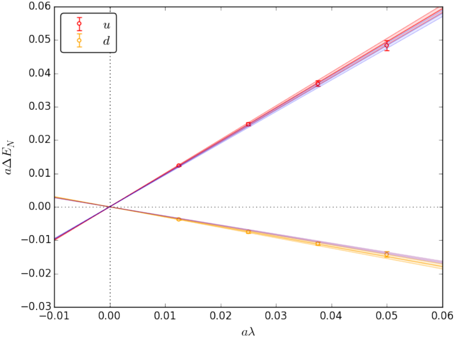


- Quadratic terms in λ are completely removed

	FH
$\Delta u_{\text{conn.}}$	0.832(18)
$\Delta d_{\text{conn.}}$	-0.260(13)
$\Delta\Sigma_{\text{conn.}}$	0.572(22)
$g_{A\text{conn.}}$	1.092(22)

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$

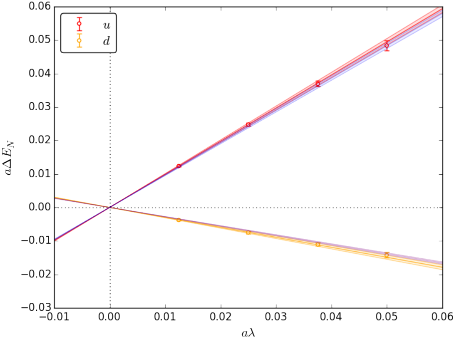


- Quadratic terms in λ are completely removed

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.832(18)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.260(13)	-0.2353(48)
$\Delta \Sigma_{\text{conn.}}$	0.572(22)	0.586(15)
$g_{A\text{conn.}}$	1.092(22)	1.056(15)

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



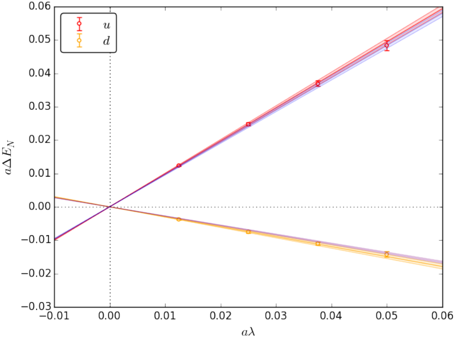
- Quadratic terms in λ are completely removed

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.832(18)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.260(13)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	0.572(22)	0.586(15)
$g_{A\text{conn.}}$	1.092(22)	1.056(15)

Possible discrepancy in Δd

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



- Quadratic terms in λ are completely removed

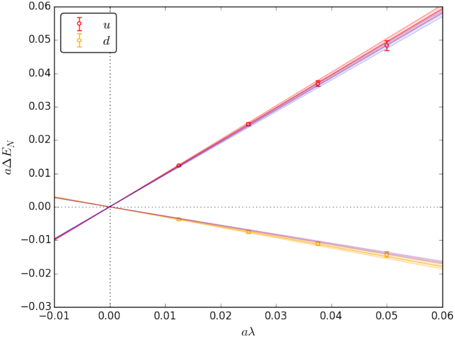
	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.832(18)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.260(13)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	0.572(22)	0.586(15)
$g_{A\text{conn.}}$	1.092(22)	1.056(15)

Possible discrepancy in Δd

- Excited state contamination in 3-pt. results?

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



- Quadratic terms in λ are completely removed

	FH	3-pt.
$\Delta u_{\text{conn.}}$	0.832(18)	0.821(14)
$\Delta d_{\text{conn.}}$	-0.260(13)	-0.2353(48)
$\Delta\Sigma_{\text{conn.}}$	0.572(22)	0.586(15)
$g_{A\text{conn.}}$	1.092(22)	1.056(15)

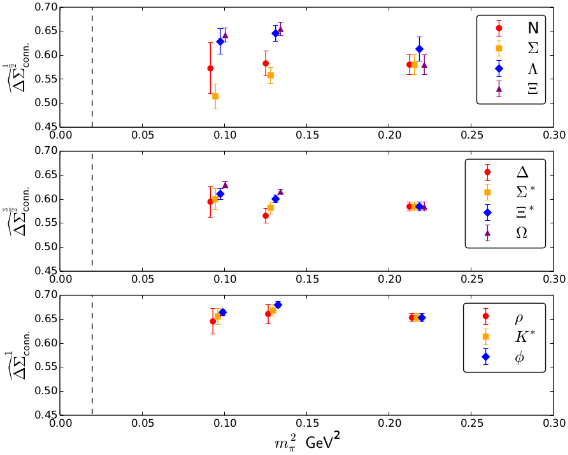
Possible discrepancy in Δd

- Excited state contamination in 3-pt. results?

We have very competitive precision, consistent results!

Quark Axial Charges in Hadrons (Connected)

All other hadron states come for free!



A. J. Chambers et al., Phys. Rev. D 90, 014510 (2014), 1405.3019

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda\bar{q}(-i\gamma_3\gamma_5)q$$

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

- The fermion matrix determinant is not real

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem. . .

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

- The fermion matrix determinant is not real
- Can't use as probabilistic weighting for HMC

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

- The fermion matrix determinant is not real
- Can't use as probabilistic weighting for HMC

Instead we modify the Lagrangian such that

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}\gamma_3\gamma_5 q$$

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

- The fermion matrix determinant is not real
- Can't use as probabilistic weighting for HMC

Instead we modify the Lagrangian such that

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}\gamma_3\gamma_5 q$$

Now the energy shifts are imaginary at first order in λ

$$E \to E(\lambda) + i\phi(\lambda) \qquad \left.\frac{\partial\phi(\lambda)}{\partial\lambda}\right|_{\lambda=0} = \pm\Delta q$$

Quark Axial Charges in the Nucleon (Disconnected)

To calculate disconnected contributions, we need new gauge ensembles

But we have a problem...

The operator we included in the Lagrangian does not satisfy γ_5-hermiticity

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}(-i\gamma_3\gamma_5)q$$

- The fermion matrix determinant is not real
- Can't use as probabilistic weighting for HMC

Instead we modify the Lagrangian such that

$$\mathcal{L} \to \mathcal{L} + \lambda \bar{q}\gamma_3\gamma_5 q$$

Now the energy shifts are imaginary at first order in λ

$$E \to E(\lambda) + i\phi(\lambda) \qquad \left.\frac{\partial\phi(\lambda)}{\partial\lambda}\right|_{\lambda=0} = \pm\Delta q$$

We need a strategy to extract this signal from the phase shifts in correlation functions

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda \Delta q]\Delta t}$$

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda \Delta q]\Delta t}$$

- The amplitude may also pick up a phase

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda\Delta q]\Delta t}$$

- The amplitude may also pick up a phase

Take combinations of real/imaginary parts of spin-up/down projections

$$R(\lambda, \Delta t) = \frac{\mathrm{Im}\, G_\uparrow(\lambda, \Delta t) - \mathrm{Im}\, G_\downarrow(\lambda, \Delta t)}{\mathrm{Re}\, G_\uparrow(\lambda, \Delta t) + \mathrm{Re}\, G_\downarrow(\lambda, \Delta t)} \xrightarrow{\text{large } t} \lambda(k - \Delta q \Delta t)$$

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda \Delta q]\Delta t}$$

- The amplitude may also pick up a phase

Take combinations of real/imaginary parts of spin-up/down projections

$$R(\lambda, \Delta t) = \frac{\mathrm{Im}\, G_{\uparrow}(\lambda, \Delta t) - \mathrm{Im}\, G_{\downarrow}(\lambda, \Delta t)}{\mathrm{Re}\, G_{\uparrow}(\lambda, \Delta t) + \mathrm{Re}\, G_{\downarrow}(\lambda, \Delta t)} \xrightarrow{\text{large } t} \lambda(k - \Delta q \Delta t)$$

Introduce effective phase shift

$$\phi_{\text{eff.}}(\lambda, \Delta t) = \frac{1}{a}\left[G(\lambda, \Delta t + a) - G(\lambda, \Delta t)\right] \xrightarrow{\text{large } t} \lambda \Delta q$$

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda\Delta q]\Delta t}$$

- The amplitude may also pick up a phase

Take combinations of real/imaginary parts of spin-up/down projections

$$R(\lambda, \Delta t) = \frac{\mathrm{Im}\, G_{\uparrow}(\lambda, \Delta t) - \mathrm{Im}\, G_{\downarrow}(\lambda, \Delta t)}{\mathrm{Re}\, G_{\uparrow}(\lambda, \Delta t) + \mathrm{Re}\, G_{\downarrow}(\lambda, \Delta t)} \xrightarrow{\text{large } t} \lambda(k - \Delta q \Delta t)$$

Introduce effective phase shift

$$\phi_{\text{eff.}}(\lambda, \Delta t) = \frac{1}{a}\left[G(\lambda, \Delta t + a) - G(\lambda, \Delta t)\right] \xrightarrow{\text{large } t} \lambda\Delta q$$

Ground state saturation will be indicated by phase plateau

Quark Axial Charges in the Nucleon (Disconnected)

To first order in λ, correlation function takes the form

$$G(\lambda, \Delta t) \xrightarrow{\text{large } t} A(\lambda) e^{-[E + i\lambda \Delta q]\Delta t}$$

- The amplitude may also pick up a phase

Take combinations of real/imaginary parts of spin-up/down projections

$$R(\lambda, \Delta t) = \frac{\mathrm{Im}\, G_{\uparrow}(\lambda, \Delta t) - \mathrm{Im}\, G_{\downarrow}(\lambda, \Delta t)}{\mathrm{Re}\, G_{\uparrow}(\lambda, \Delta t) + \mathrm{Re}\, G_{\downarrow}(\lambda, \Delta t)} \xrightarrow{\text{large } t} \lambda(k - \Delta q \Delta t)$$

Introduce effective phase shift

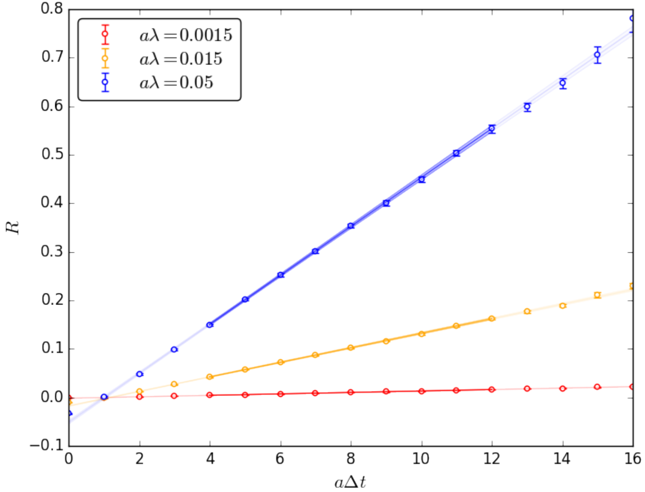
$$\phi_{\text{eff.}}(\lambda, \Delta t) = \frac{1}{a}\left[G(\lambda, \Delta t + a) - G(\lambda, \Delta t)\right] \xrightarrow{\text{large } t} \lambda \Delta q$$

Ground state saturation will be indicated by phase plateau

We can demonstrate this procedure by recalculating <u>connected</u> contributions to quark axial charges in the nucleon

Quark Axial Charges in the Nucleon (Connected - Imag.)

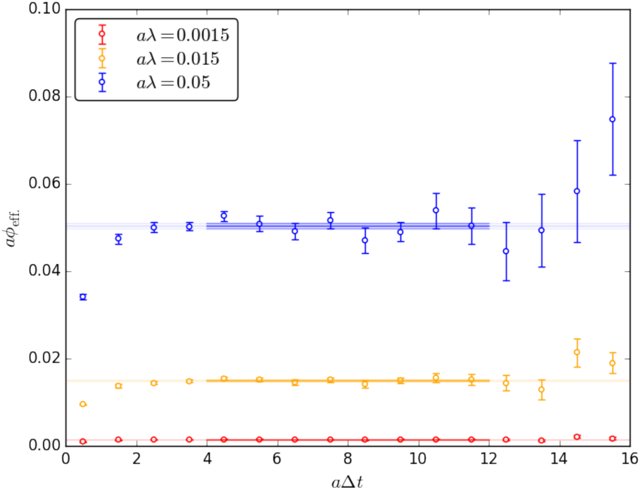
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



$$R(\lambda, \Delta t) \xrightarrow{\text{large } t} \lambda(k - \Delta q \Delta t)$$

Quark Axial Charges in the Nucleon (Connected - Imag.)

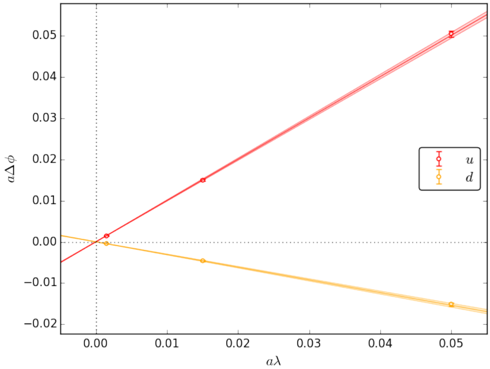
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



$$\phi_{\text{eff.}}(\lambda, \Delta t) \xrightarrow{\text{large } t} \lambda \Delta q$$

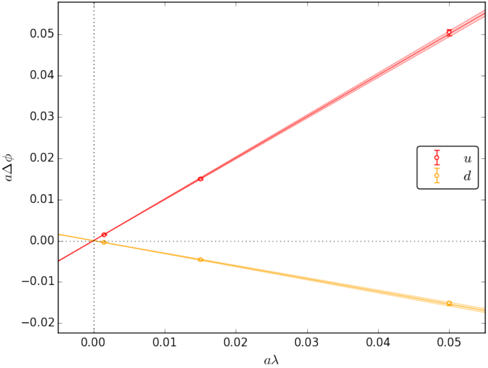
Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



Quark Axial Charges in the Nucleon (Connected)

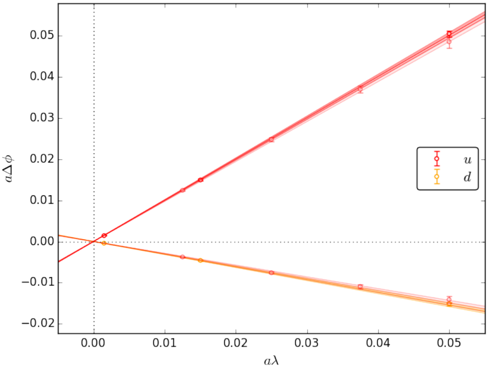
$m_\pi \approx 470$ MeV 350 configurations $32^3 \times 64$



	Imag.
$\Delta u_{\text{conn.}}$	0.851(12)
$\Delta d_{\text{conn.}}$	-0.261(8)
$\Delta\Sigma_{\text{conn.}}$	0.589(13)
$g_{A\text{conn.}}$	1.112(15)

Quark Axial Charges in the Nucleon (Connected)

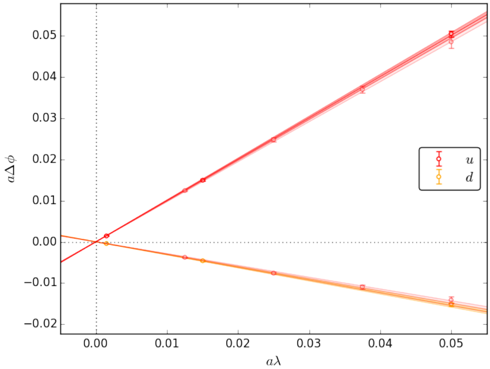
$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



	Imag.	Real
$\Delta u_{\text{conn.}}$	0.851(12)	0.841(17)
$\Delta d_{\text{conn.}}$	-0.261(8)	-0.253(10)
$\Delta\Sigma_{\text{conn.}}$	0.589(13)	0.588(16)
$g_{A\text{conn.}}$	1.112(15)	1.095(23)

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$

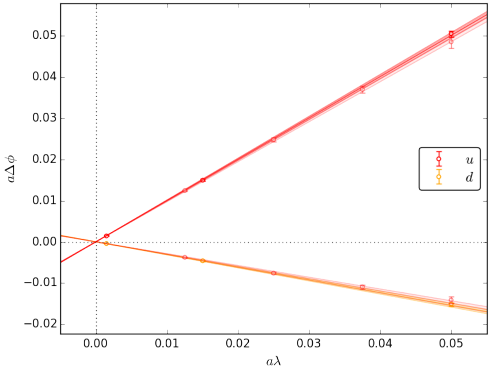


	Imag.	Real
$\Delta u_{\text{conn.}}$	0.851(12)	0.841(17)
$\Delta d_{\text{conn.}}$	-0.261(8)	-0.253(10)
$\Delta\Sigma_{\text{conn.}}$	0.589(13)	0.588(16)
$g_{A\text{conn.}}$	1.112(15)	1.095(23)

- Excellent agreement between both Feynman-Hellmann approaches

Quark Axial Charges in the Nucleon (Connected)

$m_\pi \approx 470$ MeV   350 configurations $32^3 \times 64$



	Imag.	Real
$\Delta u_{\text{conn.}}$	0.851(12)	0.841(17)
$\Delta d_{\text{conn.}}$	-0.261(8)	-0.253(10)
$\Delta\Sigma_{\text{conn.}}$	0.589(13)	0.588(16)
$g_{A\text{conn.}}$	1.112(15)	1.095(23)

- Excellent agreement between both Feynman-Hellmann approaches

We can reliably extract a signal from the correlator phase!

Quark Axial Charges in the Nucleon (Disconnected)

Let's do the disconnected for real now

Let's do the disconnected for real now

Generate new ensembles with a modification to the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \sum_q \bar{q}\gamma_3\gamma_5 q$$

Quark Axial Charges in the Nucleon (Disconnected)

Let's do the disconnected for real now

Generate new ensembles with a modification to the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \sum_q \bar{q}\gamma_3\gamma_5 q$$

- We're testing all flavours simultaneously

Quark Axial Charges in the Nucleon (Disconnected)

Let's do the disconnected for real now

Generate new ensembles with a modification to the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \sum_q \bar{q}\gamma_3\gamma_5 q$$

- We're testing all flavours simultaneously
- The stronger combined signal should be easier to extract

Quark Axial Charges in the Nucleon (Disconnected)

Let's do the disconnected for real now

Generate new ensembles with a modification to the Lagrangian

$$\mathcal{L} \to \mathcal{L} + \lambda \sum_q \bar{q}\gamma_3\gamma_5 q$$

- We're testing all flavours simultaneously
- The stronger combined signal should be easier to extract

Feynman-Hellmann relation gives phase shifts

$$E \to E(\lambda) + i\phi(\lambda) \qquad\qquad \left.\frac{\partial \phi}{\partial \lambda}\right|_{\lambda=0} = \Delta\Sigma_{\text{disc.}}$$

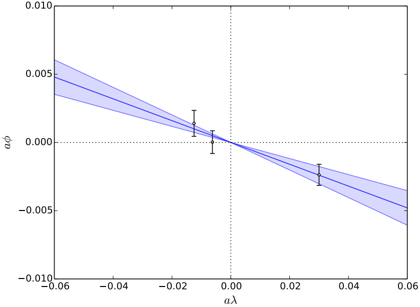
Shift in phase with respect to λ gives total disconnected quark spin contribution

Quark Axial Charges in the Nucleon (Disconnected)

Flavour symmetric point

$m_\pi \approx 470$ MeV $\quad 32^3 \times 64$

500 configurations each

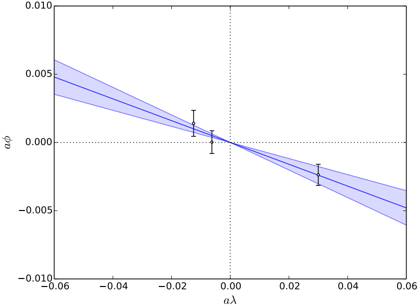


Quark Axial Charges in the Nucleon (Disconnected)

Flavour symmetric point

$m_\pi \approx 470$ MeV $\quad 32^3 \times 64$

500 configurations each



$$\Delta\Sigma_{\text{disc.}} = -0.055(18)$$

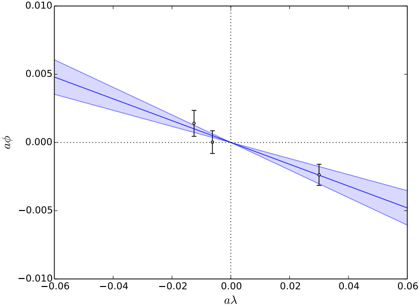
$$\Delta q_{\text{disc.}} = -0.018(6)$$

Quark Axial Charges in the Nucleon (Disconnected)

Flavour symmetric point

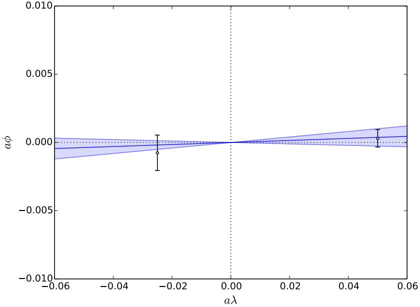
$m_\pi \approx 470$ MeV $32^3 \times 64$

500 configurations each



$m_\pi \approx 310$ MeV $32^3 \times 64$

750 configurations each



$$\Delta\Sigma_{\text{disc.}} = -0.055(18)$$

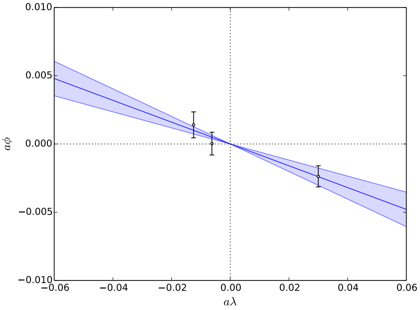
$$\Delta q_{\text{disc.}} = -0.018(6)$$

Quark Axial Charges in the Nucleon (Disconnected)

Flavour symmetric point

$m_\pi \approx 470$ MeV $\quad 32^3 \times 64$

500 configurations each

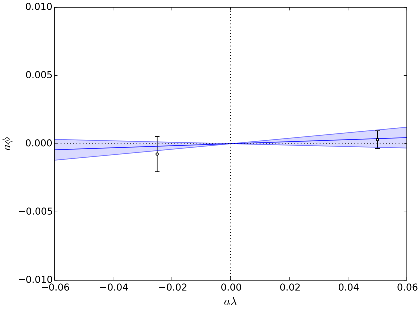


$$\Delta\Sigma_{\text{disc.}} = -0.055(18)$$

$$\Delta q_{\text{disc.}} = -0.018(6)$$

$m_\pi \approx 310$ MeV $\quad 32^3 \times 64$

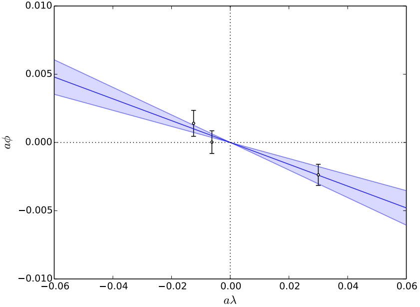
750 configurations each



$$\Delta\Sigma_{\text{disc.}} = 0.026(14)$$

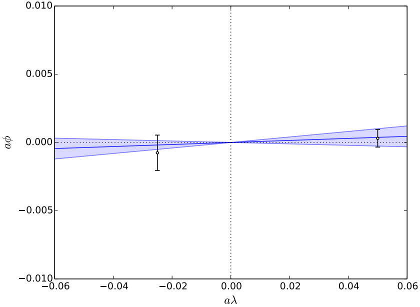
Quark Axial Charges in the Nucleon (Disconnected)

Flavour symmetric point
$m_\pi \approx 470$ MeV $\quad 32^3 \times 64$
500 configurations each



$$\Delta\Sigma_{\text{disc.}} = -0.055(18)$$
$$\Delta q_{\text{disc.}} = -0.018(6)$$

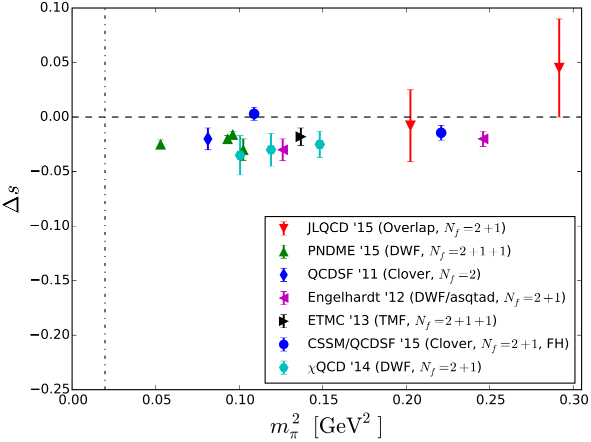
$m_\pi \approx 310$ MeV $\quad 32^3 \times 64$
750 configurations each



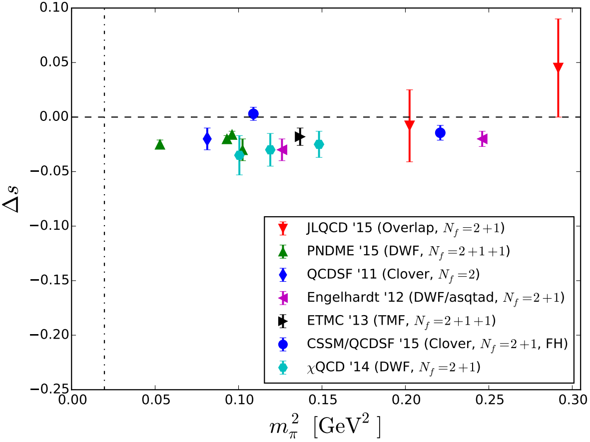
$$\Delta\Sigma_{\text{disc.}} = 0.026(14)$$

Statistics too low, λ too small?

Quark Axial Charges in the Nucleon (Disconnected)

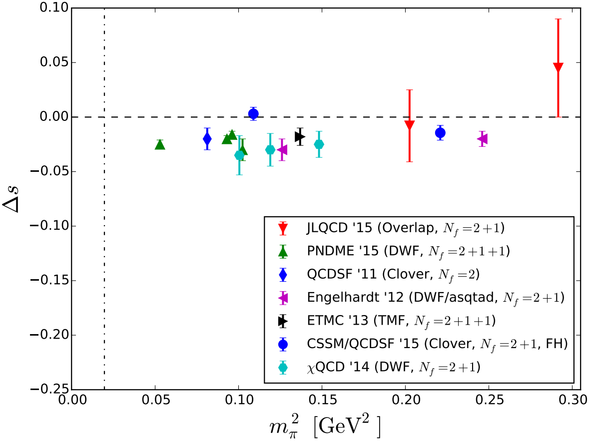


Quark Axial Charges in the Nucleon (Disconnected)



Difficult to estimate, but computational cost same order of magnitude

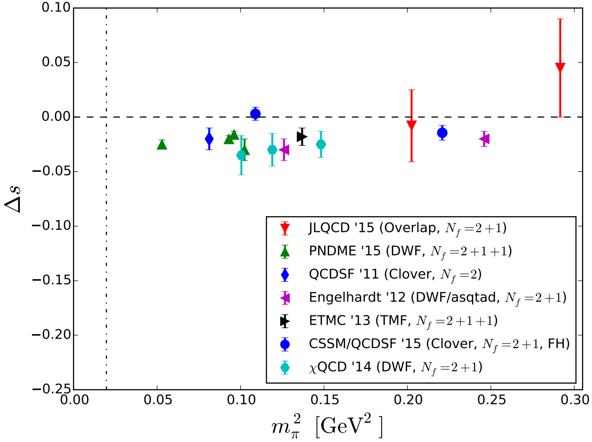
Quark Axial Charges in the Nucleon (Disconnected)



Difficult to estimate, but computational cost same order of magnitude

- Despite generation of new gauge fields

Quark Axial Charges in the Nucleon (Disconnected)



Difficult to estimate, but computational cost same order of magnitude

- Despite generation of new gauge fields

A. J. Chambers et al., Phys. Rev. D 92, 114517 (2015), 1508.06856

Overview

Want to demonstrate a Feynman-Hellmann approach to tackling two important problems in lattice QCD

Disconnected contributions to matrix elements

High-momentum form factors

Pion and Nucleon Form Factors

Hadrons are not point-like particles

Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but. . .

Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but. . .

High-momentum form factor calculations are limited by low signal-to-noise ratios

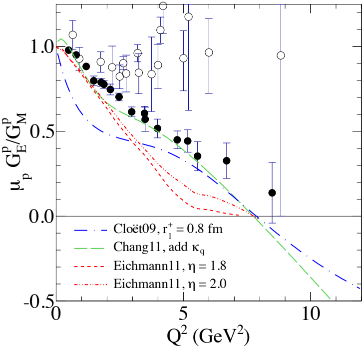
Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but...

High-momentum form factor calculations are limited by low signal-to-noise ratios



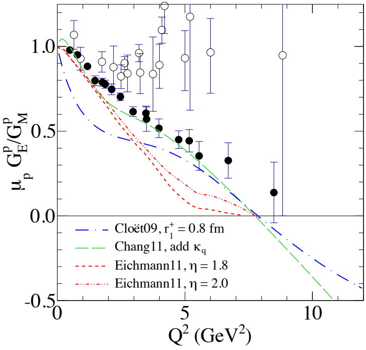
High momentum form factors are interesting for a number of reasons

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but. . .

High-momentum form factor calculations are limited by low signal-to-noise ratios



High momentum form factors are interesting for a number of reasons

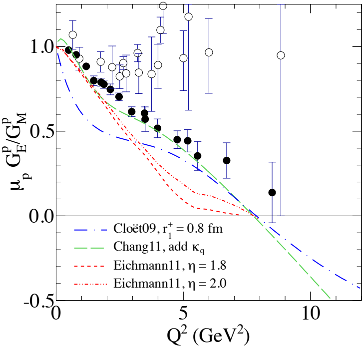
- Nucleon form factor zero crossing

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but...

High-momentum form factor calculations are limited by low signal-to-noise ratios



High momentum form factors are interesting for a number of reasons

- Nucleon form factor zero crossing
- Pion form factor asymptotic behaviour

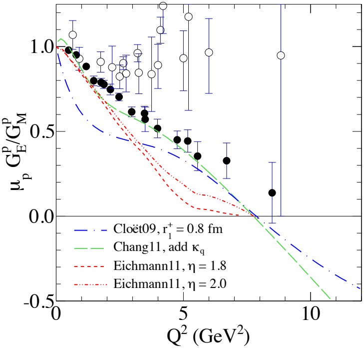
Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but. . .

High-momentum form factor calculations are limited by low signal-to-noise ratios



High momentum form factors are interesting for a number of reasons

- Nucleon form factor zero crossing
- Pion form factor asymptotic behaviour
- Chance for lattice to lead experiment

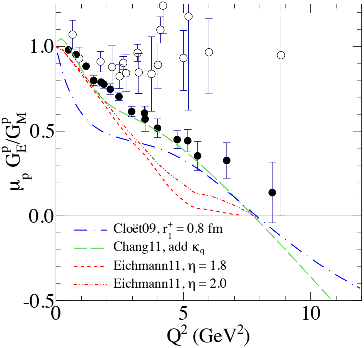
Pion and Nucleon Form Factors

Hadrons are not point-like particles

- They have a spatial distribution encoded by form factors

There is precise experimental and lattice data for low-momentum form-factors, but. . .

High-momentum form factor calculations are limited by low signal-to-noise ratios



High momentum form factors are interesting for a number of reasons

- Nucleon form factor zero crossing
- Pion form factor asymptotic behaviour
- Chance for lattice to lead experiment

We can extend the Feynman-Hellmann approach to non-forward matrix elements

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\mathcal{O}(x)$$

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle \, H(\vec{p}\,') \, | \, \mathcal{O}(0) \, | \, H(\vec{p}) \, \rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\mathcal{O}(x)$$

- Choose the momentum $\vec{q}$, fix source location to x_0

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} + e^{-i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \right) \mathcal{O}(x)$$

- Choose the momentum $\vec{q}$, fix source location to x_0
- $-\vec{q}$ insertion $\to$ well-defined transfer matrix

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} + e^{-i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \right) \mathcal{O}(x)$$

   - Choose the momentum $\vec{q}$, fix source location to x_0
   - $-\vec{q}$ insertion $\to$ well-defined transfer matrix

2. **Determine energy of hadron state while changing λ**

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} + e^{-i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \right) \mathcal{O}(x)$$

   - Choose the momentum $\vec{q}$, fix source location to x_0
   - $-\vec{q}$ insertion $\to$ well-defined transfer matrix

2. **Determine energy of hadron state while changing λ**
   - Restrict to Breit frame kinematics $E(\vec{p}) = E(\vec{p}\,')$

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle H(\vec{p}') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} + e^{-i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \right) \mathcal{O}(x)$$

   - Choose the momentum $\vec{q}$, fix source location to x_0
   - $-\vec{q}$ insertion $\to$ well-defined transfer matrix

2. **Determine energy of hadron state while changing λ**
   - Restrict to Breit frame kinematics $E(\vec{p}) = E(\vec{p}')$
   - Avoid energy transfer through current $\to$ non-trivial t dependence

Feynman-Hellmann Recipe (Non-Forward Case)

Want to calculate some non-forward matrix element

$$\langle\, H(\vec{p}\,') \,|\, \mathcal{O}(0) \,|\, H(\vec{p}) \,\rangle$$

1. **Include an extra term in the QCD Lagrangian**

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \left(e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} + e^{-i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \right) \mathcal{O}(x)$$

   - Choose the momentum $\vec{q}$, fix source location to x_0
   - $-\vec{q}$ insertion $\to$ well-defined transfer matrix

2. **Determine energy of hadron state while changing λ**
   - Restrict to Breit frame kinematics $E(\vec{p}) = E(\vec{p}\,')$
   - Avoid energy transfer through current $\to$ non-trivial t dependence

3. **Calculate matrix element from energy shifts**

$$\left.\frac{\partial E_H(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\vec{p})} \langle\, H(\vec{p}\,') \,|\mathcal{O}(0)|\, H(\vec{p}) \,\rangle$$

Breit Frame Kinematics

Quick digression about Breit Frame kinematics...

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}'^{\,2}$$

Breit Frame Kinematics

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}\,') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}\,'^{\,2}$$

Only certain momentum insertions are allowed

$$\vec{q}^{\,2} = 2n\left(\frac{2\pi}{L}\right)^2 \qquad n \in \mathbb{Z}^+$$

Breit Frame Kinematics

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}\,') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}\,'^{\,2}$$

Only certain momentum insertions are allowed

$$\vec{q}^{\,2} = 2n\left(\frac{2\pi}{L}\right)^2 \qquad n \in \mathbb{Z}^+$$

- We aren't able to access very small Q^2, however ...

Breit Frame Kinematics

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}'^{\,2}$$

Only certain momentum insertions are allowed

$$\vec{q}^{\,2} = 2n\left(\frac{2\pi}{L}\right)^2 \qquad n \in \mathbb{Z}^+$$

- We aren't able to access very small Q^2, however ...

We get excellent signal-to-noise ratios!

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}\,') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}\,'^{\,2}$$

Only certain momentum insertions are allowed

$$\vec{q}^{\,2} = 2n\left(\frac{2\pi}{L}\right)^2 \qquad n \in \mathbb{Z}^+$$

- We aren't able to access very small Q^2, however ...

We get excellent signal-to-noise ratios!

Kinematics corresponding to $\vec{p}\,' = -\vec{p}$ minimise sink momentum

Breit Frame Kinematics

Quick digression about Breit Frame kinematics...

Restricted to Breit frame kinematics for Feynman-Hellmann method

$$E(\vec{p}) = E(\vec{p}\,') \quad \longrightarrow \quad \vec{p}^{\,2} = \vec{p}\,'^{\,2}$$

Only certain momentum insertions are allowed

$$\vec{q}^{\,2} = 2n\left(\frac{2\pi}{L}\right)^2 \qquad n \in \mathbb{Z}^+$$

- We aren't able to access very small Q^2, however ...

We get excellent signal-to-noise ratios!

Kinematics corresponding to $\vec{p}\,' = -\vec{p}$ minimise sink momentum

- We are able to access much higher momentum than previously possible

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle\, \pi(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \,\rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle \pi(\vec{p}') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \bar{q}(x)\,\gamma_\mu\, q(x)$$

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle\, \pi(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \,\rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \bar{q}(x)\, \gamma_\mu\, q(x)$$

Feynman-Hellmann relation gives

$$\left.\frac{\partial E(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \left[\frac{p_\mu + p'_\mu}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle\, \pi(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \,\rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \bar{q}(x)\, \gamma_\mu\, q(x)$$

Feynman-Hellmann relation gives

$$\left.\frac{\partial E(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \left[\frac{p_\mu + p'_\mu}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

For $\mu = i = 1, 2, 3$

$$\left.\frac{\partial E(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} = \left[\frac{p_i + p'_i}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle \pi(\vec{p}') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \bar{q}(x)\,\gamma_\mu\, q(x)$$

Feynman-Hellmann relation gives

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_\mu + p'_\mu}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

For $\mu = i = 1, 2, 3$

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_i + p'_i}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

No signal when $\vec{p}' = -\vec{p}$

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle\, \pi(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \,\rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)} \bar{q}(x)\, \gamma_\mu\, q(x)$$

Feynman-Hellmann relation gives

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_\mu + p'_\mu}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

For $\mu = i = 1,2,3$

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_i + p'_i}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

For $\mu = 4$

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = F_{\pi q}(Q^2)$$

No signal when $\vec{p}\,' = -\vec{p}$

Pion Electromagnetic Form Factor

Demonstrate technique by calculating pion form factor

Flavour contributions to pion form factor defined by matrix elements

$$\langle\, \pi(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, \pi(\vec{p}) \,\rangle = (p_\mu + p'_\mu) F_{\pi q}(Q^2)$$

Make modification to the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\bar{q}(x)\,\gamma_\mu\, q(x)$$

Feynman-Hellmann relation gives

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_\mu + p'_\mu}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

For $\mu = i = 1,2,3$

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = \left[\frac{p_i + p'_i}{2E_\pi(\vec{p})}\right] F_{\pi q}(Q^2)$$

No signal when $\vec{p}\,' = -\vec{p}$

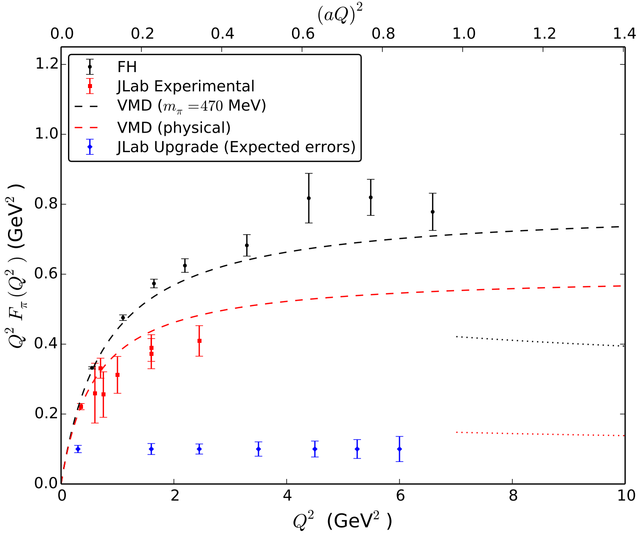
For $\mu = 4$

$$\left.\frac{\partial E(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} = F_{\pi q}(Q^2)$$

Choose this option

Pion Electromagnetic Form Factor

$m_\pi \approx 470$ MeV   500 configurations $32^3 \times 64$



Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu} \frac{q_\nu}{2M} F_{2q}(Q^2) \right] u_s(\vec{p})$$

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}\,'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2)$$

Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle N_s(\vec{p}') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \rangle = \bar{u}_s(\vec{p}') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\, \bar{q}(x)\, \gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda, \vec{p})}{\partial \lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p})\,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}\,'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

For **Spatial Current** project spin-up/down states

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}\,'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

For **Spatial Current** project spin-up/down states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0}^{\Gamma_\pm} \xrightarrow{\vec{p}\,'=-\vec{p}} \pm\frac{\hat{e}\times\vec{q}}{2E_N(\vec{p})} G_M(Q^2)$$

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\,\bar{q}(x)\,\gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}\,'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

For **Spatial Current** project spin-up/down states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0}^{\Gamma_\pm} \xrightarrow{\vec{p}\,'=-\vec{p}} \pm\frac{\hat{e}\times\vec{q}}{2E_N(\vec{p})} G_M(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Nucleon Electromagnetic Form Factors

Carry out similar analysis for nucleon form factors

Flavour contributions to Dirac and Pauli form factors defined by

$$\langle\, N_s(\vec{p}\,') \,|\, \bar{q}(0)\gamma_\mu q(0) \,|\, N_s(\vec{p}) \,\rangle = \bar{u}_s(\vec{p}\,') \left[\gamma_u F_{1q}(Q^2) + \sigma_{\mu\nu}\frac{q_\nu}{2M} F_{2q}(Q^2)\right] u_s(\vec{p})$$

Make identical modification to the action as for the pion case

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda e^{i\vec{q}\cdot(\vec{x}-\vec{x}_0)}\, \bar{q}(x)\, \gamma_\mu\, q(x)$$

For **Temporal Current** project unpolarised states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0} \xrightarrow{\vec{p}\,'=-\vec{p}} \frac{M}{E_N(\vec{p})} G_E(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

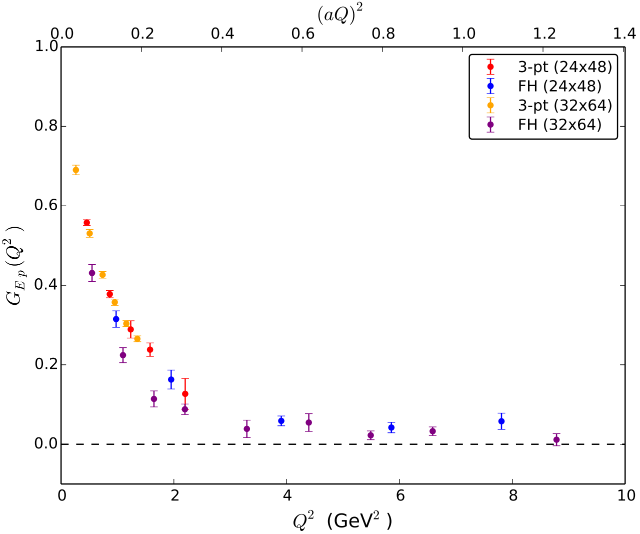
For **Spatial Current** project spin-up/down states

$$\left.\frac{\partial E_N(\lambda,\vec{p})}{\partial\lambda}\right|_{\lambda=0}^{\Gamma_\pm} \xrightarrow{\vec{p}\,'=-\vec{p}} \pm\frac{\hat{e}\times\vec{q}}{2E_N(\vec{p})} G_M(Q^2) \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Other Breit frame kinematics $\to$ combinations of G_E and G_M.

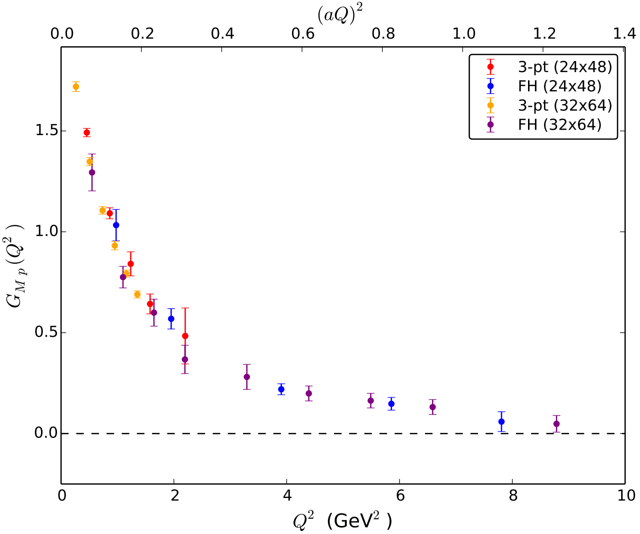
Nucleon Electromagnetic Form Factors

$m_\pi \approx 470$ MeV 500-1000 configurations $32^3 \times 64$



Nucleon Electromagnetic Form Factors

$m_\pi \approx 470$ MeV 500-1000 configurations $32^3 \times 64$



Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Calculations of pion and nucleon form factors at <u>unprecedented</u> momentum scales

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Calculations of pion and nucleon form factors at <u>unprecedented</u> momentum scales

Other work

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Calculations of pion and nucleon form factors at <u>unprecedented</u> momentum scales

Other work

Determinations of singlet/non-singlet renormalisation factors

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Calculations of pion and nucleon form factors at unprecedented momentum scales

Other work

Determinations of singlet/non-singlet renormalisation factors

Many exciting developments ongoing

Summary

Extraordinary success and exciting results from application of Feynman-Hellmann techniques

Precise determinations of disconnected quantities

- Discussed axial charges here
- Calculations of tensor charges have also been performed

Calculations of pion and nucleon form factors at unprecedented momentum scales

Other work

Determinations of singlet/non-singlet renormalisation factors

Many exciting developments ongoing

Quadratic Feynman-Hellmann for transition matrix elements

Nucleon Electromagnetic Form Factors

$m_\pi \approx 470$ MeV   500-1000 configurations $32^3 \times 64$